THE

MECHANIC'S, MACHINIST'S, AND ENGINEER'S

PRACTICAL

BOOK OF REFERENCE:

CONTAINING TABLES AND FORMULÆ FOR USE IN SUPERFICIAL AND SOLID MENSURATION; STRENGTH AND WEIGHT OF MATERIALS; MECHANICS; MACHINERY; HYDRAULICS, HYDRODYNAMICS; MARINE ENGINES, CHEMISTRY; AND MISCELLANEOUS RECIPES.

ADAPTED TO AND FOR THE USE OF

ALL CLASSES OF PRACTICAL MECHANICS.

TOGETHER WITH THE

ENGINEER'S FIELD BOOK:

CONTAINING FORMULÆ FOR THE VARIOUS METHODS OF RUNNING AND CHANGING LINES, LOCATING SIDE TRACKS AND SWITCHES, &C., &C.

TABLES OF RADII AND THEIR LOGARITHMS,

NATURAL AND LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS,

NATURAL SINES AND TANGENTS TO EVERY DEGREE AND MINUTE OF THE QUADRANT,

AND

LOGARITHMS OF NATURAL NUMBERS FROM 1 TO 10,000.

BY CHARLES HASLETT,
Civil Engineer.

EDITED BY CHARLES W. HACKLEY,
Professor of Mathematics in Columbia College, N Y

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PREFACE.

No more useful little works have ever been presented to the public than the various pocket companions of a character analogous to that here offered. These have been a good deal, though not yet too much, multiplied of late; and where the formulas, rules, and tables which they contain have been skilfully framed under the guidance of scientific men, they have afforded to the Practical Engineer, Architect, and Mechanic, the most welcome aid in the constructions and computations which make part of their daily occupation, and which, without the ever-at-hand suggestions and directions of these unpretending little servants, might consume hours and days in the turning over of large volumes, or in painful investigations based on general principles of science where the individual happened to be competent to conduct them.

The wants to be supplied in such a work are discovered by experience and observation in the different callings for which they are more especially intended. That these wants have not all been met in the works of a similar kind which have already appeared will be made evident, by a simple inspection of the amount and variety of new matter contained in the present volume.

It is not every one, however practically expert he may be in his own pursuit, that is capable of arranging and digesting in the best manner the knowledge necessary for his own use which he may have been years in acquiring, so as to render it available for the use

of others. Such a task, to be well performed, requires a combination of mental qualities not always, perhaps not often, found in the same individual.

A happy concurrence of circumstances has by accident secured for the composition of the present work the labors of several skilful hands, both as compilers from the best foreign sources, and as original producers of valuable material never before in print. The result of so much well directed industry is the rich collection, not a line of which is not invaluable, which, in the aptest form for immediate use, has been crowded into the space of a single small volume.

Steam and its application play so important a part in the economy of life at the present day, that the most useful practical rules and formulas for all the ordinary cases occurring, cannot with propriety be omitted in a work of this kind. A due attention will be found to have been paid to the matter, and some of the newest modes of managing in steam supplied with the means of the requisite computation.

The laying out of Railroad curves is one of the most important and at the same time laborious and troublesome duties which the Civil Engineer has to perform. So much of this occurring on every line of Railroad, any, however slight, improvement of method which may serve to facilitate or lessen the labor of this process is a real boon to that large and eminently useful and accomplished body of men to whom the supervision of such operations is committed.

The use of the more common trigonometric functions, to wit, sines, cosines, tangents, and cotangents, which ordinary tables furnish, is not well adapted to the peculiar problems which are presented in the construction of Railroad curves. The additional columns of secants and cosecants in the tables of Dr. Bowditch sometimes afford a slight additional facility, which would be much increased had we also columns of natural secants as well as logarithmic.

Still there would be much labor of computation which may be saved by the use of tables of external secants and versed sines, which have been employed with great success recently by the Engineers on the Ohio and Mississippi Railroad, and which, with the formulas and rules necessary for their application to the laying down of curves, drawn up by MR. HASLETT, one of the Engineers of that Road, are now for the first time given to the public. This portion of the volume alone, by the great abridgment of labor for which it provides the means, and the simplicity and convenience of the matter which it furnishes, will give it an extensive circulation among Practical Engineers.

But besides this, the Architect, the Shipbuilder, the Mason, the Carpenter, the Joiner, the Manufacturer and Artisan in iron *and every species of material*, will find rules and recipes for all kinds of estimates, computations, constructions, compositions, mixtures, et cetera, which will excite surprise at their number, novelty, and value to every one.

The contents of this volume are of so varied a nature that it was not deemed necessary to make any strenuous efforts to arrange them systematically. Being solely intended for a book of reference, the relative order of the subjects is immaterial; and the copious Table of Contents and Index afford all the assistance that can be desired by those who wish to consult its pages.

THE EDITOR.

COLUMBIA COLLEGE,
Sept. 1855.

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THE

PRACTICAL BOOK OF REFERENCE.

ARITHMETICAL SIGNS.

The following definitions of arithmetical signs which are employed in mechanical calculations, will be found of great value to those who do not yet understand them, and of some interest to those who are already familiar with their meanings.

$=$ This is the sign of *equality*, and signifies *equal to*. For example: 12 inches $=$ 1 foot (12 inches *is equal to* 1 foot).

$+$ This is the sign of *addition*, and signifies *plus*, or *more*. For example: $5 + 3 = 8$ (5 *added to* 3 *is equal to* 8).

$-$ This is the sign of *subtraction*, and signifies *minus*, or *less*. For example: $10 - 8 = 2$ (10 *minus* 8 *leaves* or *is equal to* 2).

$\times$ This is the sign of *multiplication*, and signifies *multiplied by*, or *into*. For example: $10 \times 3 = 30$ (10 *multiplied by* 3 *is equal to* 30).

$\div$ This is the sign of *division*, and signifies *divided by*. For example: $156 \div 6 = 26$ (156 *divided by* 6 *is equal to* 26); or, $24 \div 4 = 6$ (24 *divided by* 4 *is equal to* 6); or $\frac{24}{4} = 6$ (24 *fourths are equal to* 6 *wholes*).

$:\ ::\ :$ This is the sign of *proportion*, and signifies *proportion*. For example: $4 : 6 :: 8 : 12$ (*as* 4 *is to* 6, *so is* 8 *to* 12); or $3 : 5 :: 9 : 15$ (that is, *as* 3 *is to* 5, *so is* 9 *to* 15); $\frac{3}{5} = \frac{9}{15}$.

$\sqrt{}$ This is the sign of the SQUARE *root*. When it is placed before a number (as thus, $\sqrt{5} = 25$), it means that the square root of that number is required. For example: $\sqrt{25} = 5$, because $5 \times 5 = 25$; or, $\sqrt{9} = 3$, because $3 \times 3 = 9$; or, $\sqrt{64} = 8$, because $8 \times 8 = 64$.

$\sqrt[3]{}$ This is the sign of the CUBE *root*. When it is placed before a number, it means that the cube root of that number is required. For example: $\sqrt[3]{64} = 4$ (that is, $4 \times 4 = 16$, and $4 \times 16 = 64$); or, $\sqrt[3]{216} = 6$ (that is, $6 \times 6 = 36$, and $6 \times 36 = 216$).

^{2} When this mark is added to a number (thus, 6^2), it means that that number is to be *squared*. For example: $5^2 = 25$ (that is, $5 \times 5 = 25$); or $6^2 = 36$ (that is, $6 \times 6 = 36$).

^{3} When this mark is added to a number, it means that that number is to be *cubed*. For example; $5^3 = 5 \times 5 \times 5 = 125$ (that is, $5 \times 5 = 25$, and $5 \times 25 = 125$; or, $7^3 = 343$ (that is, $7 \times 7 = 49$, and $7 \times 49 = 243$). The *index* or *power* (as the small figure annexed is called) shows how many times a number is to be multiplied by itself.

—— This is called the *bar*. It signifies that all the numbers or quantities under it are to be taken together. For example: $\overline{3 + 5} \times 4 = 32$ (3 plus 5 are equal to 8, and *that*, multiplied by 4, is equal to 32); or, $\overline{7 - 3} + 8 = 12$ (7 less 3 is equal to 4, and *that*, if added to 8, is equal to 12); or, $5 \times \overline{4 + 3} = 35$ (that is, 4 and 3 are 7, which, if multiplied by 5, is equal to 35); or, $5 \times \overline{6 + 4} = 50$ (that is, 6 and 4 are 10, and ten times 5 are 50). The parenthesis ( ) is sometimes used in place of the bar, thus: $(6 + 4) \times 5 = 50$.

$\therefore$ The meaning of this sign is *therefore.*

$\because$ This sign signifies *because.*

$\perp$ The meaning of this sign is *perpendicular.*

$\angle$ This sign signifies an *angle.*

$\sim$ This sign denotes *difference*, and is placed between two quantities (as $x \sim y$) when it is not known which of them is the greater.

$>$ or $\sqsupset$ The meaning of these signs is GREATER *than*. For example: $AB > CD$ (that is, AB is *greater than* CD).

$<$ or $\sqsubset$ The meaning of these signs is LESS *than*. For example: $AB < CD$ (that is, AB is *less than* CD).

· This is a decimal point. When placed before a number (thus, .1), it means that that number has a unit (1) for its denominator. For example: .1 is the same as $\frac{1}{10}$; .125 is the same as $\frac{125}{1000}$; .01 is the same as $\frac{1}{100}$; .001 is the same as $\frac{1}{1000}$; .0001 is the same as $\frac{1}{10000}$; 42.85 is the same as $42\frac{85}{100}$; 57.217 is the same as $57\frac{217}{1000}$.

° This is a *degree* mark. It is written and printed as follows: 25° (that is, 25 degrees).

′ This is a *minute* sign.

″ These two accents signify *seconds.*

‴ These three accents signify *thirds.* They read thus: 57° 17′ 43″ 39‴ (that is, 57 degrees, 17 minutes, 43 seconds, and 39 thirds).

ALGEBRAIC SYMBOLS.

The advantage of these, in a work like the present, may be thus illustrated:

Let l denote the length, b the breadth, and d the depth of an iron beam. If it be desired to express the product of the length and breadth, divided by the depth, it is done as follows:

$$\frac{l\,b}{d}$$

That is to say, multiplication is expressed by simply writing the letters which represent numbers one after the other; division, by drawing a line under the dividend, and writing the divisor below.

The sum of the length and breadth, divided by the depth, would be expressed briefly thus:

$$\frac{l+b}{d}$$

The square of the length, multiplied by the cube of the breadth, thus:

$$l^2\,b^3$$

The square root of the length, divided by the fourth root of the breadth, thus:

$$\frac{\sqrt{l}}{\sqrt[4]{b}}$$

The square root of the difference of the length and breadth, divided by the depth, thus:

$$\frac{\sqrt{l-b}}{d}$$

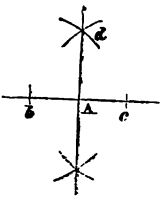
The square root of the quotient of the sum and difference of the length and breadth, thus:

$$\sqrt{\frac{l+b}{l-b}}$$

Any other letters—as a, b, c, &c.—may stand for the given dimensions.

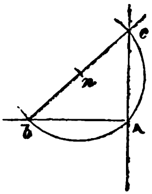
These explanations will serve to give the sense of the symbols which will be met with throughout the work.

PRACTICAL GEOMETRY.



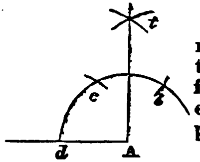
1. *From any given point, in a straight line, to erect a perpendicular; or, to make a line at right angles with a given line.*

On each side of the point A, from which the line is to be made, take equal distances, as A *b*, A *c*; and from *b* and *c* as centres, with any distance greater than *b* A or *c* A, describe arcs cutting each other at *d*; then will the line A *d* be the perpendicular required.



2. *When a perpendicular is to be made at or near the end of a given line.*

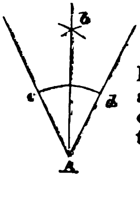
With any convenient radius, and with any distance from the given line A *b*, describe a portion of a circle, as *b* A *c*, cutting the given point in A; draw, through the centre of the circle *n*, the line *b n c*; and a line from the point A, cutting the intersections at *c*, is the perpendicular required.



3. *To do the same otherwise.*

From the given point A, with any convenient radius, describe the arc *d c b*; from *d* cut the arc in *c*, and from *c* cut the arc in *b*; also from *c* and *b* as centres, describe arcs cutting each other in *t*; then will the line A *t* be the perpendicular as required.

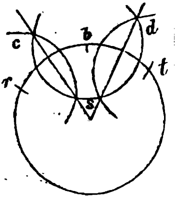
Note.—When the three sides of a triangle are in the proportion of 3, 4, and 5 equal parts, respectively, two of the sides form a right angle; and observe that in each of these or the preceding problems, the perpendiculars may be continued below the given lines, if necessarily required.



4. *To bisect any given angle.*

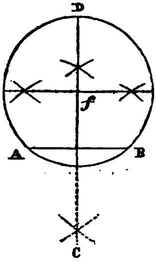
From the point A as a centre, with any radius less than the extent of the angle, describe an arc, as *c d*; and from *c* and *d* as centres, describe arcs cutting each other at *b*; then will the line A *b* bisect the angle as required.

5. *To find the centre of a circle, or radius, that shall cut any three given points, not in a direct line.*



From the middle point b as a centre, with any radius, as $b\ c$, $b\ d$, describe a portion of a circle, as $c\ s\ d$; and from r and t as centres, with an equal radius, cut the portion of the circle in $c\ s$ and $d\ s$; draw lines through where the arcs cut each other; and the intersection of the lines at s is the centre of the circle as required.

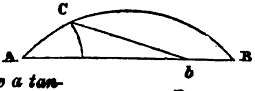
6. *To find the centre of a given circle.*



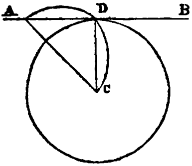
Bisect any chord in the circle, as A B, by a perpendicular, C D; bisect also the diameter E D in f; and the intersection of the lines at f is the centre of the circle required.

7. *To find the length of any given arc of a circle.*

With the radius A C, equal to ¼th the length of the chord of the arc A B, and from A as a centre, cut the arc in c; also from B as a centre, with equal radius, cut the chord in b; draw the line C b; and twice the length of the line is the length of the arc nearly.

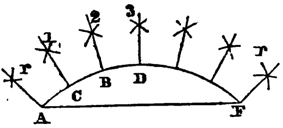


8. *Through any given point, to draw a tangent to a circle.*



Let the given point be at A; draw the line A C, on which describe the semicircle A D C; draw the line A D B, cutting the circumference in D, which is the tangent as required.

9. *To draw from or to the circumference of a circle lines tending towards the centre, when the centre is inaccessible.*



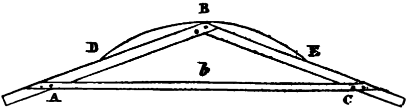
Divide the whole or any given portion of the circumference into the desired number of equal parts; then, with any radius less than the distance of two divisions, describe arcs cutting each other, as A 1, B 1, C 2, D 2, &c.; draw the lines C 1, B 2, D 3, &c., which lead to the centre, as required.

To draw the end lines.

As A r, F r, from C describe the arc r, and with the radius C 1,

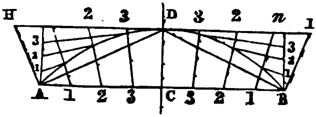
from A or F as centres, cut the former arcs at r, or r, and the lines A r, F r, will tend to the centre as required.

10. *To describe an arc, or segment of a circle, of large radii.*



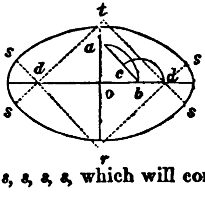
Of any suitable material, construct a triangle, as A B C; make A B, B C, each equal in length to the chord of the arc D E, and height, twice that of the arc B b. At each end of the chord D E fix a pin, and at B, in the triangle, fix a tracer (as a pencil), move the triangle along the pins as guides; and the tracer will describe the arc required.

11. *Or otherwise.*



Draw the chord A C B; also draw the line H D I, parallel with the chord, and equal to the height of the segment; bisect the chord in C, and erect the perpendicular C D; join A D, D B; draw A H perpendicular to A D, and B I perpendicular to B D; erect also the perpendiculars A n, B n; divide A B and H I into any number of equal parts; draw the lines 1 1, 2 2, 3 3, &c.; likewise divide the lines A n, B n, each into half the number of equal parts; draw lines to D from each division in the lines A n, B n, and, through where they intersect the former lines, describe a curve, which will be the arc or segment required.

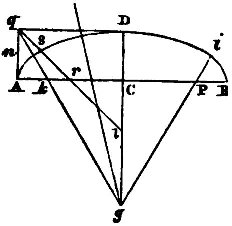
12. *To describe an ellipse, having the two diameters given.*



On the intersection of the two diameters as a centre, with a radius equal to the difference of the semi-diameters, describe the arc a b; and from b as a centre, with half the chord b c a, describe the arc c d; from o, as a centre, with the distance o d, cut the diameters in d r, d t; draw the lines r, s, s, and t, s, s; then from r and t describe the arcs s, s, s, s; also from d and d, describe the smaller arcs s, s, s, s, which will complete the ellipse as required.

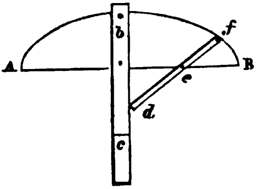
13. *To describe an elliptic arch, the width and rise of span being given.*

Bisect with a line at right angles the chord or span A B; erect



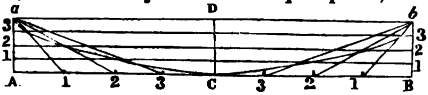
the perpendicular A q, and draw the line q D equal and parallel to A C; bisect A C and A q in r and n; make C l equal to C D, and draw the line $l\,r\,q$; draw also the line $n\,s$ D; bisect s D with a line at right angles, and meeting the line C D in g; draw the line $g\,q$, make C P equal to C k, and draw the line g P i; then from g as a centre, with the radius g D, describe the arc s D i; and from k and P as centres, with the radius A k, describe the arcs A s and B i, which completes the arch as required. *Or,*

14. Bisect the chord A B, and fix at right angles any straight guide, as $b\,c$; prepare, of any suitable material, a rod or staff, equal to half the chord's length, as $d\,e\,f$; from the end of the staff, equal to the height of the arch, fix a pin e, and at the extremity a tracer f; move the staff, keeping its end to the guide and the fixed pin to the chord, and the tracer will describe one half the arc required.



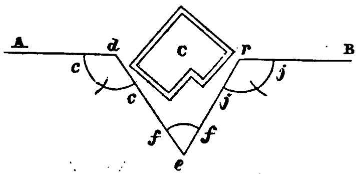
15. *To describe a parabola, the dimensions being given.*

Let A B equal the length, and C D the breadth of the required parabola; divide C A, C B into any number of equal parts; also divide the perpendiculars Aa and Bb into the same number of equal parts; then from a and b draw lines meeting each division on the line A C B; and a curve line drawn through each intersection will form the parabola required.



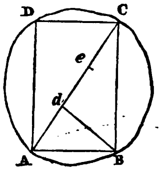
16. *To obtain by measurement the length of any direct line, though intercepted by some material object.*

Suppose the distance between A and B is required, but the right line is intercepted by the object C. On the point d, with any convenient radius, describe the arc $e\,c$, make the arc twice the radius in length, through which draw the line $d\,c\,e$; and on e describe another

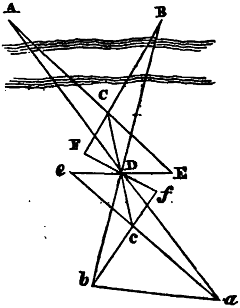


are equal in length to once the radius, as eff; draw the line efr equal to efd; on r describe the arc jj, in length twice the radius; continue the line through rj, which will be a right line, and de, or er, equal the distance between dr, by which the distance between A and B is obtained as required.

17. *A round piece of timber being given, out of which to cut a beam of strongest section.*

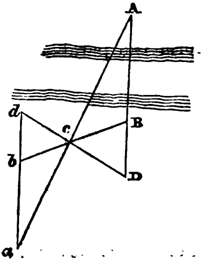


Divide into three equal parts any diameter in the circle, as A d, e C; from d or e, erect a perpendicular meeting the circumference of the circle, as d B; draw A B and B C, also A D equal to B C, and D C equal to A B, and the rectangle will be a section of the beam as required.



18. *To measure the distance between two objects, both being inaccessible.*

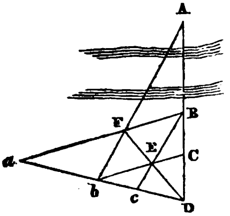
From any point C draw any line C c, and bisect it in D; take any point E in the prolongation of A C, and draw the line E e, making D e equal to D E; in like manner take any point F in the prolongation of B C, and make D f equal to F D. Produce A D and $e c$ till they meet in d, and also B D and $f c$ till they meet in b; then $a b$ equal A B, or the distance between the objects, as required.



19. *To ascertain the distance, geometrically, of any inaccessible object on an equal plane.*

Let it be required to find the distance between A and B, A being inaccessible; produce the line in the direction of A B to any point, as D; draw the line D d at any angle to the line A B; bisect the line D d, through which draw the line B b, making $c b$ equal to B c; draw the line $d b a$; also through c, in the direction c A, draw the line $a c$ A, intersecting the line $d b a$; then $b a$ equal B A, the distance required.

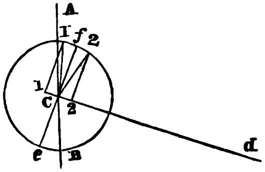
20. *Otherwise.*



Prolong A B to any point D, making B C equal to C D; draw the line D *a* at any angle with D A, and the line C *b* similar to B *c*; draw also the line D E F, which intersects the line B *a*; then *a b* equal B A, or the distance required.

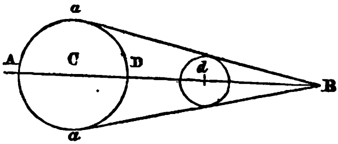
21. *To find the proper position for an eccentric, in relation to the crank in a steam engine, the angle of eccentric rod, and travel of the valve, being given.*

Draw the right line A B, as the situation of the crank at commencement of the stroke; draw also the line C *d*, as the proper given angle of eccentric rod with the crank; then from C as centre, describe a circle equal to the travel of the valve; draw the line *e f* at right angles to the line C *a*, draw also the lines 1 1, and 2 2, parallel to the line *e f*; and at a distance from *e f* on each side, equal to the lap and lead of the valve, draw the angular lines C 1, C 2, which are the angles of eccentric with the crank, for forward or backward motion, as may be required.



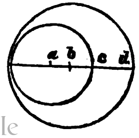
22. *The throw of an eccentric, and the travel of the valve in a steam-engine, also the length of one lever for communicating motion to the valve, being given, to determine the proper length for the other.*

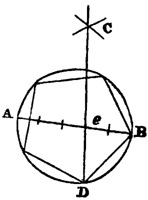
On any right line, as A B, describe a circle A D, equal to the throw of eccentric and travel of valve; then from C as a centre, with a radius equal to the length of lever given, cut the line A B, as at *d*, on which describe a circle, equal to the throw of eccentric or travel of valve, as may be required; draw the tangents B *a*, B *a*, cutting each other in the line A B, and *d* B is the length of the lever as required.



Note.—The throw of an eccentric is equal to the sum of twice the distance between the centres of formation and revolution, as *a b*, or to the degree of eccentricity it is made to describe, as *c d*. And

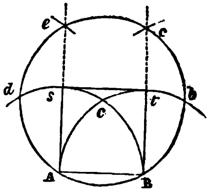
The travel of a valve is equal to the sum of the widths of the two steam openings, and the valve's excess of length more than just sufficient to cover the openings.





23. *To inscribe any regular polygon in a given circle.*

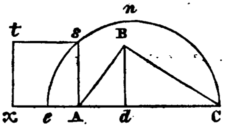
Divide any diameter, as A B, into so many equal parts as the polygon is required to have sides; from A and B as centres, with a radius equal to the diameter, describe arcs cutting each other in C; draw the line C D through the second point of division on the diameter *e*, and the line D B is one side of the polygon required.



24. *To construct a square upon a given right line.*

From A and B as centres, with the radius A B, describe the arcs A *c b*, B *c d*, and from *c*, with an equal radius, describe the circle or portion of a circle *e d*, A B, *b c*; from *b d* cut the circle at *e* and *c*; draw the lines A *e*, B *c*, also the line *s t*, which completes the square as required.

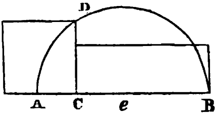
25. *To form a square equal in area to a given triangle.*



Let A B C be the given triangle; let fall the perpendicular B *d*, and make A *e* half the height *d* B; bisect *e* C, and describe the semicircle *e n* C; erect the perpendicular A *s*, or side of the square, then A *s t x* is the square of equal area as required.

26. *To form a square equal in area to a given rectangle.*

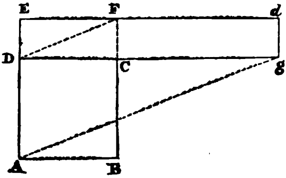
Let the line A B equal the length and breadth of the given rectangle; bisect the line in *e*, and describe the semicircle A D B; then from A with the breadth, or from B with the length, of the rectangle, cut the line A B at C, and erect the perpendicular C D, meeting the curve at D, and C D equal a side of the square required.

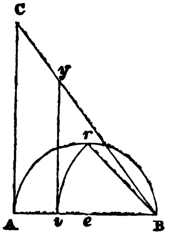


27. *To find the length for a rectangle whose area shall be equal to that of a given square, the breadth of the rectangle being also given.*

Let A B C D be the given square and D E the given breadth of

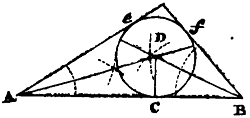
rectangle; continue the line B C to F, and draw the line D F; also continue the line D C to *g*, and draw the line A *g* parallel to D F; from the intersection of the lines at *g*, draw the line *g d* parallel to D E, and E *d* parallel to D *g*; then E D *d g* is the rectangle as required.





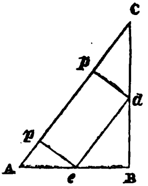
28. *To bisect any given triangle.*

Suppose A B C the given triangle; bisect one of its sides, as A B in *e*, from which describe the semicircle A *r* B; bisect the same in *r*, and from B, with the distance B *r*, cut the diameter A B in *v*; draw the line *v y* parallel to A C, which will bisect the triangle as required.



29. *To describe a circle of greatest diameter in a given triangle.*

Bisect the angles A and B, and draw the intersecting lines A D, B D, cutting each other in D; then from D as centre, with the distance or radius D C, describe the circle C *e f*, as required.



30. *To form a rectangle of greatest surface, in a given triangle.*

Let A B C be the given triangle; bisect any two of its sides, as A B, B C, in *e* and *d*; draw the line *e d*; also, at right angles with the line *e d*, draw the lines *e p*, *d p*, and *e p p d* is the rectangle required.

RATIO OF THE HARDNESS OF METALS.

1. Iron,	4. Silver,	6. Tin,
2. Platina,	5. Gold.	7. Lead.
3. Copper.		

STRENGTH OF WOOD.

All woods are from 7 to 20 times stronger transversely than longitudinally. They become stronger both ways when dry.

DECIMAL ARITHMETIC.

Decimal Arithmetic is the most simple and explicit mode of performing practical calculations, on account of its doing away with the necessity of fractional parts in the fractional form, thereby reducing long and tedious operations to a few figures arranged and worked in all respects according to the usual rules of common arithmetic.

Decimals simply signify tenths; thus, the decimal of a foot is the tenth part of a foot, the decimal of that tenth is the hundredth of a foot, the decimal of that hundredth is the thousandth of a foot, and so might the divisions be carried on and lessened to infinity: but in practice it is seldom necessary to take into account any degree of less measure than a one-hundredth part of the integer or whole number. And, as the entire system consists in supposing the whole number divided into tenths, hundredths, thousandths, &c., no peculiar notation is required, otherwise than placing a mark or dot to distinguish between the whole and any part of the whole, thus: 34.25 gallons signify 34 gallons, 2 tenths, and 5 hundredths of a gallon; 11.04 yards signify 11 yards and 4 hundredths of a yard; 16.008 shillings signify 16 shillings and 8 thousandth parts of a shilling; from which it must appear plain that ciphers on the right hand of decimals are of no value whatever, but placed on the left hand they diminish the decimal value in a tenfold proportion: for .6 signify 6 tenths; .06 signify 6 hundredths; and .006 signify 6 thousandths of the integer or whole number.

Reduction.

Reduction means the converting or changing of vulgar fractions to decimals of equal value; also finding the fractional value of any decimal given.

Rule 1. Add to the numerator of the fraction any number of ciphers at pleasure, divide the sum by the denominator, and the quotient is the decimal of equivalent value.

Rule 2. Multiply the given decimal by the various fractional denominations of the integer, or whole number, cutting off from the right hand of each product, for decimals, a number of figures equal to the given number of decimals, and thus proceed until the lowest degree, or required value, is obtained.

Ex. 1. Required the decimal equivalent, or decimal of equal value, to $\frac{3}{12}$ of a foot.

$$\frac{3.00}{12} = \cdot 25, \text{ the decimal required.}$$

Ex. 2. Reduce the fraction $\frac{1}{8}$ of an inch to a decimal of equal value.

$$\frac{1.000}{8} = \cdot 125, \text{ the decimal required.}$$

Ex. 3. What is the decimal equivalent to $\frac{7}{8}$ of a gallon?

$\frac{7.000}{8} = \cdot875$, the decimal equivalent.

Ex. 4. Required the fractional value of the decimal .40625 of an inch.

$$\begin{array}{lr} & \cdot40625 \\ \text{Multiply by } \frac{1}{8} & 8 \\ \hline & 3\cdot25000 \\ \times \frac{2}{16} = \frac{1}{8} & 2 \\ \hline & \cdot50000 \\ \times \frac{2}{32} = \frac{1}{16} & 2 \\ \hline & 1\cdot00000 \end{array}$$

$\frac{3}{8}$ and $\frac{1}{32}$ of an inch, the value required.

Ex. 5. What is the fractional value of ·625 of a cwt.?

$$\begin{array}{lr} & \cdot625 \\ \text{Multiply by 4 qrs} & 4 \\ \hline & 2\cdot500 \\ \times \text{ 28 lbs.} & 28 \\ \hline & 14\cdot000 \end{array}$$

= 2 quarters and 14 lbs., the value required.

Ex. 6. Ascertain the fractional value of ·875 of an imperial gallon.

$$\begin{array}{lr} & \cdot875 \\ \text{Multiply by 4 quarts} & 4 \\ \hline & 3\cdot500 \\ \times \text{ 2 pints} & 2 \\ \hline & 1\cdot000 \end{array}$$

= 3 quarts and 1 pint, the value required.

Ex. 7. What is the fractional value of ·525 of a £. sterling?

$$\begin{array}{lr} & \cdot525 \\ \text{Multiply by 20 sh.} & 20 \\ \hline & 10\cdot500 \\ \times \text{ 12 pence} & 12 \\ \hline & 6\cdot000 \end{array}$$

= 10 shillings and 6 pence, the value required.

Independent of the mark or dot which distinguishes between integers and decimals, the fundamental rules—viz. Addition, Subtraction, Multiplication, and Division—are in all respects the same as in Simple Arithmetic; and an example in each, illustrative of placing the separating point, will no doubt render the whole system sufficiently intelligible, even to the dullest capacity.

Ex. 1. Add into one sum the following integers and decimals:

16·625; 11·4; 20·7831; 12·125; 8·04; and 7·002.

```
16·625
11·4
20·7831
12·125
 8·04
 7·002
75·9751 = the sum required.
```

Ex. 2. Subtract 119·80764 from 234·98276.

```
234·98276
119·80764
115·17512 = the remainder required.
```

Ex. 3. Multiply 62·10372 by 16·732.

```
      62·10372
        16·732
     12420744
    18631116
   43472604
  37262232
  6210372
1039·11944304 = the product required.
```

Observe that the number of figures in the product from the right hand, accounted as decimals, are equal to the number of decimals in the multiplier and multiplicand taken together.

Ex. 4. Divide 39·375 by 9·25.

```
9·25) 39·375 (4·256 = the quotient required.
      3700
       2375
       1850
        5250
        4625
         6250
         5550
          700
```

Observe that the number of decimals, in the divisor and quotient together, must be equal to the number in the dividend.

Note.—The operation might be still continued, so as to reduce the quotient to a degree of greater exactitude; but in practice it is quite unnecessary, being even now reduced to a measure of greater nicety than is commonly required.

MENSURATION.

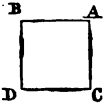
Mensuration is the method of calculating the comparative magnitudes of figures, and it is divided into two parts—Mensuration of Superficies or Surfaces, and Mensuration of Solids.

The magnitude of a surface is called its area, and is the space inclosed between its boundary lines.

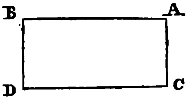
The magnitude of a body is called its solid contents, and is expressed in cubic feet, inches, &c.

Mensuration of Superficies.

Square.



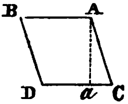
Rectangle.



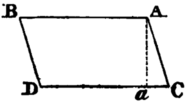
A Square is a quadrilateral figure, which has all its sides equal, and all its angles right angles.

A Rectangle is a four-sided figure, which has its angles, right angles, and its opposite sides parallel.

Rhombus.



Rhomboid.

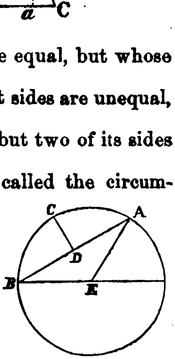


A Rhombus is a parallelogram, whose sides are equal, but whose angles are not right angles.

A Rhomboid is a parallelogram, whose adjacent sides are unequal, and whose angles are not right angles.

A Trapezoid is a four-sided figure, which has but two of its sides parallel.

A Circle is a figure bounded by one line, called the circumference, and is such that all lines drawn to the circumference from a certain point within the figure, called the centre, are equal to each other. Any of these lines is called a radius; and a line drawn through the centre, terminating both ways in the circumference, is called a diameter. The portion of circle cut off by a diameter is called a semicircle.



An Arc of a circle is any portion of the circumference.

A SEGMENT of a circle is a figure contained by an arc and its chord.

A VERSED SINE is a line drawn from the middle of a chord perpendicular to the circumference.

A SECTOR of a circle is a figure contained by two radii and an arc, as A C B E.

PROBLEM I.

To find the area of any parallelogram.

RULE. Multiply the length by the perpendicular height, and the product will be the area.

EXAMPLE. Required the area of a rhomboid whose length A B = 20·5, and perpendicular height a A = 11·75.

$$20·5 \times 11.75 = 240·875, \text{ the area.}$$

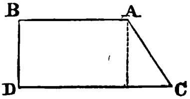
Note.—In a square, or rectangle, the perpendicular height is the breadth: therefore, to find the areas of a square and rectangle, multiply the length by the breadth.

PROBLEM II.

To find the area of a trapezoid.

RULE. Add together the two parallel sides, multiply their sum by the breadth or height, and half the product is the area.

EXAMPLE. Required the area of a trapezoid whose sides A B and C D are 14·5 and 10·25, and breadth, a A = 7·25.



$$\frac{\overline{14·5 + 10·25} \times 7·25}{2} = 89·71875,$$

the area.

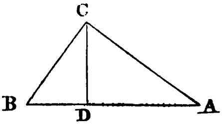
PROBLEM III.

To find the area of a triangle.

RULE. Multiply one of its sides as a base by a perpendicular let fall from the opposite angle, and take half the product for the area.

Or, from half the sum of the three sides subtract each side separately, and multiply the three remainders so obtained and the half sum together, and the square root of the product will be the area.

EXAMPLE 1. Required the area of a triangle A B C, whose base A B = 16·5, and perpendicular D C = 10·25.



$$\frac{16·5 \times 10·25}{2} = 84·5625,$$

the area.

EXAMPLE 2. What is the area of that triangle whose three sides are 8, 12, and 16 respectively?

$$\frac{8 + 12 + 16}{2} = 18, \text{ the half sum of the sides;}$$

then,

$$\begin{array}{ccc} 18 & 18 & 18 \\ 8 & 12 & 16 \\ \hline 10 & 6 & 2 \end{array} \text{ and } \sqrt{18 \times 10 \times 6 \times 2} = 46{\cdot}47, \text{ the area.}$$

PROBLEM IV.

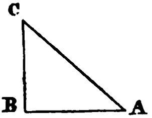
If any two sides of a right-angled triangle be given, the third side may be found by the following rules.

1.—To the square of the base add the square of the perpendicular; and the square root of the sum will be the hypothenuse or longest side.

2.—Multiply the sum of the hypothenuse, and one side by their difference; and the square root of the product will be the other side.

EXAMPLE 1. Given the base A B = 16, and perpendicular B C = 12; required the length of the hypothenuse A C.

$\sqrt{16^2 + 12^2} = 20$, the length of the hypothenuse A C.



EXAMPLE 2. Given the base A B = 16, and hypothenuse A C = 20; required the length of the perpendicular B C.

$$\sqrt{20 + 16 \times 4} = 12, \text{ length of the perpendicular B C.}$$

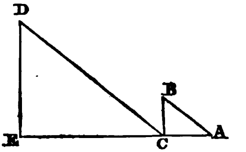
Note.—The diagonal line, or hypothenuse in a square, is equal to the square root of twice the square of the side. And the side of a square is equal to the square root of half the square of its diagonal.

Thus suppose each side of a square equal 12 feet:

$$12^2 \times 2 = \sqrt{288} = 16{\cdot}9705 \text{ feet, the diagonal. Or,}$$

$$\frac{16{\cdot}9705^2}{2} = \sqrt{144} = 12 \text{ feet, the length of each side.}$$

Similar triangles, or those which are equi-angular to each other, have the sides about their equal angles proportional; thus, in the annexed figure the triangles A B C and C D E are similar, and therefore have the sides about the equal angles proportional:



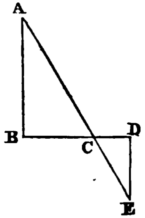
A C : B C : : C E : D E;
A B : B C : : C D : D E, &c.

The utility, then, of the above triangles for practical purposes, as, for instance, ascertaining the heights of buildings, &c., will be seen from the following:

Suppose D E to be an eminence, of which it is required to find the height, and E C the length of the shadow cast by the sun; then, in order to find D E, we may erect perpendicularly at C a pole of any known length, as B C, and after measuring the length of its shadow A C, state—as the length of the pole's shadow is to the height of the pole itself, so is the length of the shadow of D E to the height of D E; or,

As A C : C B : : C E : E D;

and supposing A C = 6 feet, B C = 4 feet, and C E = 30 feet, then E D would be 20 feet.



Again, supposing we wished to find the distance between two objects A and B; draw D B of any length at right angles to A B, and in D B take any point C, through which draw A E; also, at D, at right angles to D B, draw D E, making the triangle D E C, and state,

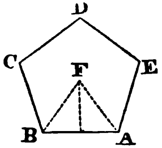
As D C : D E : : B C : B A.

PROBLEM V.

To find the area of any regular polygon.

RULE. Multiply the sum of its sides by a perpendicular drawn from its centre to one of its sides, and take half the product for the area.

Or, multiply the square of the side of a polygon (from three to twelve sides) by the numbers in the fourth column of the table for polygons, opposite the number of sides required, and the product will be the area nearly.



EXAMPLE 1. Required the area of the regular pentagon A B C D E, each side being 7·5, and perpendicular F G = 6·4.

$$\frac{7{\cdot}5 \times 5 \times 6{\cdot}4}{2} = 120, \text{ the area.}$$

EXAMPLE 2. What is the area of a regular hexagon, each side being 8·75 in length?

$$8{\cdot}75^2 \times 2{\cdot}598 = 199{\cdot}009375, \text{ the area.}$$

TABLE *of multipliers for polygons from three to twelve sides.*

Names.	Sides.	Multipliers.	Multipliers.	Multipliers.	Areas.
Trigon	3	2	1·73	·579	·433
Tetragon...	4	1·41	1·412	·705	1·000
Pentagon...	5	1·238	1·174	·852	1·72
Hexagon...	6	1·156	= Radius.	= Length of side.	2·598
Heptagon ..	7	1·11	·867	1·16	3·634
Octagon ...	8	1·08	·765	1·307	4·828
Nonagon...	9	1·062	·681	1·47	6·1818
Decagon....	10	1·05	·616	1·625	7·694
Undecagon .	11	1·04	·561	1·777	9·365
Dodecagon..	12	1·037	·515625	1·94	11·196

1. *The breadth of a polygon given, to find the radius of a circle to contain that polygon.*

RULE. Multiply half the breadth of the polygon by the numbers in the first column opposite to its name, or number of sides, and the product will be the radius of a circle to contain that polygon.

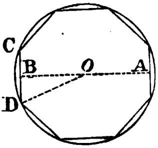
And if the polygon have an unequal number of sides, the half breadth is accounted from its centre to one of its sides.

2. *The radius of a circle given, to find the length of side.*

RULE. Multiply the radius of any circle by the numbers in the second column opposite the polygon required, and the product will be the length of side nearly that will divide that circle into the proposed number of sides. And,

3. *The length of side given, to find the radius.*

RULE. Multiply the given length of side by the numbers in the third column opposite the polygon required, and the product will be the radius of a circle to contain that polygon.



EXAMPLE 1. Required the radius of a circle to contain an octagon, whose breadth A B = 18·5 inches.

Half of 18·5 = 9·25, and 9·25 × 1·08 = 9·99 or ten inches nearly, the radius of the circle O D.

EXAMPLE 2. Given the radius O D = 9·99 inches, required the length of side D C.

9·99 × ·765 = 7·64235, the length of side.

EXAMPLE 3. Given the length of side D C = 7·64235; required the radius D O.

$$7 \cdot 64235 \times 1 \cdot 307 = 9 \cdot 98855145, \text{ or } 9 \cdot 99 \text{ in. nearly.}$$

PROBLEM VI.

Having the diameter of a circle given, to find the circumference; or the circumference given, to find the diameter.

RULE 1. As 7 is to 22, so is the diameter to the circumference. Or, as 22 is to 7, so is the circumference to the diameter.

2. As 1 is to 3·1416, so is the diameter to the circumference. Or, as 3·1416 is to 1, so is the circumference to the diameter.

EXAMPLE 1. Required the circumference of a circle when the diameter is 23·5.

$$\frac{23 \cdot 5 \times 22}{7} = 73\tfrac{6}{7}, \text{ the circumference.}$$

EXAMPLE 2. The circumference of a circle is $73\frac{6}{7}$, required the diameter.

$$\frac{73\frac{6}{7} \times 7}{22} \times 23 \cdot 5, \text{ the diameter.}$$

EXAMPLE 3. Required the circumference of a circle whose diameter is 30.

$$3 \cdot 1416 \times 30 = 94 \cdot 248, \text{ the circumference.}$$

EXAMPLE 4. What is the diameter of a circle when the circumference is 94·248?

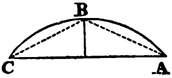
$$94 \cdot 248 \div 3 \cdot 1416 = 30, \text{ the diameter.}$$

PROBLEM VII.

To find the length of any arc of a circle.

RULE. Subtract the chord of the whole arc from eight times the chord of half the arc; and $\frac{1}{3}$ of the remainder is the length of the arc nearly.

EXAMPLE. Required the length of the arc A B C; the chord of half the arc A B = 19·8, and chord of the whole arc A C = 34·4.



$$19 \cdot 8 \times 8 = 158 \cdot 4, \text{ and}$$

$$\frac{158 \cdot 4 - 34 \cdot 4}{3} = 41 \cdot 33, \text{ the length of the arc.}$$

PROBLEM VIII.

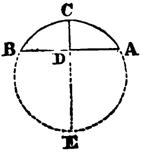
To find the diameter of a circle, by having the chord and versed sine given.

RULE. Divide the square of half the chord by the versed sine, to

the quotient of which add the versed sine, and the sum will be the diameter.

Or, if the sum of the squares of the semichord and versed sine be divided by the versed sine, the quotient will be the diameter of the circle to which that segment corresponds.

EXAMPLE. Given the chord $AB = 24$, and versed sine $CD = 8$; required the diameter of the circle CE.



Half the chord $= 12$, and $12^2 \div 8 = 18 + 8 =$ 26, the diameter.

Or, $\frac{12^2 + 8^2}{8} = 26$, as before.

PROBLEM IX.

To find the area of an ellipsis, or oval.

RULE. Multiply the longest diameter by the shortest, and the product by ·7854; the result is the area.

An oval is 25 inches by 16·5: what are its superficial contents?

$$25 \times 16{\cdot}0 = 412{\cdot}5 \times {\cdot}7854 = 323{\cdot}9775 \text{ inches, the area.}$$

Note.—Multiply half the sum of the two diameters by 3·1416, and the product is the circumference of the oval or ellipsis.

PROBLEM X.

To find the area of a parabola, or its segment.

RULE. Multiply the base by the perpendicular height, and two-thirds of the product is the area.

What is the area of a parabola whose base is 20 feet and height 12?

$$20 \times 12 = \frac{240 \times 2}{3} = 160 \text{ feet, the area.}$$

Some of the properties of a circle.

1. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter or outline.
2. The areas of circles are to each other as the squares of their diameters, or of their radii.
3. Any circle whose diameter is double that of another, contains four times the area of the other.
4. The area of a circle is equal to the area of a triangle whose base is equal to the circumference, and perpendicular equal to the radius.

5. The area of a circle is equal to the rectangle of its radius, and a right line equal to half its circumference.

6. The area of a circle is found by squaring the diameter, and multiplying by the decimal ·7854; or by multiplying the circumference by the radius, and dividing the product by 2?

EXAMPLE 1. Required the area of a circle, the diameter being 30·5.

$$30{\cdot}5^2 \times {\cdot}7854 = 730{\cdot}618350, \text{ the area required.}$$

EXAMPLE 2. What is the area of a circle when the diameter is 1?

In this case the circumference is 3·1416, half of which is 1·5708, and half of 1 = ·5; then 1·5708 × ·5 = ·7854, the area.

Having the area of a circle given, to find the diameter.

RULE. As 355 is to 452, so is the area to the square of the diameter.

Or, multiply the square root of the area by 1·12837, and the product will be the diameter.

Or, divide the area by the decimal ·7854, and extract the square root.

EXAMPLE. Required the diameter of that circle whose area is 122·71875.

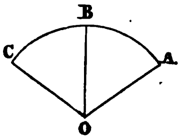
$$\frac{\sqrt{122{\cdot}71875 \times 452}}{355} = 12{\cdot}5, \text{ diameter.}$$

Or, $\sqrt{122{\cdot}71875} = 11{\cdot}077$; and $11{\cdot}077 \times 1{\cdot}12837 = 12{\cdot}49895$, or 12·5, diameter.

PROBLEM XI.

To find the area of a sector of a circle.

RULE. Multiply the length of the arc by the radius of the circle, and half the product will be the area.



EXAMPLE. Required the area of a sector of a circle whose arc A B C = 26·666, and radius B O = 16·9.

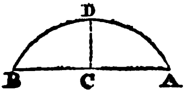
$$\frac{26{\cdot}666 \times 16{\cdot}9}{2} = 225{\cdot}3277, \text{ the area.}$$

PROBLEM XII.

To find the area of a segment of a circle.

RULE. Multiply the versed sine by the decimal ·626, to the square of the product add the square of half the chord; multiply twice the square root of the sum by ⅔ of the versed sine, and the product will be the area.

EXAMPLE. Required the area of a segment of a circle whose chord A B = 48, and versed sine C D = 18.



$18 \times \cdot 626 = 11\cdot 268^2 = 126\cdot 967824$; which add to 576, being the square of half the chord = 702967824, twice the square root of which is $53\cdot 026 \times 12$; being ⅔ of the versed sine = 636·312, the area.

The following is a near approximate to the preceding rule:

To the cube of the versed sine, divided by twice the length of the chord, add ⅔ of the product of the chord, multiplied by the versed sine; and the sum will be the area of the segment nearly. Take the last example:

Versed sine = 18, and chord 48, then, $\frac{18^3}{48 \times 2} = 60\cdot 7$; and

$\frac{48 \times 18 \times 2}{3} = 576 + 60\cdot 7 = 636\cdot 7$, the area nearly.

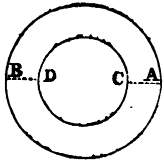
Or, the area of a segment may be found by finding the area of a sector having the same radius as the segment; then deducting the area of the triangle, leaves the area of the segment.

PROBLEM XIII.

To find the area of a circular ring or space included between two concentric circles.

RULE. Add the inside and outside diameters together, multiply the sum by their difference, and by ·7854, and the product will be the area.

EXAMPLE. The diameters of two concentric circles, A B and C D, are 10 and 6; required the area of the ring or space contained between them.



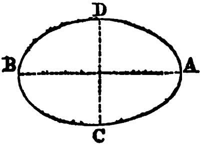
$\overline{10 + 6} \times 4 \times \cdot 7854 = 50\cdot 2656$, the area.

PROBLEM XIV.

To find the area of an ellipsis.

RULE. Multiply the transverse or longer diameter by the conjugate or shorter diameter, and by ·7854, and the product will be the area.

EXAMPLE. Required the area of an ellipsis whose longer diameter A B = 12, and shorter diameter C D = 9.



$12 \times 9 \times \cdot 7854 = 84\cdot 8232$, the area.

Note.—If half the sum of the two diameters be multiplied by 3·1416, the product will be the circumference of the ellipsis.

Thus $12 + 9 = 21$, and $\frac{3\cdot 1416 \times 21}{2} = 36\cdot 1384$, the circumference.

Mensuration of Solids.

By solids are meant all bodies, whether solid, fluid, or bounded space, that can be comprehended within length, breadth, and thickness.

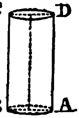
PROBLEM I.

To find the convex surface and solid content of a cylinder.

RULE 1. Multiply the circumference of the base by the height of the cylinder, and the product is the convex surface.

RULE 2. Multiply the area of the base by the height of the cylinder, and the product is the solid content.

EXAMPLE 1. Required the convex surface of the cylinder A B C D, whose base A B = 32 inches, and perpendicular height B C = 6 feet.



$3{\cdot}1416 \times 32 \times 72$ inches $= 7238{\cdot}2464$ square or superficial inches, and $7238{\cdot}2464 \div 144 = 50{\cdot}2658$ superficial feet.

EXAMPLE 2. Required the solid content, in cubic inches and cubic feet, of the cylinder as above.

$32^2 \times {\cdot}7854 \times 72 = 57905{\cdot}9712$ cubic inches, and
$57905{\cdot}9712 \div 1728 = 33{\cdot}5104$ cubic feet.

EXAMPLE 3. Suppose the cylinder A B C D be intended to contain a fluid, and that the sides and bottom are each one inch in thickness, how many imperial gallons would it contain?

$32 - 2 = 30$ inches diameter; and $72 - 1 = 71$ inches deep; then $\dfrac{30^2 \times {\cdot}7854 \times 71}{277{\cdot}274} = 181$ gallons.

Or, $50187{\cdot}06 \times {\cdot}003607 = 181$, as before.

PROBLEM II.

To determine the dimensions of any cylindrical vessel, whereby to contain the greatest cubical contents, bounded by the least superficial surface.

RULE. Multiply the given cubical contents by 2·56, and the cube root of the product equal the diameter, and half the diameter equal the depth.

EXAMPLE. Suppose a cylindrical vessel is to be made so as to contain 600 cubic feet, and of such dimensions as to require the least possible materials by which it is constructed, what must be its depth and diameter?

$600 \times 2{\cdot}56 = \sqrt[3]{1536} = 11{\cdot}5379$ feet diameter,
and $11{\cdot}5379 \div 2 = 5{\cdot}76895$ feet in depth.

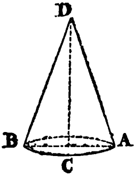
Note.—If the vessel is to be constructed with two ends, then the cube root of four times the solidity divided by 3·1416 equal both the length and diameter, so as to expose the least possible surface, or be composed of the least possible materials, of which to be constructed.

PROBLEM III.

To find the surface and solid content of a cone or pyramid.

RULE 1. Multiply the circumference of the base by the slant height, and half the product will be the slant surface; to which add the area of the base, and the product will be the whole surface.

RULE 2. Multiply the area of the base by the perpendicular height, and $\frac{1}{3}$ of the product will be the solid content.



EXAMPLE 1. Required the convex surface of a cone whose base A B = 20 inches, and slant height B D = 29·5.

$\frac{3·1416 \times 20 \times 29·5}{2} = 926·772$ square inches, and divided by 144 = 6·435 superficial feet.

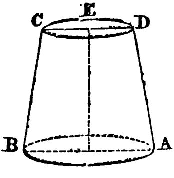
EXAMPLE 2. Required the solidity of the cone as above, the perpendicular C D being 28 inches.

$\frac{20^2 \times ·7854 \times 28}{3} = 2932·16$ cubic inches, and divided by 1728 = 1·697 cubic feet.

PROBLEM IV.

To find the surface of the frustum of a cone or pyramid.

RULE. Multiply the sum of the perimeters of the two ends by the slant height, and half the product will be the slant surface; to which add the areas of the two ends, and the product will be the whole surface.



EXAMPLE. Required the convex surface of the frustum of a cone A B C D, whose base A B = 20 inches, the slant height B C = 19, and top end C D = 11.

$\frac{\overline{3·1416 \times 20 + 3·1416 \times 11} \times 19}{2}$ = 925·2012 square inches, and divided by 144 = 6·425 feet nearly.

PROBLEM V.

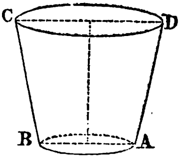
To find the solid content of the frustum of a cone.

RULE. To the product of the diameters of the two ends add the sum of their squares; multiply this sum by the perpendicular height and by ·2618; the product is the solid content.

EXAMPLE 1. Required the solid content of the frustum in Problem IV., whose perpendicular E F = 18 inches.

$20 \times 11 = 220$, and $220 + 20^2 + 11^2 \times 18 \times ·2618 = 3491·8884$ cubic inches, and divided by 1728 = 2·0208 cubic feet nearly.

EXAMPLE 2. Required the content, in imperial gallons, of the inverted frustum of a cone A B C D, whose inner dimensions are $3\frac{1}{2}$ feet deep, 18 inches diameter at bottom, and 22 inches diameter at top.



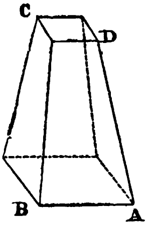
$22 \times 18 = 396$, and $\overline{396 + 22^2 + 18^2} \times 42 \times \cdot 2618 = \dfrac{13238\cdot 7024}{277\cdot 274} = 47\cdot 745$ galls. nearly.

Or, $13238\cdot 7024 \times 0\cdot 00360654 = 47\cdot 75$ gallons nearly, as before.

PROBLEM VI.

To find the solid content of the frustum of a pyramid.

RULE. To the sum of the areas of the two ends add the square root of their product; multiply this sum by the perpendicular height, and $\frac{1}{3}$ of the product is the solid content.



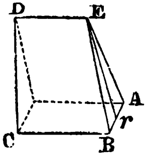
EXAMPLE. Required the solid content of the frustum of a pyramid A B C D, whose perpendicular height = 24 inches, the area of the base = 144 inches, and area of the top end = 64.

$144 + 64 = 208$, and $\sqrt{144 \times 64} = 96$; then $\dfrac{208 + 96 \times 24}{3} = 2432$ cubic inches, and $\div 1728 = 1\cdot 4074$ cubic feet nearly.

PROBLEM VII.

To find the solidity of a wedge.

RULE. To the length of the edge add twice the length of the base; multiply that sum by the height, and by the breadth of the base, and one-sixth of the product will be the solidity.



EXAMPLE. Required the content in cubic inches of the wedge A B C D E, whose base A B C = 12 inches long and 4 inches broad, the length of the edge D E = 10 inches, and perpendicular height r E = 20 inches.

$\dfrac{\overline{10 + 24} \times 20 \times 4}{6} = 452\cdot 33$ cubic inches.

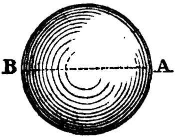
PROBLEM VIII.

To find the convex surface and solid content of a sphere or globe.

RULE 1. Multiply the square of the diameter by $3\cdot 1416$; the product will be the convex superficies.

RULE 2. Multiply the cube of the diameter by ·5236, and the product is the solid content.

EXAMPLE 1. Required the convex surface of a sphere, whose diameter A B = $25\frac{1}{2}$ inches.



$25{\cdot}5^2 \times 3{\cdot}1416 = 2042{\cdot}8254$ square inches, $\div$ 144 = 14·1862 square or superficial feet.

EXAMPLE 2. Required the solid content of a sphere whose diameter A B = $25\frac{1}{2}$ inches.

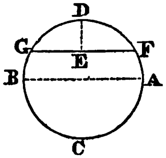
$25{\cdot}5^3 \times {\cdot}5236 = 8682{\cdot}00795$ cubic inches; $\div$ 1728 = 5·0243 cubic feet.

PROBLEM IX.

To find the convex surface and solid content of the segment of a sphere.

RULE 1. Multiply the height of the segment by the whole circumference of the sphere, and the product is the curved surface.

RULE 2. Add the square of the height to three times the square of the radius of the base; multiply that sum by the height, and by ·5236, and the product is the solid content.



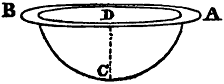
EXAMPLE 1. The diameter A B of the sphere A B C D = 20 inches; what is the convex surface of that segment of it whose height E D = 8 inches?

$3{\cdot}1416 \times 20 \times 8 = 502{\cdot}656$ square inches; $\div$ 144 = 3·49 superficial feet.

EXAMPLE 2. The base F G of the segment F D G = 18 inches, and perpendicular E D = 8; what is the solid content?

$8^2 = 64$, and $9^2 \times 3 = 243$; then $\overline{243 + 64} \times 8 \times {\cdot}5236 = 1285{\cdot}9616$ cubic inches, $\div$ 1728 = ·7441 cubic feet.

EXAMPLE 3. Suppose A B C D to be a sugar-pan, and that the diameter of the mouth A B is 4 feet, the depth D C being 25 inches, how many imperial gallons will it contain?



$25^2 = 625$, and $24^2 \times 3 = 1728$; then

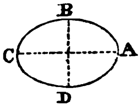
$$\overline{1728 + 625} \times 25 \times {\cdot}5236 = \frac{30800{\cdot}77}{277{\cdot}274} =$$

111·084 gallons.

PROBLEM X.

To find the solidity of a spheroid.

RULE. Multiply the square of the revolving axis by the fixed axis, and by ·5236, and the product will be the solidity.



EXAMPLE 1. Required the solid content of the prolate spheroid A B C D, whose fixed axis A C is 50, and revolving axis B D 30.

$30^2 \times 50 \times \cdot5236 = 23562$, the solidity.

EXAMPLE 2. What is the solid content of an oblate spheroid, the fixed axis being 30, and revolving axis 50?

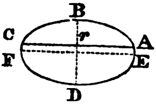
$50^2 \times 30 \times \cdot5236 = 39270$, the solid content.

PROBLEM XI.

To find the solidity of the segment of a spheroid when the base is circular or parallel to the revolving axis.

RULE. From triple the fixed axis take double the height of the segment; multiply the difference by the square of the height, and by ·5236; then say, as the square of the fixed axis is to the square of the revolving axis, so is the former product to the solidity.

EXAMPLE 1. Required the solid content of the segment A B C, whose height B r is 10; the revolving axis E F being 40, and fixed axis B D 25.



$\overline{25 \times 3} - \overline{10 \times 2} = 55$, and $55 \times 10^2 \times \cdot5236 = 2879\cdot8$. Then, as $25^2 : 40^2 :: 2879\cdot8 : 7872\cdot3$ nearly.

EXAMPLE 2. What is the solid content of the segment of a spheroid whose height = 20 inches, the revolving axis being 25, and fixed axis 50?

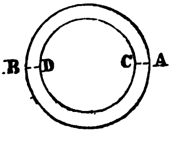
$\overline{50 \times 3} - \overline{20 \times 2} = 110$, and $110 \times 20^2 \times \cdot5236 = 23038\cdot4$; then, as $50^2 : 25^2 :: 23038\cdot4 : 5759\cdot6$ inches, the solid content.

PROBLEM XII.

To find the convex surface and solid content of a cylindric ring.

RULE 1. Multiply the thickness of the ring added to the inner diameter by the thickness and by 9·8698, and the product will be the convex surface.

RULE 2. To the thickness of the ring add the inner diameter; multiply that sum by the square of the thickness and by 2·4674, and the product will be the solid content.



EXAMPLE 1. The thickness of a cylindric ring A C or D B = 2 inches, and inner diameter = 18, required the convex superficies.

$\overline{18 + 2} \times 2 \times 9\cdot8698 = 394\cdot792$ square inches, and $\div\ 144 = 2\cdot741$ superficial feet nearly.

EXAMPLE 2. Required the solid content of the ring as above.

$18 + 2 \times 2^2 \times 2{\cdot}4674 = 197{\cdot}392$ cubic inches, and $\div\ 1728 = {\cdot}114$ cubic feet.

Note.—A cubic foot is equal to 1728 cubic inches,
or 2200 cylindrical inches,
or 3300 spherical inches,
or 6600 conical inches.

Also, the cubic foot being considered unity, or 1,
A cylinder 1 foot in diameter and 1 foot in length .... = ·7854
A sphere 1 foot in diameter .... = ·5236
And a cone 1 foot in diameter at the base and 1 foot in height .... = ·2618

Decimal Approximations,

FOR FACILITATING CALCULATIONS IN MENSURATION.

Lineal feet	multiplied by	·00019	=	miles.
“ yards,	“	·000568	=	“
Square inches,	“	·007	=	square feet.
“ yards,	“	·0002067	=	acres.
Circular inches,	“	·00546	=	square feet.
Cylindrical inches,	“	·0004546	=	cubic feet.
“ feet,	“	·02909	=	cubic yards.
Cubic inches,	“	·00058	=	cubic feet.
“ feet,	“	·03704	=	cubic yards.
“ “	“	6·232	=	imperial gallons.
“ inches,	“	·003607	=	“ “
Cylindrical feet,	“	4·895	=	“ “
“ inches,	“	·002832	=	“ “
Cubic inches,	“	·263	=	lbs. avs. of cast iron.
“ “	“	·281	=	“ wrought do.
“ “	“	·283	=	“ steel.
“ “	“	·3225	=	“ copper.
“ “	“	·3037	=	“ brass.
“ “	“	·26	=	“ zinc.
“ “	“	·4103	=	“ lead.
“ “	“	·2636	=	“ tin.
“ “	“	·4908	=	“ mercury.
Cylindrical inches,	“	·2065	=	“ cast iron.
“	“	·2168	=	“ wrought iron.
“	“	·2223	=	“ steel.
“	“	·2533	=	“ copper.
“	“	·2385	=	“ brass.
“	“	·2042	=	“ zinc.
“	“	·3223	=	“ lead.
“	“	·207	=	“ tin.
“	“	·3854	=	“ mercury.
Avoirdupois lbs.,	“	·009	=	cwts.
“	“	·00045	=	tons.

INSTRUMENTAL ARITHMETIC;

OR, UTILITY OF THE SLIDE RULE.

The slide rule is an instrument by which the greater portion of operations in arithmetic and mensuration may be advantageously performed, provided the lines of division and gauge points be made properly correct, and their several values familiarly understood.

The lines of division are distinguished by the letters A B C D, A B and C being each divided alike, and containing what is termed a double radius, or double series of logarithmic numbers, each series being supposed to be divided into 1000 equal parts, and distributed along the radius in the following manner:

From 1 to	2	contains	301	of these parts, being the log. of	2.
"	3	"	477	"	3.
"	4	"	602	"	4.
"	5	"	699	"	5.
"	6	"	778	"	6.
"	7	"	845	"	7.
	8	"	903	"	8.
	9	"	954	"	9.

1000 being the whole number.

The line D, on the improved rules, consists of only a single radius; and although of larger radius, the logarithmic series is the same, and disposed of along the line in a similar proportion, forming exactly a line of square roots to the numbers on the lines B C.

Numeration.

Numeration teaches us to estimate or properly value the numbers and divisions on the rule in an arithmetical form.

Their values are all entirely governed by the value set upon the first figure, and, being decimally reckoned, advance tenfold from the commencement to the termination of each radius: thus, suppose 1 at the joint be one, the 1 in the middle of the rule is ten, and 1 at the end one hundred. Again, suppose 1 at the joint ten, 1 in the middle is 100, and 1 or 10 at the end is 1000, &c., the intermediate divisions on which complete the whole system of its notation.

To Multiply Numbers by the Rule.

Set 1 on B opposite to the multiplier on A; and against the number to be multiplied on B is the product on A.

Multiply 6 by 4.

Set 1 on B to 4 on A: and against 6 on B is 24 on A. The slide thus set, against

7	on B	is	28	on A
8	"		32	"
9	"		36	"
10	"		40	"
11	"		44	"
12	"		48	"
15	"		60	"
25	"		100, &c., &c.	

To divide Numbers upon the Rule.

Set the divisor on B to 1 on A, and against the number to be divided on B is the quotient on A.

Divide 63 by 3.

Set 3 on B to 1 on A, and against 63 on B is 21 on A.

Proportion, or Rule of Three Direct.

Rule. Set the first term on B to the second on A, and against the third upon B is the fourth upon A.

1. If 4 yards of cloth cost 38 shillings, what will 30 yards cost at the same rate?

Set 4 on B to 38 on A, and against 30 on B is 285 shillings on A.

2. Suppose I pay 31*s.* 6*d.* for 3 cwt. of iron, at what rate is that per ton? 1 *ton* = 20 *cwt.*

Set 3 upon B to 31·5 upon A, and against 20 upon B is 210 upon A.

Rule of Three Inverse.

Rule. Invert the slide, and the operation is the same as direct proportion.

1. I know that six men are capable of performing a certain given portion of work in eight days, but I want the same performed in three: how many men must there be employed?

Set 6 upon C to 8 upon A, and against 3 upon C is 16 upon A.

2. The lever of a safety valve is 20 inches in length, and 5 inches between the fixed end and centre of the valve: what weight must there be placed on the end of the lever to equipoise a force or pressure of 40 lbs. tending to raise the valve?

Set 5 upon C to 40 upon A, and against 20 on C is 10 on A.

3. If 8¾ yards of cloth, 1½ yards in width, be a sufficient quantity, how much will be required of that which is only ⅞ths in width, to effect the same purpose?

Set 1·5 on C to 8·75 on A, and against 8·75 upon C is 15 yards upon A.

Square and Cube Roots of Numbers.

On the engineer's rule, when the lines C and D are equal at both ends, C is a table of squares, and D a table of roots, as—

Squares,	1	4	9	16	25	36	49	64	81	on C.
Roots,	1	2	3	4	5	6	7	8	9	on D.

To find the geometrical mean proportion between two numbers.

Set one of the numbers upon C to the same number upon D, and against the other number upon C is the mean number or side of an equal square upon D.

Required the mean proportion between 20 and 45.

Set 20 upon C to 20 upon D, and against 45 upon C is 30 on D.

To cube any number, set the number upon C to 1 or 10 upon D, and against the same number upon D is the cube number upon C.

Required the cube of 4.

Set 4 upon C to 1 or 10 upon D, and against 4 upon D is 64 upon C.

To extract the cube root of any number, invert the slide, and set the number upon B to 1 or 10 upon D, and where two numbers of equal value coincide, on the lines B D, is the root of the given number.

Required the cube root of 64.

Set 64 upon B to 1 or 10 upon D, and against 4 upon B is 64 upon D, or root of the given number.

On the common rule, when 1 in the middle of the line C is set opposite to 10 on D, then C is a table of squares, and D a table of roots.

To cube any number by this rule, set the number upon C to 10 upon D, and against the same number upon D is the cube upon C.

Mensuration of Surface.

1. *Squares, Rectangles, &c.*

Rule. When the length is given in feet, and the breadth in inches, set the breadth on B to 12 on A; and against the length on A is the content in square feet on B.

If the dimensions are all inches, set the breadth on B to 144 upon A; and against the length upon A is the number of square feet on B.

Required the content of a board 15 inches broad and 14 feet long.

Set 15 upon B to 12 upon A; and against 14 upon A is 17·5 square feet on B.

2. *Circles, Polygons, &c.*

Rule. Set ·7854 upon C to 1 or 10 upon D · then will the lines C and D be a table of areas and diameters.

Areas,	3·14	7·06	12·56	19·63	28·27	38·48	50·26	63·61	upon C.
Diameters,	2	3	4	5	6	7	8	9	upon D.

In the common rule, set ·7854 on C to 10 on D; then C is a line or table of areas, and D of diameters, as before.

Set 7 upon B to 22 upon A; then B and A form or become a table of diameters and circumferences of circles.

Circumferences,	3·14	6·28	9·42	12·56	15·7	18·85	22	25·13	28·27	upon A.
Diameters,	1	2	3	4	5	6	7	8	9	upon B.

Polygons from 3 *to* 12 *sides.* Set the gauge-point upon C to 1 or 10 upon D; and against the length of one side upon D is the area upon C.

Sides,	3	5	6	7	8	9	10	11	12.
Gauge-points,	·433	1·7	2·6	3·63	4·82	6·18	7·69	9·87	11·17.

Required the area of an equilateral triangle, each side 12 inches in length.

Set ·433 upon C to 1 upon D; and against 12 upon D are 62·5 square inches upon C.

TABLE OF GAUGE-POINTS FOR THE ENGINEER'S RULE.

Names.	F, F, F.	F, I, I.	I, I, I.	F, I.	I, I.	F.	I.
Cubic inches, - -	578	83	1728	106	1273	105	121
Cubic feet, - - -	1	144	1	1833	22	121	83
Imperial gallons, -	163	231	277	294	353	306	529
Water in lbs., - -	16	23	276	293	352	305	528
Gold " - -	814	1175	141	149	178	155	269
Silver " - -	15	216	261	276	334	286	5
Mercury " - -	118	169	203	216	258	225	389
Brass " - -	193	177	338	354	424	369	637
Copper " - -	18	26	319	331	397	345	596
Lead " - -	141	203	243	258	31	27	465
Wro't iron " - -	207	297	357	338	453	394	682
Cast " " - -	222	32	384	407	489	424	733
Tin " - -	219	315	378	401	481	419	728
Steel " - -	202	292	352	372	448	385	671
Coal " - -	127	183	22	33	28	242	42
Marble " - -	591	85	102	116	13	118	195
Freestone " - -	632	915	11	1162	14	141	21

FOR THE COMMON SLIDE RULE.

Names.	F, F, F	F, I, I.	I, I, I.	F, I.	I, I.	F.	I.
Cubic inches, - -	36	518	624	660	799	625	118
Cubic feet, - - -	625	9	108	114	138	119	206
Water in lbs., - -	10	144	174	184	22	191	329
Gold " - -	507	735	88	96	118	939	180
Silver " - -	938	136	157	173	208	173	354
Mercury " - -	738	122	127	132	162	141	242
Brass " - -	12	174	207	221	265	23	397
Copper " - -	112	163	196	207	247	214	371
Lead " - -	880	126	152	162	194	169	289
Wro't iron " - -	129	186	222	235	283	247	423
Cast " " - -	139	2	241	254	304	265	458
Tin " - -	137	135	235	25	300	261	454
Steel " - -	136	183	22	233	278	239	418
Coal " - -	795	114	138	146	176	151	262
Marble " - -	370	53	637	725	81	72	121
Freestone " - -	394	57	69	728	873	755	132

Mensuration of Solidity and Capacity.

General Rule. Set the length upon B to the gauge-point upon A; and against the side of the square, or diameter on D, are the cubic contents, or weight in lbs. on C.

1. Required the cubic contents of a tree 30 feet in length, and 10 inches quarter girt.

Set 20 upon B to 144 (the gauge-point) upon A; and against 10 upon D is 20·75 feet upon C.

2. In a cylinder 9 inches in length and 7 inches diameter, how many cubic inches?

Set 9 upon B to 1273 (the gauge-point) upon A; and against 7 on D is 346 inches on C.

3. What is the weight of a bar of cast iron 3 inches square, and 6 feet long?

Set 6 upon B to 32 (the gauge-point) upon A; and against 3 upon D is 168 lbs. upon C.

By the common rule.

4. Required the weight of a cylinder of wrought iron 10 inches long, and 5½ diameter.

Set 10 upon B to 283 (the gauge-point) upon A; and against 5½ upon D is 66·65 lbs. on C.

5. What is the weight of a dry rope 25 yards long, and 4 inches circumference?

Set 25 upon B to 47 (the gauge-point) upon A; and against 4 on D is 53·16 lbs. on C.

6. What is the weight of a short linked chain 30 yards in length, and $\frac{6}{16}$ths of an inch in diameter?

Set 30 upon B to 52 (the gauge-point) upon A; and against 6 on D is 123·5 lbs. on C.

Land Surveying.

If the dimensions taken are in chains, the gauge-point is 1 or 10; if in perches, 160; and if in yards, 4840.

Rule. Set the length upon B to the gauge-point on A; and against the breadth upon A is the content in acres upon B.

1. Required the number of acres or contents of a field 20 chains 50 links in length, and 4 chains 40 links in breadth.

Set 20·5 on B to 1 on A; and against 4·4 on A is 9 acres on B.

2. In a piece of ground 440 yards long, and 44 broad, how many acres?

Set 440 upon B to 4840 on A; and against 44 on A is 4 acres on B.

Power of Steam-Engines.

Condensing Engines—Rule. Set 3·5 on C to 10 on D; then D is a line of diameters for cylinders, and C the corresponding number of horse power; thus,

Horse power, .	3½	4	5	6	8	10	12	16	20	25	30	40	50	on C.
C. D.	10 in.	10¾	12	13⅛	15⅛	17	18½	21⅜	24	26¾	29¼	33¾	37¾	on D.

The same is effected on the common rule by setting 5 on C to 12 on D.

Non-condensing Engines.—Rule. Set the pressure of steam in lbs. per square inch on B to 4 upon A; and against the cylinder's diameter on D is the number of horse power upon C.

Required the power of an engine, when the cylinder is 20 inches diameter, and steam 30 lbs. per square inch.

Set 30 on B to 4 on A; and against 20 on D is 30 horse power on C.

The same is effected on the common rule by setting the force of the steam on B to 250 on A.

Of Engine Boilers.

How many superficial feet are contained in a boiler 23 feet in length and $5\frac{1}{2}$ in width?

Set 1 upon B to 23 upon A; and against 5·5 upon B is 126·5 square feet upon A.

If 5 square feet of boiler surface be sufficient for each horse power, how many horse power of engine is the boiler equal to?

Set 5 upon B to 126·5 upon A; and against 1 upon B is 25·5 upon A.

The Laws of Motion.

If M = mass of a material body,
And W = the weight of it.

$$\therefore W = M \times 32{\cdot}19;$$

Or the mass of a body is equal to its weight divided by 32·19.

EXAMPLE. Find the weight of a body whose mass is $3\frac{1}{2}$:

$$\therefore W = 3{\cdot}5 \times 32{\cdot}19 = 112{\cdot}66 \text{ lbs.}$$

The gravity of a material body is its weight. Falling bodies fall through the same space in the same time, whatever may be their weight. A body one ton will fall to the ground no faster than a body one pound.

The velocity of a body is the number of feet passed over in one second.

Put v = the velocity of a falling body, at the end of t seconds,

$$\therefore v = 32{\cdot}19 \times t.$$

The quantity 32 19 is the velocity of a falling body at the end of one second.

Rule, to find the Velocity of a Falling Body at the end of any Number of Seconds.

Multiply the number of seconds by 32·19, the product will be the velocity.

EXAMPLE. Find the velocity of a body falling from a height in nine seconds:

$$\text{Velocity} = 32{\cdot}19 \times 9 = 289{\cdot}71.$$

Put s for the number of feet a falling body falls through in t seconds:

$$\therefore s = \frac{32{\cdot}19\, t^2}{2}.$$

Rule to find the Space passed over by a Falling Body in any Number of Seconds.

Square the number of seconds, and multiply the result by 16·09, the product will be the distance passed over in feet.

EXAMPLE. A stone fell from the top of a chimney to the bottom in four seconds; find the height of the chimney:

Height of chimney $= 16{\cdot}09 \times 16 = 257{\cdot}44$ feet.

$$s = \frac{v^2}{64{\cdot}39}, \text{ where } v \text{ is the velocity.}$$

Rule to find the Space passed over by a Falling Body when the Velocity is given.

Square the velocity, and divide by 64·39; the quotient will be the number of feet passed over.

The quantity 32·19 is frequently called the accelerating force of gravity, and is denoted by f. The following formulæ include all cases that can occur in falling bodies.

$$s = \text{space passed over} = \frac{f t^2}{2} = \frac{t v}{2} = \frac{v^2}{2 f};$$

$$v = \text{velocity at the end of } (t) \text{ seconds} = f t = \frac{2 s}{t} = \sqrt{2 f s};$$

$$t = \text{time} = \frac{v}{f} = \frac{2 s}{v} = \sqrt{\frac{2 s}{f}};$$

$$f = \frac{v}{t} = \frac{v^2}{2 s} = \frac{2 s}{t^2}.$$

The above formulæ and rules are applicable only to the case when the body is acted upon by the force of gravity.

Rules and Formulæ when a body is acted on by any force.

Put M = mass acted on by a force of F pounds.

a = velocity at the end of a second, which is called accelerating force.

s = space passed over in (t) seconds, producing a velocity (v).

$$\therefore a = \frac{F}{M} = \frac{v}{t};$$

$$\text{And } 2s = \frac{Ft^2}{M} = \frac{Mv^2}{F}$$

Rule for finding the accelerating force of a body.

Divide the force by the mass (remembering that mass is equal t weight divided by 32·19) or the velocity by the time, eithe quotient will give the accelerating force.

EXAMPLE. A force of 25 lbs. acts on a body whose weight is 84 lbs. Find the accelerating force.

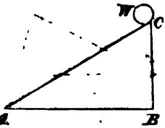
$$\text{The mass} = \frac{84}{32 \cdot 19} = 2 \cdot 6 \text{ nearly};$$

$$\therefore a = \frac{25}{2 \cdot 6} = 9 \cdot 62 \text{ nearly.}$$

The velocity at the end of 10 seconds $= 9 \cdot 62 \times 10 = 96 \cdot 2$.

Time of a Body falling down an Inclined Plane.

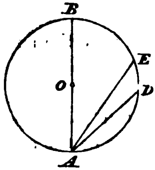
Let $A\,B\,C$ be an inclined plane, $B\,C$ perpendicular, and $A\,B$ parallel to the horizon.



The velocity at A in falling down $A\,C$ is the same as it would be in falling perpendicularly down the height $B\,C$.

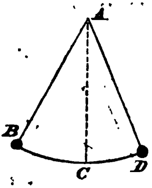
Put t = time in falling from C to A.
$l = A\,C$ the length of the inclined plane.
$h = B\,C$ the height of ditto.

$$\therefore t = \sqrt{\frac{2\,l^2}{f\,h}}.$$



Let $A\,D\,E\,B$ be a circle whose diameter $A\,B$ is perpendicular to the horizon. The times of a body falling down any chords $A\,D$, $A\,E$ are equal, and equal to the time in falling vertically through $A\,B$.

The Time of Oscillation of a Simple Pendulum.



Let $A\,B$ the length of the pendulum $= l$,
And $\pi = 3 \cdot 14159$, &c.; $g = 32 \cdot 19$
T = time in seconds oscillating from the point B to D.

The arc $\overset{\frown}{B\,C} = \overset{\frown}{C\,D}$ is small.

$$\therefore T = \pi\left(\frac{l}{g}\right)^{\frac{1}{2}}$$

Rule to find the Time of one Oscillation of a Simple Pendulum.

Divide the length of the pendulum by 32·19; extract the square root of this quotient, and multiply the result by 3·1416, and the product will be the time of oscillation in seconds.

If L be the length of a pendulum which oscillates in one second,

$$\therefore T = \left(\frac{l}{L}\right)^{\frac{1}{2}}.$$

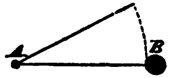
The value of L for the latitude of London is 39·1386 inches. A pendulum $9\frac{25}{32}$, $4\frac{25}{72}$, $2\frac{57}{128}$ inches long, will oscillate in a half, a third, a quarter seconds respectively.

If n be the number of oscillations made by a pendulum in one hour, then

$$l = 3600^2 \times \frac{L}{n^2}$$

The time of oscillation is not dependent on the weight of the bob.

Centrifugal Force.



Let the weight W, placed at B, be connected with a cord, or wire, with the fixed point A round which it revolves with a uniform velocity.

Put V = velocity of rotation.

$r = A\,B$, the length of the cord in feet.

F = centrifugal force, or the force which is exerted to break the cord in the direction of its length.

$$\therefore F = \frac{W\,V^2}{32{\cdot}19 \times r}.$$

If n be the number of revolutions in one minute,

$$\therefore F = \frac{331}{1000000} \times W\,r\,n^2.$$

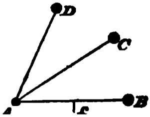
If W be measured in tons, then F will be in tons also.

If w be the angular velocity,

$$\therefore F = \frac{W\,r\,w^2}{g}$$

If T be the time of the weight making a complete revolution,

$$\therefore w = \text{angular velocity} = \frac{2\,\pi}{T} = \frac{V}{r}.$$



If there be several bodies at B, C, D, and revolving round the axis passing through A, and perpendicular to the plane $A\,D\,B\,C$,

$$\therefore F = \frac{w^2}{g}\left\{ r^1 W^1 + W^2 + r^3 W^3 + \&c. \right\}$$

Where w = angular velocity, W^1; W^2; W^3, &c.: the weights at $B\ C\ D$, &c., and r^1, r^2, r^3, &c., the distances $A\ B$, $A\ C$, $A\ D$, &c.

EXAMPLE. Let the weights at B and C be 80 and 90 lbs. respectively, revolving at a distance $A\ B = 8$ feet, $A\ C = 12$ feet, with a velocity making 40 revolutions per minute. Find the centrifugal force, or the pressure on the axis passing through A.

$$w = \frac{2\pi \times 40}{60} = \frac{4\pi}{3};$$

$$\therefore F = \frac{16\pi^2}{9}\left\{ 8 \times 80 + 12 \times 90 \right\} = 30178 \text{ lbs.}$$

The moment of inertia.

If $(W_1 + W_2 + W_3 + \&c.)\,k^2 = W_1 r_1 + W_2 r_2^2 + W_3 r_3^2 + \&c.$

Each side of this equation is called the moment of inertia, and the distance k is called the radius of gyration of the revolving system.

Let a constant force F act at a distance $A f = a$ from the axis of motion.

The angular velocity at the end of a second

$$= \frac{g F a}{(W_1 + W_2 + W_3 + \&c.)\,k^2}.$$

The angular velocity at the end of one revolution

$$= \frac{2\sqrt{g F a \pi}}{\sqrt{W_1 + W_2 + W_3 + \&c.} \times k}.$$

If a point O be determined from the equation

$$A\ O = \frac{k^2}{A\ G},$$

where G is the centre of gravity of the system, then O is called the centre of oscillation.

The values of k in Geometrical Solids.

A rectangular parallelopipedon revolving about an axis passing through its centre of gravity, and parallel to either of its edges.

$$k^2 = \frac{b^2 + c^2}{12},$$

where $b\,c$ are the length and breadth at right angles to the axis of revolution.

An upright triangular prism about a vertical axis passing through its centre of gravity.

$$k^2 = \frac{a^2}{48} + \frac{c^2}{36}.$$

The section of the prism perpendicular to the revolving axis is an isosceles triangle; the base being denoted by (a), and the perpendicular upon it from the angle contained by the equal sides by (c).

In a cylinder, whose radius is (r), revolving about its axis,

$$k^2 = \frac{r^2}{2}.$$

In a hollow cylinder, whose internal and external radii are a and b respectively, revolving about its axis,

$$k^2 = \frac{a^2 + b^2}{2}.$$

In a cylinder, whose radius is r and length l, revolving round a line at right angles to its axis, and passing through its middle,

$$k^2 = \frac{l^2}{12} + \frac{r^2}{4}.$$

In a sphere, whose radius is r, revolving about its diameter,

$$k^2 = \frac{2\,r^2}{5}.$$

In a hollow sphere, whose internal and external radii are (a) and (b) respectively, revolving about its diameter,

$$k^2 = \frac{2\,(b^5 - a^5)}{5\,(b^3 - a^3)}.$$

In a cone, whose base is a circle, radius r,

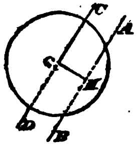
$$k^2 = \frac{3\,r^2}{10}.$$

In a cone, whose radius of base is r and height h, revolving about a line at right angles to its axis, and passing through its centre of gravity,

$$k^2 = \frac{3\,(4\,r^2 + h^2)}{80}.$$

The square of the radius of gyration about any line in a revolving system, is equal to the square of the radius of gyration about a line parallel to it passing through the centre of gravity and the square of the distance from the centre of gravity to the line about which the system revolves.

Let G be the centre of gravity of any body; draw $A\,B$ any line about which the system revolves. Let $C\,D$ be parallel to $A\,B$, and draw $G\,H$ perpendicular to $A\,B$.



Let $K =$ radius of gyration when revolving about $A\,B$.

$k =$ radius of gyration when revolving about $C\,D$.

$\therefore K^2 = k^2 + G\,H^2$.

This important theorem will readily extend the theorems which are given above to most practical cases.

The Centre of Gyration

is that part of a body revolving about an axis, into which, if the whole quantity of matter were collected, the same moving force would generate the same angular velocity.

To find the centre of Gyration, multiply the weight of the several particles by the squares of their distances from the centre of motion, and divide the sum of the products by the weight of the whole mass; the square root of the quotient will be the distance of the centre of gyration, from the centre of motion.

The distances of the centre of gyration from the centre of motion, of different revolving bodies, are as follows:

In a straight rod revolving about one end, the length × ·5773.

In a circular plate, revolving on its centre, the radius × ·7071.

In a circular plate, revolving about one diameter, the radius × ·5.

In a thin circular ring, revolving about one diameter, radius × ·7071.

In a solid sphere, revolving about one diameter, the radius × ·6325.

In a thin hollow sphere, revolving about one diameter, the radius × ·8164.

In a cone, revolving about its axis, the radius of the base × ·5477.

In a right-angled cone, revolving about its vertex, the height × ·836.

In a paraboloid, revolving about its axis, the radius of the base × ·5773.

The Centre of Percussion

is that point in a body revolving about a fixed axis, into which the whole of the force or motion is collected.

It is, therefore, that point of a revolving body which would strike any obstacle with the greatest effect; and, from this property, it has received the name of the centre of percussion.

The centres of oscillation and percussion are in the same point.

If a heavy straight bar, of uniform density, be suspended at one extremity, the distance of its centre of percussion is two-thirds of its length.

In a long slender rod of a cylindrical or prismatic shape, the centre of percussion is nearly two-thirds of the length from the axis of suspension.

In an isosceles triangle, suspended by its apex, the distance of the centre of percussion is three-fourths of its altitude. In a line or rod whose density varies as the distance from the point of suspension, also in a fly-wheel, and in wheels in general, the centre of percussion is distant from the centre of suspension three-fourths of the length.

In a very slender cone or pyramid, vibrating about its apex, the distance of its centre of percussion is nearly four-fifths of its length.

On Work.

A unit of work is one pound avoirdupois raised vertically one foot.

If U denotes the units of work in raising W lbs. h feet—

$$\therefore U = Wh.$$

Rule to find the Units of Work in Raising a given Weight a given Height.

Multiply the height in feet by the weight in pounds, the product will be the units of work done.

Example. Find the units of work in raising half a ton 30 feet high.

$$\therefore U = 1120 \times 30 = 33600 \text{ units of work.}$$

It is important to observe, in the application of the above formula to practical cases, that the height (h) is the vertical distance through which the centre of gravity of the body whose weight is (W) is raised.

Example. Find the units of work in lowering the surface of water in a well one yard; the depth to the surface of water being 40, and diameter 3 feet.

The weight of a cubic foot of water is 62½ lbs.

The weight of water $= 9 \times \cdot 7854 \times 3 \times 62 \cdot 5 = 1325 \cdot 36$ lbs.

The height through which the centre of gravity is raised $= 41 \cdot 5$ feet.

$$\therefore U = 1325 \cdot 36 \times 41 \cdot 5 = 55002 \text{ units of work.}$$

The work done in raising a body up an inclined plane, or any curved surface, is equal to the work done in raising the body vertically through the height of the inclined plane.

There are 29000 units of work done in sawing a square foot of green oak.

Horse Power.

A *horse power* is 33000 units of work done in one minute.

Put H, equal to the horse power, and U, the units of work done in T hours:

$$\therefore 33000\,H = \frac{U}{60\,T}.$$

The following results are taken from MORIN:

A Man laboring Eight Hours per Day will perform the following Units of Work.

Raising his own body,	4250
Drawing, or pushing horizontally,	3120
Pushing and drawing alternately in a vertical direction,	2380
Turning a handle,	2600
Working with his arms and legs, as in rowing,	4000

A Man laboring Six Hours per Day.

Raising material with a pulley,	1560
Raising material with the hands,	1470
Raising material upon the back, and returning empty,	1126

A Man laboring Ten Hours per Day.

Raising material with a wheelbarrow on ramps,	720
Throwing earth to the height of five feet,	470

Useful Work of a Man raising Water—Duration of Labor, Eight Hours per Day.

With a windlass from deep wells,	2560
With an upright chain pump,	1730
With a Chinese wheel,	2167
With an Archimedean screw,	1505
Raising water from a well with a pail and rope,	1054

Work of Animals.

A horse, in a common pumping engine,	17550
A mule, ditto,	11700
An ass, ditto,	3510

EXAMPLE. Required the horse power of an engine that will saw 368 planks, each being 30 feet by 2 feet 6 inches, in twelve hours.

There are 29000 units of work done in sawing one square foot;

Then $30 \times 2{\cdot}5 \times 368 \times 29000 =$ units of work done in sawing the planks.

Put $x =$ the horse power of the engine;

Then $60 \times 12 \times 33000 \times x =$ units of work done by the engine in twelve hours.

$$\text{Hence, } x = \frac{30 \times 2{\cdot}5 \times 368 \times 29000}{60 \times 12 \times 33000} = 33{\cdot}7 \text{ horse power.}$$

EXAMPLE. How many tons of coals would two men raise, working with a wheel and axle, from a pit whose depth is 20 yards, in 12 hours?

From the Table, a man working with a wheel and axle will do 2600 units of work in one minute.

Then, $2600 \times 60 \times 12 \times 2 =$ work done by the two men.

Put $x =$ the tons of coals raised.

Then, $2240 \times 20 \times 3 \times x =$ work done by the two men.

$$\therefore x = \frac{2600 \times 60 \times 12 \times 2}{2240 \times 20 \times 3} = 27{\cdot}85 \text{ tons raised.}$$

The Traction of Horses at various rates of Travelling.

It is a well known fact, that the traction or force which a horse can exert decreases with the increase of speed.

Rate in miles per hour,	2	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5
Force exerted by the horse,	166 lbs.	125	104,	83,	$62\frac{1}{2}$,	$41\frac{2}{3}$.

Accumulated Work.

If a force be applied to move a body subject to no resistance whatever, it will be wholly occupied in increasing the speed of the body. In this case the work which is done by the action of the force applied is accumulated in the body, therefore it is called accumulated work.

Put $V =$ the velocity of the body or feet per second.

And $W =$ the weight of the body in pounds.

$$\text{Accumulated work} = \frac{W V^2}{64}.$$

If W be measured in tons, and V be measured in miles per hour,

$$\text{Accumulated work} = \frac{3388}{45} W V^2$$

A railway train 80 tons moves uniformly at the rate of 30 miles per hour, find the accumulated work.

$$\text{Accumulated work} = \frac{3388 \times 80 \times 900}{45} = 5420800.$$

$$\text{The horse power of the engine} = \frac{5420800}{33000} = 164 \text{ nearly.}$$

Generally the horse power of the engine $= \dfrac{77\, W V^2}{33750}$ where W is in tons and V in miles per hour.

The friction of a railway train is from 8 to 10 lbs. per ton.

Work done by Machines.

The moving power, which is applied to any machine moving uniformly, is employed in overcoming the resistance of friction, and useful work done at the working points of the machine. Hence,

the aggregate number of units of useful work yielded by any machine at its working point is less than the number received upon the machine directly from the moving power, by the number of units expended upon the resistance of friction. (The machine moving uniformly.)

General Rule to find the Work done by any Machine.

Find the distance through which the power (P) applied to the machine has travelled in one minute, and let this distance be called (a).

Find the distance through which the weight (W), producing useful work, has travelled in one minute, and let this distance be (b).

Then $a\,P - b\,W =$ work done by friction per minute.

And $a\,P =$ work applied per minute.

$b\,W =$ useful work done per minute.

The Horse Power of an Engine.

Let P be the mean effective pressure of the steam on the piston.

l be the length of the stroke in feet.

n be the number of strokes per minute.

$$\therefore \text{Horse power of the engine} = \frac{n\,l\,P}{33000}.$$

The nominal horse power $= \dfrac{7\,n\,l}{33000}$ as adopted by the Admiralty.

On the Strength of Animals.

Let P be the force in lbs. that any animal can exert when moving at (v) miles per hour.

Put $K =$ the greatest effort the animal can exert when standing.

And $c =$ the greatest number of miles per hour the animal can give itself when unimpeded by any weight.

According to Bouguer, $P = \left(1 - \dfrac{v}{c}\right) . K.$

" Euler, $P = \left(1 - \dfrac{v^2}{c^2}\right) . K.$

" Euler, $P = \left(1 - \dfrac{v}{c}\right)^2 . K.$

It is readily seen that (v) miles per hour is equal to (88 v) feet per minute. Put U the units of work done by the animal per minute,

Then, according to Bouguer, $U = 88\left(v - \dfrac{v^2}{c}\right) . K.$

According to Euler, $U = 88\left(v - \frac{v^3}{c^2}\right) . K.$

" Euler, $U = 88v\left(1 - \frac{v}{c}\right)^2$

The values of U will be the greatest when

$v = \frac{c}{2}$. According to Bouguer.

$v = \frac{c}{\sqrt{3}}$. " Euler.

$v = \frac{c}{3}$. " Euler.

Substitute these values in the formula for P and U, then there will result:

$\frac{K}{2}$ = the load of the animal when producing the greatest effect.

$\frac{2K}{3}$ = " " "

$\frac{4K}{9}$ = " " "

$22\,c\,K$ = the greatest effect, by first formula.

$\frac{176\,c\,K}{3\sqrt{3}}$ = " by second formula.

$\frac{352\,c\,k}{27}$ = " by third formula.

To Calculate the Different Parts of a Crane as respects Mechanical Advantage.

(1.) *The number of revolutions of the pinion to one of the wheel, the length of the handle, and the force applied being given, to find the diameter of the barrel.*

RULE. Multiply the diameter of the circle described by the winch, or handle, in inches, by the power applied in lbs., and by the number of revolutions of the pinion to one of the wheel; divide this product by the weight to be raised in lbs., and the quotient is the diameter of the barrel in inches.

(2.) *The diameter of the barrel, the length of the handle, and the force applied given, to find the number of revolutions of the pinion to one of the wheel.*

RULE. Multiply the weight to be raised in lbs. by the diameter of the barrel in inches, and divide the product by the diameter of the circle described by the handle in inches, multiplied by the power applied in lbs., and the quotient is the revolutions of the pinion to one of the wheel.

(3.) *The diameter of the barrel, the number of revolutions of the pinion to one of the wheel, and the power applied given, to find the length of the handles.*

RULE. Multiply the weight to be raised in lbs. by the barrel's diameter in inches, and divide the product by the power applied in lbs., multiplied by the number of revolutions of the pinion to one of the wheel, and half the quotient is the length of the handles.

(4.) *The diameter of the barrel, the revolutions of the pinion to one of the wheel, and length of handles given, to find the power required.*

RULE. Multiply the weight to be raised in lbs. by the diameter of the barrel in inches, and divide the product by the diameter of the circle described by the handle multiplied by the revolutions of the pinion to one of the wheel, and the quotient is the power applied.

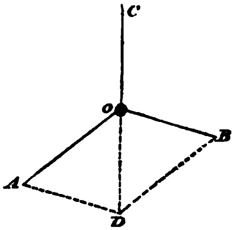
The handles of a crane should not be less than 2 *feet* 11 *inches or* 3 *feet from the ground, and the jib to stand at an angle of about* 45 *degrees.*

Equilibrium and Pressure of Beams.

The Parallelogram of Forces.

It has been proved by experiment that three forces, proportional to the two sides of a parallelogram and its diagonal, are in a state of equilibrium when their directions are in the direction of these lines.

Let two forces, represented in direction and magnitude by the lines $A\,O$ and $B\,O$, act at the point O, then a third force $C\,O$ in direction and magnitude can be found, so that the three forces are in a state of equilibrium.

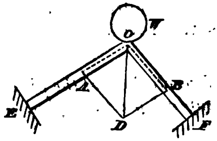


Draw $A\,D$, $B\,D$, parallel to $O\,B$, $O\,A$, respectively; join $D\,O$, and produce it to C, making $C\,O$ equal to $O\,D$, then $O\,C$ is the force required.

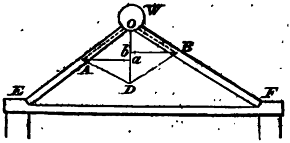
The two forces $A\,O$, $B\,O$ are called *components*, and $C\,O$ the *resultant* of the

components. The components and resultant are called the parallelogram of forces.

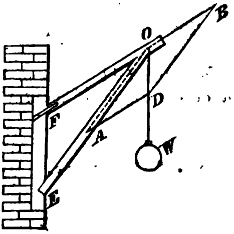
Any resultant force can be readily decomposed into two components, which will be the sides of a parallelogram whose diagonal is the resultant.



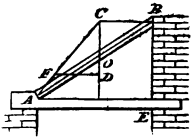
Let the beams *O E*, *O B* sustain a weight (*W*) tons at the point *O*; draw *O D* vertical, and make it equal to (*W*) inches, then draw *D A*, *D B* parallel to *O F* and *O E* respectively; measure *D A*, *D B* in inches which will be the pressure in tons in the directions *O F* and *O E*.



In this case *E F* is a tie beam to prevent the lower ends of the beams *O E*, *O F* from spreading. Draw *O D* vertically equal to (*W*) inches, then draw *D A*, *D B* parallel to *O F*, *O E* and *a A*, *b B*, parallel to *E F*, then *A D* will be the thrust in *O F*, and *D B* in *O E*, and *A a* equal to *b B* will be the thrust in the direction of the tie beam *E F*.



Draw *O D* vertically equal to (*W*) inches, and draw *D A* parallel to *O F*, and *D B* parallel to *O E*, then *O B*, *O A* will represent the pressures in the directions *O F*, *O E*.



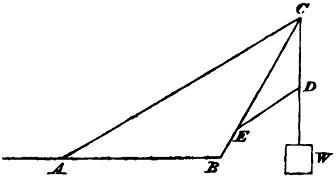
Let *A B* be a beam whose centre of gravity is *O*, and resting against an upright wall *B E*, the lower end resting on an abutment cut in the beam *A E* at *A*.

Through the centre of gravity *O* draw the line *O D* vertically equal to the weight of the beam, draw *B C*, *D F* parallel to *E A*, join *C A*; then *C F* represents the thrust at *A* in the direction *C F*, and *F D* represents the thrust at *B*, and also the horizontal thrust at *A*.

To Compute the Tension of the 'guise' and Shear-leg of a pair of Shears.

Let *B C* be the shear-leg and *A C* the guise, and (*W*) weight in tons supported at *C*.

Make C as many inches as (W) contains tons, draw DE parallel to AC, then DE measured in inches will be the tension in tons of the *guise* AC, and CE measured in inches will be the pressure in the direction of the shear-leg CE.



To Compute the Tension on the guise arithmetically.

Put $AB = c$, $BC = a$, and $AC = b$.

Then, tension in $AC = \dfrac{b\,(b^2 - a^2 - c^2)\,W}{c\,\sqrt{(a+b+c)\,(b+c-a)\,(a+c-b)\,(a+b-c)}}$

And the pressure in $CB = \dfrac{a\,(b^2+c^2-a^2)\,W}{2c\,\sqrt{(a+b+c)(a+b-c)(b+c-a)(a+c-b)}}$

SPECIFIC GRAVITY.

THE comparative density of various substances, expressed by the term *Specific Gravity*, affords the means of readily determining the bulk from the known weight, or the weight from the known bulk; and this will be found more especially useful, in cases where the substance is too large to admit of being weighed, or too irregular in shape to allow of correct measurement. The standard with which all solids and liquids are thus compared, is that of distilled water, one cubic foot of which weighs 1000 ounces avoirdupois; and the specific gravity of a *solid* body is determined by the difference between its weight in the air and in water. Thus,

If the body be *heavier* than water, it will displace a quantity of fluid equal to it in *bulk*, and will lose as much weight on immersion as that of an equal bulk of the fluid. Let it be weighed first, therefore, in the air, and then in water, and its weight in the air be divided by the difference between the two weights, and the quotient will be its specific gravity, that of water being unity.

Example. A piece of copper ore weighs $56\frac{1}{4}$ ounces in the air, and $43\frac{3}{4}$ ounces in water: required its specific gravity.

$56{\cdot}25 - 43{\cdot}75 = 12{\cdot}5$ and $56{\cdot}25 \div 12{\cdot}5 = 4{\cdot}5$, the specific gravity.

If the body be *lighter* than water it will float, and displace a quantity of fluid equal to it in *weight*, the bulk of which will be equal to that only of the part immersed. A heavier substance

must therefore be attached to it, so that the two may sink in the fluid. Then, the weight of the lighter substance in the air must be added to that of the heavier substance in water, and the weight of both united, in water, be subtracted from the sum; the weight of the lighter body in the air must then be divided by the difference, and the quotient will be the specific gravity of the lighter substance required.

Example. A piece of fir weighs 40 ounces in the air, and, being immersed in water attached to a piece of iron weighing 30 ounces, the *two* together are found to weigh 3·3 ounces in water, and the iron alone 25·8 ounces in the water: required the specific gravity of the wood.

$40 + 25{\cdot}8 = 65{\cdot}8 - 3{\cdot}3 = 62{\cdot}5$; and $40 \div 62{\cdot}5 = 0{\cdot}64$, the specific gravity of the fir.

The specific gravity of a *fluid* may be determined by taking a solid body, heavy enough to sink in the fluid, and of known specific gravity, and weighing it both in the air and in the fluid. The difference between the two weights must be multiplied by the specific gravity of the solid body, and the product divided by the weight of the solid in the air; the quotient will be the specific gravity of the fluid, that of water being unity.

Example. Required the specific gravity of a given mixture of muriatic acid and water; a piece of glass, the specific gravity of which is 3·, weighing 3¾ ounces when immersed in it, and 6 ounces in the air.

$6 - 3{\cdot}75 = 2{\cdot}25 \times 3 = 6{\cdot}75 \div 6 = 1{\cdot}125$, the specific gravity.

Since the weight of a cubic foot of distilled water, at the temperature of 60 degrees (Fahrenheit), has been ascertained to be 1000 avoirdupois ounces, it follows that the specific gravities of all bodies compared with it, may be made to express the weight, in ounces, of a cubic foot of each, by multiplying these specific gravities (compared with that of water as unity) by 1000. Thus, that of water being 1, and that of silver, as compared with it, being 10·474, the multiplication of each by 1000 will give 1000 ounces for the cubic foot of water, and 10474 ounces for the cubic foot of silver.

TABLE OF SPECIFIC GRAVITIES—WATER = 1000.

Metals.		Mercury,	13·586	Crown glass,	2·488
Antimony,	6·712	***Organic Bodies.***		Flint glass,	3·329
Zinc,	7·100	Oak wood,	0·925	Rock crystal,	2·653
Cast Iron,	7·207	Cork,	0·240	Diamonds,	3·501
Tin,	7·291	Ivory,	1·826	***Liquids.***	
Steel,	7·816	White wax,	0·960	Ether,	0·715
Cast copper,	8·788	***Inorganic Non-Metallic Bodies.***		Alcohol,	0·792
Bismuth,	9·882			Oil of turpentine,	0·870
Silver,	10·474			Sea water,	1·026
Lead,	11·352	Agate,	2·590	Milk,	1·030
Gold,	19·258	Amber,	1·078	Nitric acid,	1·506
Platinum,	20·337	Sulphur,	2·033	Sulphuric acid,	1·845

Weights of given bulks of water and air for calculating the absolute weights from the specific gravities of bodies.

			Logarithms.
Cubic inch of distilled water (bar. 30, therm. 62)	in grains	252·458	2·40219
..... foot	in ounces avoir.	997·1369691	2·99875
	in pounds do.	62·3210606	1·79463
Weight of 100 cubic in. of air	in grains do.	30·49	1·48416

THE MECHANICAL POWERS, AND THEIR APPLICATION.

The simple Mechanical Powers are six in number, viz. the *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw*. All machines are formed by combinations to a greater or less extent of these six elements. The mechanical effects, however, of the whole, are ultimately resolvable into that of the lever.

By means of the Mechanical Powers a great weight may be sustained, or a great resistance slowly overcome, by the application of a small force. Or, a great velocity may be imparted to a small weight or resistance, by the use of a great force or power.

The Lever.

Levers are of three orders:

In the first order, the fulcrum is between the weight and the power.

In the second order, the weight is between the fulcrum and the power

In the third order, the power is between the weight and the fulcrum.

The bent lever has no peculiarity except that of form, which is given to it for convenience in use. Its properties are those of the first order.

In order to preserve an equilibrium between the power and the weight, they must be to each other inversely as their distances from the fulcrum.

Case 1. *When the Lever is of the first order, or when the fulcrum is between the power and the weight.*

Rule. Divide the weight to be raised by the power to be applied;

the quotient will give the difference of leverage necessary to support the weight in equilibrio. Hence, a small addition either of leverage or weight will cause the power to preponderate.

EXAMPLE 1. A ball weighing 3 tons is to be raised by 4 men, who can exert a force of 12 cwt. Required the proportionate length of lever.

$$3 \text{ tons} = 60 \text{ cwt.}; \text{ and } \frac{60}{12} = 5.$$

In this example, the proportionate lengths of the lever to maintain the weight in equilibrio, are as 5 to 1. But, although the ball is sustained by a force of only one fifth of its weight, no power is gained, for the weight passes through only one fifth of the space passed through by the power.

EXAMPLE 2. A weight of 1 ton is to be raised with a lever 8 feet in length, by a man who can exert, for a short time, a force of rather more than 4 cwt. Required at what part of the lever the fulcrum must be placed.

$\frac{20 \text{ cwt.}}{4 \text{ cwt.}} = 5$; *i. e.*, the weight is to the power as 5 to 1; therefore,

$\frac{8}{5 \times 1} = 1$ foot and a third from the weight.

EXAMPLE 3. A weight of 40 lbs. is placed one foot from the fulcrum of a lever. Required the power to raise the same when the length of the lever on the other side of the fulcrum is five feet.

$$\frac{40 \times 1}{5} = 8 \text{ lbs., the power.}$$

CASE 2. *When the lever is of the second order, or when the fulcrum is at one end of the lever and the power at the other, with the weight between them.*

RULE. As the distance between the power and the fulcrum is to the distance between the weight and the fulcrum, so is the effect to the power.

EXAMPLE 1. Required the power necessary to raise 120 lbs. when the weight is placed six feet from the power and two feet from the fulcrum.

As 8 : 2 :: 120 : 30 lbs., the power.

EXAMPLE 2. A beam 20 feet in length, and supported at both ends, bears a weight of two tons at the distance of eight feet from one end. Required the weight on each support.

$\frac{40 \text{ cwt.} \times 8 \text{ feet}}{20 \text{ feet}} = 16$ cwt. on the support that is furthest from the weight; and $\frac{40 \times 12}{20 \text{ feet}} = 24$ cwt. on the support nearest to the weight.

Case 3. *When the lever is of the third order, or the weight is at one end of the lever, the fulcrum at the other, and the power is applied between them.*

RULE. As the distance between the power and the fulcrum is to the length of the lever, so is the weight to the power.

EXAMPLE. The length of the lever being eight feet, and the weight at its extremity 60 lbs., required the power to be applied six feet from the fulcrum to raise it.

As 6 : 8 :: 60 : 80 lbs., Ans.

The Pulley.

Pulleys are of two kinds, fixed and movable.

The fixed pulley affords no economy of power, but merely changes its direction. The movable pulley changes its position with that of the weight, and effects a saving equal to half the power. An equilibrium is preserved between the power and weight, when the weight is equal to the product of the power and twice the number of movable pulleys.

RULE. Divide the weight to be raised by twice the number of pulleys in the lower block; the quotient will give the power necessary to raise the weight.

EXAMPLE. Required the power to raise 600 lbs. when the lower block contains six pulleys.

$$\frac{600}{6 \times 2} = 50 \text{ lbs., the power.}$$

The Wheel and Axle.

The wheel and axle act as a revolving lever; and in order to obtain an equilibrium between the power acting on the circumference of the wheel, and the weight or resistance acted on by the circumference of the axle, the power must be to the weight as the radius of the axle is to that of the wheel. One or more radii of the wheel, or winches, are often substituted for the wheel in the simple machine; and in compound machines the action is communicated by teeth or cogs, forming wheel-and-pinion work.

RULE. As the radius of the wheel is to the radius of the axle, so is the effect to the power.

EXAMPLE. A weight of 50 lbs. is exerted on the periphery of a wheel whose radius is 10 feet. Required the weight raised at the extremity of a cord wound round the axle, the radius being 20 inches.

$$\frac{50 \text{ lbs.} \times 10 \text{ feet} \times 12 \text{ inches}}{20 \text{ inches}} = 300 \text{ lbs., the weight.}$$

The Inclined Plane.

The inclined plane acts as a mechanical power by sustaining a portion of the weight to be raised, while the direction of the applied force is changed from the perpendicular to one more or less horizontal, and the weight moves upwards on it in a diagonal between them. Equilibrium is sustained when the power is to the weight as the perpendicular height of the inclined plane is to its inclined length or hypothenuse, when the power acts in a direction parallel to the inclination of the plane; but as the height is to the base when in a direction parallel to the base.

RULE. As the length of the plane is to its height, so is the weight to the power.

EXAMPLE. Required the power necessary to raise 540 lbs. up an inclined plane 5 feet long and 2 feet high.

As 5 : 2 :: 540 : 216 lbs., the power.

The *length*, in the above rule, must represent that of the inclined surface, or of the base, accordingly as the power acts parallel to either of these surfaces.

The Wedge.

The wedge may be regarded as two inclined planes, united by a common base, acting on two weights or resistances at once, or on a fulcrum and a weight, between which it moves, generally, in practice, by the impulse of successive blows.

As in the inclined plane, equilibrium consists in the power being to the resistance as the back of the wedge is to its length, or to the length of its side, accordingly as the resistance acts perpendicularly to the central line of length or to that of the side.

Case 1. *When two bodies are forced from one another by means of a wedge, in a direction parallel to its back.*

RULE. As the length of the wedge is to half its back or head, so is the resistance to the power.

EXAMPLE. The breadth of the back or head of the wedge being 3 inches, and the length of either of its inclined sides 10 inches, required the power necessary to separate two substances with a force of 150 lbs.

As 10 : 1½ :: 150 : 22½ lbs., the power.

Case 2. *When only one of the bodies is movable.*

RULE. As the length of the wedge is to its back or head, so is the resistance to the power.

EXAMPLE. The breadth, length, and force, the same as in the last example.

As 10 : 3 :: 150 : 45 lbs., the power.

The Screw.

The screw is an inclined plane, and may be supposed to be generated by wrapping a triangle, or an inclined plane, round a cylinder. The base of the triangle is the circumference of the cylinder; its height, the distance between two consecutive cords or threads; and the hypothenuse forms the spiral cord or inclined plane.

RULE. To the square of the circumference of the screw, add the square of the distance between two threads, and extract the square root of the sum: this will give the length of the inclined plane. Its height is the distance between two consecutive cords or threads.

When a winch or lever is applied to turn the screw, the power of the screw is as the circle described by the handle of the winch, or lever, to the internal or distance between the spirals.

Case 1. *When the weight to be raised is given, to find the power.*

RULE. Multiply the weight by the distance between two threads of the screw, and divide the product by the circumference of the circle described by the lever. The quotient is the power.

EXAMPLE. Required the power to be applied to the end of a lever three feet long, to raise a weight of five tons with a screw of $1\frac{1}{4}$ inch between the threads.

$$\frac{11200 \text{ lbs.} \times 1{\cdot}25}{36 \text{ inches} \times 2 \times 3{\cdot}1416} = 61{\cdot}9 \text{ lbs., the power.}$$

Case 2. *When the power is given, to find the weight it will raise.*

RULE. Multiply the power by the circumference of the circle described by the lever, and divide the product by the distance between two threads of the screw: the quotient will be the weight. The example is the converse of that in the former case.

TO HARDEN AND POLISH ALABASTER.—1. Take a strong solution of alum, strain it, and put it into a wooden trough sufficiently large to contain the figure, which must be suspended in it by means of a thread of silk; let it rest until a sufficient quantity of the salt is crystallized on the cast, then withdraw it, and polish it with a clean cloth and water.

2. Take white wax, melt it in a convenient vessel, and dip the cast or figure into it; withdraw, and repeat the operation of dipping until the liquid wax rests upon the surface of the cast; then let it cool and dry, when it must be polished with a clean brush.

TOOTHED WHEELS.

The *pitch* (or the distance between the centres of two contiguous teeth) of cog-wheels is measured on the *pitch-line*, or extreme circumference of the wheel; and the distance between that line and the centre of the circle is reckoned as the *radius* of the wheel.

The following rules have been laid down for the diameters and number of teeth for wheels and pinions.

RULE 1.

As the number of teeth in the wheel + 2·25,
Is to the diameter of the wheel,
So is the number of teeth in the pinion + 1·5,
To the diameter of the pinion.

EXAMPLE. Given the number of teeth in the wheel = 210, the diameter of the wheel = 25 inches, and the number of teeth in the pinion = 30, to find the diameter of the pinion.

As 210 + 2·25 : 25 :: 30 + 1·5 : 3·7102, = the diameter of the pinion.

RULE 2.

As the number of teeth in the wheel + 2·25,
Is to the diameter of the wheel,
So is (No. of teeth in pinion + No. of teeth in wheel) ÷ 2,
To the distance of their centres.

EXAMPLE. Given the number of teeth in the wheel = 210, the diameter of the wheel = 25 inches, and the number of teeth in the pinion = 30, to find the distance at which their centres should be placed.

As 210 + 2·25 : 25 :: $\frac{30 \times 210}{2}$: 14·1842 inches, = the distance of their centres.

On the Velocity of Wheels, Drums, Pulleys, &c.

When wheels are applied to communicate motion from one part of a machine to another, their teeth act alternately on each other; consequently, if one wheel contains 60 teeth and another 20, the one containing 20 teeth will make three revolutions, while the other makes but one; and if drums or pulleys are taken in place of wheels, the result will be the same, because their circumferences, describing equal spaces, render their revolutions unequal; from this the rule is derived, namely,

Multiply the velocity of the driver by the number of teeth it contains, and divide by the velocity of the driven: the quotient will be the number of teeth it ought to contain. Or, multiply the velocity of the driver by its diameter, and divide by the velocity of the driven: the quotient will be the diameter of the driven.

If the velocities of driver and driven are given with the distance of their centres,

Then the sum of the velocities : $\begin{Bmatrix}\text{velocity of driver}\\ \text{velocity of driven}\end{Bmatrix}$: : distance of centres : $\begin{Bmatrix}\text{radius of driven.}\\ \text{radius of driver.}\end{Bmatrix}$

EXAMPLE 1. If a wheel that contains 75 teeth makes 16 revolutions per minute, required the number of teeth in another to work in it, and make 24 revolutions in the same time.

$$\text{Here } \frac{75 \times 16}{24} = 50 \text{ teeth.} = \textit{Ans.}$$

EXAMPLE 2. A wheel, 64 inches diameter, and making 42 revolutions per minute, is to give motion to a shaft at the rate of 77 revolutions in the same time; required the diameter of a wheel suitable for that purpose.

$$\text{Here } \frac{64 \times 42}{77} = 34{\cdot}9 \text{ inches.} = \textit{Ans.}$$

EXAMPLE 3. Required the number of revolutions per minute made by a wheel or pulley 20 inches diameter, when driven by another of 4 feet diameter, and making 46 revolutions per minute.

$$\text{Here } \frac{48 \times 46}{20} = 110{\cdot}4 \text{ revolutions.} = \textit{Ans.}$$

EXAMPLE 4. A shaft, at the rate of 22 revolutions per minute, is to give motion, by a pair of wheels, to another shaft at the rate of $15\frac{1}{2}$; the distance of the shafts from centre to centre is $45\frac{1}{2}$ inches; the diameters of the wheels at the pitch lines are required.

$$\text{Here } 22 + 15{\cdot}5 : 22 :: 45{\cdot}5 \textit{ in.} : \frac{22 \times 45{\cdot}5}{22 + 15{\cdot}5} = 26{\cdot}69 \textit{ in.}$$

the radius of the driven wheel; which, doubled, gives 53·38 inches, the diameter. = 1st *Ans.*

Therefore 45·5 inches − 26·69 inches = 18 81 inches, the radius of the driver; which, doubled, gives 37·62 inches, the diameter. = 2d *Ans.*

EXAMPLE 5. Suppose a drum to make 20 revolutions per minute, required the diameter of another to make 58 revolutions in the same time.

Here 58 ÷ 20 = 2·9, that is, their diameters must be as 2·9 to 1; thus, if the one making 20 revolutions be called 30 inches, the other will be 30 ÷ 2·9 = 10·345 inches diameter.

EXAMPLE 6. Required the diameter of a pulley, to make $12\frac{1}{2}$ revolutions in the same time as one of 32 inches making 26.

$$\text{Here } \frac{32 \times 26}{12{\cdot}5} = 66{\cdot}56 \text{ inches diameter.}$$

EXAMPLE 7. A shaft, at the rate of 16 revolutions per minute, is to give motion to a piece of machinery, at the rate of 81 revolutions in the same time; the motion is to be communicated by means of two gearing wheels and two pulleys, with an intermediate shaft; the driving wheel contains 54 teeth, and the driving pulley on the axis of the driven wheel is 25 inches diameter; required the number of teeth in the other wheel, and the diameter of the other pulley.

Let the driven wheel have a velocity of 36, a mean proportional between the extreme velocities 16 and 81;

then, $\frac{16 \times 54}{36} = 24$, the number of teeth in the driven wheel.= 1st *Ans.*

And $\frac{36 \times 25}{81} = 11{\cdot}11$ inches, diameter of the driven pulley.= 2d *Ans.*

EXAMPLE 8. Suppose in the last example the revolutions of one of the wheels to be given, the number of teeth in both, and likewise the diameter of each pulley, to find the revolutions of the last pulley.

Here $\frac{16 \times 54}{24} = 36$, velocity of the intermediate shaft.=*Ans.*

Also, $\frac{36 \times 25}{11{\cdot}11} = 81$, the velocity of the machine.

GOLD LUSTRE FOR STONE-WARE.—Gold, 6 parts; aqua regia, 36 parts. Dissolve: then add, tin, 1 part. Next add balsam of sulphur, 3 parts; oil of turpentine, 1 part. Mix gradually in a mortar, and rub it in until the mixture becomes hard; then add oil of turpentine, 4 parts It is then ready to be applied to a ground prepared for the purpose.

TO PETRIFY WOOD, &c.—Take equal quantities of gem-salt, rock-alum, white vinegar, chalk, and pebbles powdered. Mix all these ingredients: there will happen an ebullition. If, after it is over, you throw into this liquor any porous matter, and leave it there soaking four or five days, they will positively turn into petrifactions.

STEAM POWER AND THE STEAM-ENGINE.

STEAM is of great utility as a productive source of motive power; in this respect, its properties are—elastic force, expansive force, and reduction by condensation. *Elastic* signifies the whole urgency or power the steam is capable of exerting with undiminished effect. By *expansive force* is generally understood the amount of diminishing effect of the steam on the piston of a steam-engine, reckoning from that point of the stroke where the steam of uniform elastic force is cut off but it is more properly the force which steam is capable of exerting, when expanded to a known number of times its original bulk. And *condensation*, here understood, is the abstraction or reduction of heat by another body, and consequently not properly a contained property of the steam, but an effect produced by combined agency, in which steam is the principal; because any colder body will extract the heat and produce condensation, but steam cannot be so beneficially replaced by any other fluid capable of maintaining equal results.

The rules formed by experimenters, as corresponding with the results of their experiments on the elastic force of steam at given temperatures vary, but approximate so closely, that the following rule, because of being simple, may in practice be taken in preference to any other:

RULE. To the temperature of the steam, in degrees of Fahrenheit, add 100; divide the sum by 177; and the 6th power of the quotient will equal the force in inches of mercury.

EXAMPLE. Required the force of steam corresponding to a temperature of 312°.

$$\frac{312 + 100}{177} = 2{\cdot}3277^6 = 159 \text{ inches of mercury.}$$

To Estimate the Amount of Advantage Gained by Using Steam Expansively in a Steam-Engine.

When steam of a uniform elastic force is employed throughout the whole ascent or descent of the piston, the amount of effect produced is as the quantity of steam expended. But let the steam be shut off at any portion of the stroke—say, for instance, at one half—it expands by degrees until the termination of the stroke, and then exerts half its original force; hence an accumulation of effect in proportion to the quantity of steam.

RULE. Divide the length of the stroke by the distance or space into which the dense steam is admitted, and find the hyperbolic logarithm of the quotient, to which add 1; and the sum is the ratio of the gain.

EXAMPLE. Suppose an engine with a stroke of 6 feet, and the

steam cut off when the piston has moved through 2; required the ratio of gain by uniform and expansive force

$6 \div 2 = 3$; hyperbolic logarithm of $3 = 1{\cdot}0986 + 1 = 2{\cdot}0986$, ratio of effect; that is, supposing the whole effect of the steam to be 3, the effect by the steam being cut off at $\frac{1}{3} = 2{\cdot}0986$.

Again, let the greatest elastic force of steam in the cylinder of an engine equal 48 lbs. per square inch, and let it be cut off from entering the cylinder when the piston has moved $4\frac{1}{2}$ inches, the whole stroke being 18; required an equivalent force of the steam throughout the whole stroke.

$$18 \div 4{\cdot}5 = 4, \text{ and } 48 \div 4 = 12.$$
$$\text{Logarithm of } 4 + 1 = 2{\cdot}38629.$$
$$\text{Then } 2{\cdot}38629 \times 12 = 28{\cdot}635 \text{ lbs. per square inch.}$$

In regard to the other case of expansion, when the temperature is constant, the bulk is inversely as the pressure; thus, suppose steam at 30 lbs. per square inch, required its bulk to that of original bulk, when expanded so as to retain a pressure equal to that of the atmosphere, or 15 lbs.

$$\frac{15+30}{15} = 3 \text{ times its original bulk.}$$

It is because of the latent heat in steam, or water in an aëriform state, that it becomes of such essential service in heating, boiling, drying, &c. In the heating of buildings, its economy, efficiency, and simplicity of application are alike acknowledged; the steam being simply conducted through all the departments by pipes, by extent of circulation condenses—the latent heat being thus given to the pipes, and diffused by radiation. In boiling, its efficiency is considerably increased, if advantage be taken of sufficiently inclosing the fluid, and reducing the pressure on its surface, by means of an air-pump. Thus, water in a vacuum boils at about a temperature of 98°; and in sugar refining, where such means are employed, the syrup is boiled at 150°.

The latent heat of steam at the common pressure of the atmosphere, according to very accurate experiments, is found to be 1000°; and we know that the sensible, or thermometric heat = 212°. Now 212° − 32° = 180°, and 1000° + 180° = 1180°; therefore, steam at 212° is simply highly rarified water, and contains 1180° of heat; hence, to find the latent heat of steam at any other temperature, subtract the sensible heat from 1180°, and add 32° = the latent heat.

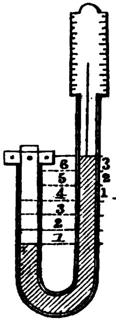
EXAMPLE. Required the latent heat of steam whose sensible heat is 224°.

$$1180° - 224° = 956°,$$
$$\text{And } 956° + 32° = 988° \text{ latent heat.}$$

A cubic inch of water produces about 1700 cubic inches of steam

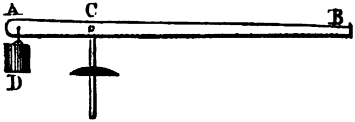
at 212°, or the common pressure of the atmosphere; but the boiling point varies considerably with the pressure on the surface of the fluid; thus, in a vacuum, water boils at about 90°; under common pressure, at 212°; and when pressed with a column of mercury 4 inches in height, at 216°; each inch of mercury producing by its pressure a rise of about 1° in the thermometer.

The pressure or force of steam in the boiler (less than the weight upon the safety-valve) is generally indicated by a column of mercury in a bent iron tube, which causes the range of the float to be only half the range of the mercury, 2 inches of mercury being nearly equal to 1 lb. pressure of steam in the boiler, thus:



To Calculate the Effect of a Lever and Weight upon the Safety-Valve of a Steam-Boiler, &c.

The lever, under all circumstances, is balanced by a known weight or weights, on the short end, making its point of rest on the valve the centre of motion; so that the weight, added to that of the lever, is the effective weight upon the valve, independent of any other additional weight, thus:



There are three different ways that it may be required to calculate the lever:

1. *When a certain pressure is required upon the valve, the distance of the weight upon the lever, and the distance of the valve from the centre of motion given, to find what weight will be required upon the lever at that distance.*

From the pressure on the valve in lbs. subtract the weight of the valve in lbs. and the effective weight of the lever, multiply the remainder by the distance between the fulcrum and the valve, and divide the product by the distance between the fulcrum and the

weight, and the quotient is the weight in lbs. required to be placed upon the lever at that distance.

2. *When a certain pressure is required upon the valve, the weight upon the lever and distance of the valve from the centre of motion given, to find where that weight must be placed.*

From the required weight upon the valve in lbs. take the weight of the valve, add the effective weight of the lever, multiply the remainder by the distance between the fulcrum and the valve, and divide the product by the weight in lbs. upon the lever, and the quotient is the distance in inches from the fulcrum that the weight must be placed.

3. *When the distance of weight, distance of valve from the centre of motion, and weight upon the lever are given, to find what pressure is upon that valve.*

Multiply the weight in lbs. upon the lever by the distance in inches to the fulcrum, divide the product by the distance between the fulcrum and the valve, and the quotient, plus the weight of the valve and effective weight of the lever, equal the weight upon the valve in lbs.

EXAMPLE 1. Suppose the lever A B (as above) to be 24 inches in length, and the valve C placed 5 inches from the centre of motion A, what weight must be placed upon the lever 20 inches from A, to equal 80 lbs., on the valve C, the weight of the lever being 2 lbs., the weight D, which balances the lever, 4½ lbs., and the weight of the valve 3 lbs.?

2	lbs.	weight of the lever.
4·5	"	to balance ditto.
3	"	weight of the valve.
9·5 lbs.		

$$\text{Then } \frac{\overline{80-9{\cdot}5}\times 5}{20} = 17{\cdot}625 \text{ lbs.}$$

EXAMPLE 2. Suppose the weight upon the lever equal 17·625 lbs., it is required at what distance from A the weight must be placed to equal 80 lbs. at C.

$$\frac{\overline{80 - 9{\cdot}5} \times 5}{17{\cdot}625} = 20 \text{ inches.}$$

EXAMPLE 3. Suppose, as before, that a weight of 17·625 lbs. is placed upon the lever 20 inches from A, required the pressure at C, the distance from the centre of motion being 5 inches, and the effective weight of the lever at that point equal 6½ lbs., also the weight of the valve 3 lbs.

$$\begin{aligned} \frac{17{\cdot}625 \times 20}{5} &= 70{\cdot}5 \\ &+\ \ 6{\cdot}5 \\ &+\ \ 3 \\ &= 80 \text{ lbs.} \end{aligned}$$

To Find the Proper Diameter for a Safety-Valve.

Multiply the bottom surface of the boiler, or surface immediately exposed to the action of the fire, in feet, by the multiplier opposite to the pressure in lbs. on each square inch of the safety-valve, and the square root of the product is the valve's diameter in inches at the narrowest part. If the boiler is to have two safety-valves, then the square root of half the product equal the diameter of each.

Pressure in lbs. per square inch.	*Multipliers.*	*Pressure in lbs. per square inch.*	*Multipliers.*
3	·356	15	·315
4	·353	20	·305
5	·348	25	·293
6	·344	30	·289
7	·339	35	·282
8	·336	40	·275
10	·329	45	·270
12	·321	50	·264

In constructing steam-engines, the following simple *rule for obtaining the nominal horse power* is now generally adopted:

The area of the cylinder in square inches multiplied by 7 lbs. pressure, multiplied into the speed of the piston in feet per minute, divided by 33000, equal the nominal horse power.

$$\text{Thus, } \frac{\text{area of cylinder} \times 7 \text{ lbs.} \times \text{feet per minute}}{33000} = \text{nominal H. P.}$$

The length of stroke and relative speed of piston, and number of revolutions per minute, will be found by the following table. In calculating the gross horse power developed in any cylinder, as shown by the *indicator*, it has been customary to allow one-tenth, and sometimes one-eighth, for friction; this is now very properly abandoned, and the following rule for calculating the indicator diagram should be always adopted: the mean pressure as shown on the card, multiplied into the area of the cylinder, multiplied into the speed of piston, in feet per minute, when the card was taken; this product, divided by 33000, will give the gross or indicated horse power;

	ft.	in.					ft.	
For	3	0	stroke	30	revolutions per minute	=	180	per minute.
	3	6	"	27	"	=	189	"
	4	0	"	$24\frac{1}{2}$	"	=	196	"
	4	6	"	$22\frac{2}{3}$	"	=	204	"
	5	0	"	21	"	=	210	"
	5	6	"	$19\frac{1}{2}$	"	=	216	"
	6	0	"	$18\frac{1}{2}$	"	=	222	"
	6	6	"	$17\frac{1}{7}$	"	=	226	"
	7	0	"	$16\frac{1}{2}$	"	=	231	"
	7	6	"	$15\frac{11}{15}$	"	=	236	"
	8	0	"	15	"	=	240	"
	8	6	"	$14\frac{6}{17}$	"	=	244	"
	9	0	"	$13\frac{2}{3}$	"	=	247	"

The *Air-Pump*. The diameter of the air-pump should be a little more than half the diameter of the cylinder, or the diameter of the cylinder in inches multiplied by ·6 will give the diameter of the air-pump in inches, the length of stroke to be one-half the length of stroke of the piston.

The *Condenser* should never be less than half the capacity of the cylinder; and in engines where the pressure on the boiler ranges from twelve to twenty pounds on the square inch, a much larger condenser should be given.

The foot and delivery-valve passages should have an area of one-third of the air-pump.

The *Steam-Ports*. The *area* of the steam-ports on the cylinder should never be less than one-twentieth of the area of the cylinder. If the speed of the piston is above 250 feet per minute, the ports should never be less than one-fourteenth the area of the cylinders.

The *Cold-Water Pump*. The capacity of the cold-water pump should be not less than one-thirty-sixth of the capacity of the cylinder.

The *Fly-Wheel*. To find the weight of the fly-wheel rim the following practical rule is generally adopted:

$$\frac{\text{Horse power of the engine} \times 2000}{(\text{velocity of circumference of wheel in feet per second})^2} =$$

the weight of the fly-wheel in cwts.

The *Fly-Wheel*, or *Crank-Shaft*. The nominal horse power of the engine and speed of the shaft being given, the diameter of this shaft, whether cast or wrought iron, will be found in the *Tables of Strength of Shafts*.

The *Governor*. To find the number of revolutions, divide 375 by the square root of the length of the pendulum; half of this quotient is the number of revolutions the balls ought to make per minute.

To find the length of the pendulum, divide 375 by twice the number of revolutions; the quotient squared is the length of the pendulum.

General Proportions of Locomotive Engines.

For the area of the steam-ports when the stroke is 18 inches, the square of the diameter of the cylinder × ·068 = the area in square inches.

For the area of the eduction ports, the square of the diameter of the cylinder in inches × ·128 = the area in square inches.

The breadth of the bridges between the eduction ports and the induction = ¾ inch and 1 inch.

The diameter of the chimney = the diameter of the cylinder.

For the area of the fire-grate, the diameter of the cylinder in inches × ·77 = the area in superficial feet.

For the effective heating-surface of the boiler, the square of the diameter of the cylinder in inches × 5 ÷ 2 = area in square feet.

For the diameter of the feed-pump ram, the square of the diameter of the cylinder in inches × ·011 = the diameter in inches.

For the cubical content of the steam-room, the square of the diameter of the cylinder in inches × 9 ÷ 40 = content in cubic feet.

For the cubical content of inside fire-box above fire-bars, the square of the diameter of the cylinder in inches ÷ 4 = content in cubic feet.

For the inside diameter of the steam-pipe, the square of the diameter of the cylinder in inches × ·03 = the diameter in inches.

For the diameter of the branch steam-pipe, the square of the diameter of the cylinder in inches × ·021 = the diameter in inches.

For the diameter of the top of the blast-pipe, the square of the diameter of the cylinder in inches × ·017 = the diameter in inches.

For the diameter of the feed-pipes, the diameter of the cylinder in inches × ·141 = the diameter in inches.

For the diameter of the piston-rod, the diameter of the cylinder in inches ÷ 7 = the diameter in inches.

For the thickness of the piston, the diameter of the cylinder in inches × 2 ÷ 7 = the thickness in inches.

For the diameter of the connecting-rod at the middle, the diameter of the cylinder in inches × ·21 = the diameter in inches.

For the diameter of the plain part and inside bearing of the crank-axle, the cube root of the square of the diameter of the cylinder in inches × ·96 = the diameter in inches.

For the diameter of the outside bearings of the crank for axle, the cube root of the product of the square of the diameter of the cylinders in inches × ·396 = the diameter in inches.

For the diameter of the crank-bearing, the diameter of the cylinder in inches × ·404 = the diameter in inches.

For the length of the crank-bearing, the diameter of the cylinder in inches × ·233 = the length in inches.

Remarks on Steam-Engine Boilers and their Proportions.

For engines designed to give a gross indicator horse power of at least twice the nominal horse power, the grate surface should be ·66 or ·69 square feet per nominal horse power, but may be increased to ·75 square feet, and should never be diminished to less than ·60 square feet as a minimum.

The area of opening over the bridges or through the tubes, should be ·125 square feet, or 18 square inches per horse power, and may be increased to ·143 square feet, or 20 square inches with advantage, particularly in tubular boilers, and should never be diminished to less than 15 square inches, or ·109 square feet per horse power.

The area of chimney should be ·076 square feet, or 11 square inches, but may be increased to 13 square inches, and should never be diminished to less than 10 square inches per horse power.

The heating surface in fire-places and flues should be 14 square feet per horse power, exclusive of all bottom surface, but may be increased to 15 square feet, and should never be diminished to less than 12 square feet per horse power.

In calculating tubular boilers the whole surface of the tubes should be taken, and there should be a total of 17 square feet per horse power in the fire-places and tubes.

In engines designed to work to a gross power in the cylinder by the indicator greater than twice the nominal horse power, these proportions must be increased; or, if the reverse be intended, they may be diminished in proportion.

Of the Pressure of Steam, in Inches of Mercury, at Different Temperatures.

I. Temperature, Fahrenheit.	II. Dalton.	III. Ure.	IV. Young.	V. Macneill.	VI. Tredgold.
0°	0·08				
10	0·12				
20	0·17		0·11		
32	0·26	0·20	0·18		0·17
40	0·34	0·25	0·20		0·24
50	0·49	0·36	0·36	0·36	0·37
60	0·65	0·52	0·53		0·55
70	0 87	0·73	0·75	0·73	0·78
80	1·16	1·01	1·05		1·11
90	1·59	1·36	1·44	1·36	1·53
100	2·12	1·86	1·95		2·08
110	2·79	2·45	2·62	2·46	2·79
120	3·63	3·30	3·46		3·68
130	4·71	4·37	4·54	4·41	4·81
140	6·05	5·78	5·88		6·21
150	7·73	7·53	7·55	7·42	7·94
160	9·79	9·60	9·62		10·05
170	12·31	12·05	12·14	12·05	12·60
180	15·38	15·16	15·23		15·67
190	18·98	19·00	18·96	18·93	19 00
200	23·51	23·60	23·44		23·71
210	28·82	28·88	28·81	28·81	28·86
212	30·00	30·00	30·00	30·00	30·00
220	35·18	35·54	35·19		34·92
230	44·60	43·10	42·27	42·63	42·00
240	53·45	51·70	51·66		50·24

Of the Temperature of Steam at different Pressures in Atmospheres.

I. Temperature Fahrenheit.	II. French Acad.	III. Dr. Ure.	IV. Young.	V. Macneill.	VI. Tredgold.
1st At.	212·0°	212°	212°	212°	212°
2d "	250·5	250·0	240·3	249	250.+
3d "	275·2	275·0	271		274.+
4th "	293·7	291·5	288	290	294.+
5th "	308·8	304·5	302		309.+
6th "	320·4	315·5			322.—
7th "	331·7	325·5			
8th "	342·0	336·0		337	342.+
9th "	350·0	345·			
10th "	358·9				
11th "	366·8				
12th "	374·0				372.—
13th "	380·6				
14th "	386·9				
15th "	392·8				
16th "	398·5				
17th "	403·8				
18th "	408·9				
19th "	413·9				
20th "	418·5				414·
30th "	457·2				
40th "	466·6				
50th "	510·6				

TO PREVENT SPONTANEOUS COMBUSTION.—It is a fact better ascertained than accounted for, that fixed oils, when mixed with any light kind of charcoal, or substances containing carbon, such as cotton, flax, or even wool, which is not of itself inflammable, heat by the process of decomposition, and after remaining in contact some time, at length burst into flame. This spontaneous combustion takes place in waste cotton which has been employed to wipe machines, and then thrown away and allowed to accumulate into a heap. We have known an instance of the kind in a manufactory for spinning worsteds, where the waste wool, or "slubbings," as it is termed in Yorkshire, was thrown into a corner and neglected. It then heated, and was on the point of bursting into flame, when the attention of the workmen was directed to the heap by the smoke and smell. In cotton mills the danger exists in a still greater degree, and it is believed that the destruction of many cotton factories has been occasioned by this means. The cause of this peculiar property of fixed oils deserves more attention than has hitherto been paid to it.

TABLE

Of the Elastic Force of Steam, and Corresponding Temperature of the Water with which it is in Contact.

Pressure on a Square Inch.*	Elastic Force in Inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.	Pressure on a Square Inch.	Elastic Force in Inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.
lbs.				lbs.			
14·7	30·00	212·0	1700	49	99·96	281·9	564
15	30·60	212·8	1669	50	102·00	283·2	554
16	32·64	216·3	1573	51	104·04	284·4	544
17	34·68	219·6	1488	52	106·08	285·7	534
18	36·72	222·7	1411	53	108·12	286·9	525
19	38·76	225·6	1343	54	110·16	288·1	516
20	40·80	228·5	1281	55	112·20	289·3	508
21	42·84	231·2	1225	56	114·24	290·5	500
22	44·88	233·8	1174	57	116·28	291·7	492
23	46·92	236·3	1127	58	118·32	292·9	484
24	48·96	238·7	1084	59	120·36	294·2	477
25	51·00	241·0	1044	60	122·40	295·6	470
26	53·04	243·3	1007	61	124·44	296·9	463
27	55·08	245·5	973	62	126·48	298·1	456
28	57·12	247·6	941	63	128·52	299·2	449
29	59·16	249·6	911	64	130·56	300·3	443
30	61·21	251·6	883	65	132·60	301·3	437
31	63·24	253·6	857	66	134·64	302·4	431
32	65·28	255·5	833	67	136·68	303·4	425
33	67·32	257·3	810	68	138·72	304·4	419
34	69·36	259·1	788	69	140·76	305·4	414
35	71·40	260·9	767	70	142·80	306·4	408
36	73·44	262·6	748	71	144·84	307·4	403
37	75·48	264·3	729	72	146·88	308·4	398
38	77·52	265·9	712	73	148·92	309·3	393
39	79·56	267·5	695	74	150·96	310·3	388
40	81·60	269·1	679	75	153·02	311·2	383
41	83·64	270·6	664	76	155·06	312·2	379
42	85·68	272·1	649	77	157·10	313·1	374
43	87·72	273·6	635	78	159·14	314·0	370
44	89·76	275·0	622	79	161·18	314·9	366
45	91·80	276·4	610	80	163·22	315·8	362
46	93·84	277·8	598	81	165·26	316·7	358
47	95·88	279·2	586	82	167·30	317·6	354
48	97·92	280·5	575	83	169·34	318·4	350

* This includes the pressure of the atmosphere.

TABLE—(Continued).

Pressure on a Square Inch.	Elastic Force in Inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.	Pressure on a Square Inch.	Elastic Force in Inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.
lbs.				lbs.			
84	171·38	319·3	346	98	199·92	330·5	301
85	173·42	320·1	342	99	201·96	331·3	298
86	175·46	321·0	339	100	204·01	332·0	295
87	177·50	321·8	335	110	224·40	339·2	271
88	179·54	322·6	332	120	244·82	345·8	251
89	181·58	323·5	328	130	265·28	3[illegible]2·1	233
90	183·62	324·3	325	140	285·61	357·9	218
91	185·66	325·1	322	150	306·08	363·4	205
92	187·70	325·9	319	160	326·42	368·7	193
93	189·74	326·7	316	170	346·80	373·6	183
94	191·78	327·5	313	180	367·25	378·4	174
95	193·82	328·2	310	190	387·61	382·9	166
96	195·86	329·0	307	200	408·04	387·3	158
97	197·90	329·8	304				

TABLE

Of the Force and Temperature of Steam in Atmospheres.

Atmos.	Temp. Fah.	Atmos.	Temp. Fah.	Atmos.	Temp. Fah.
	Deg.		Deg.		Deg.
1	212·00	10	358·88	19	413·78
2	250·52	11	366·85	20	418·46
3	275·18	12	374·00	21	422·96
4	293·72	13	380·66	22	427·28
5	307·50	14	386·94	23	431·42
6	320·36	15	392·86	24	435·56
7	331·70	16	398·48	25	439·34
8	341·78	17	403·82		
9	350·78	18	408·92	50	510·60

TO WRITE ON SILVER WITH A BLACK WHICH WILL NEVER GO OFF.—Take burnt lead and pulverize it. Incorporate it next with sulphur and vinegar, to the consistency of a painting color, and write with it on any silver plate. Let it dry, then present it to the fire so as to heat the work a little, and it is finished.

TABLE

Of the Heating Power of various Combustible Substances, exhibiting the utmost Quantity of Water evaporated by the Given Weights, and the smallest Quantity of Air capable of producing Total Combustion.

Species of Combustible.	Pounds of Water which a Pound can heat, from 0° to 212°.	Pounds of Boiling Water evaporated by a Pound.	Weight of Atmospheric air at 32°, to burn a Pound.
Wood, in its ordinary state, .	26	4·72	4·47
Wood charcoal, ...	73	13·37	11·46
Pit coal, ...	60	10·90	9·26
Coke, ...	65	11·81	11·46
Turf, ...	30	5·45	4·60
Turf charcoal, ...	64	11·63	9·86
Carburetted hydrogen, . .	76	13·81	14·58
Oil, ...			
Wax, ...	78	14·18	15·00
Tallow, ...			
Alcohol of commerce, . . .	52	9·56	11·60

TO ESTIMATE DISTANCE.—Observe how many seconds elapse between a flash of lightning and the thunder, and multiply them by 1142, the number of feet sound travels in a second, the product will be the distance in feet. The same process may be applied to the flash and report of a gun, or any other sound, provided we can ascertain the time at which it is produced, and the interval that elapses before it reaches the ear.

Illustration. Saw a flash of lightning five seconds before I heard the thunder: required the distance.

$$\frac{5 \times 1142}{3 \times 1760} = 1\tfrac{43}{528} \text{ mile distant.}$$

In the absence of a watch, the pulsations at the wrist may be counted as seconds, by deducting one from every seven or eight.

PRISMATIC DIAMOND CRYSTALS FOR WINDOWS.—A hot solution of sulphate of magnesia, and a clear solution of gum arabic, mixed together. Lay it on hot. For a margin or for figures, wipe off the part you wish to remain clear with a wet towel.

PERFECTLY BLACK HARD GLASS.—Plain paste, 600 parts; zaffre, 3 parts; manganese, 3 parts; iron, 3 parts.

TABLE

Of Nominal Horse Power of Low Pressure Engines.

Diam. of Cylinder in Inches.	LENGTH OF STROKE IN FEET.											
	1	1½	2	2½	3	3½	4	4½	5	5½	6	7
4	·34	·39	·43	·46	·49	·52	·54	·56	·58	·60	·62	·6[illegible]
5	·53	·61	·67	·72	·76	·81	·84	·88	·91	·94	·96	1·0[illegible]
6	·76	·87	·96	1·04	1·10	1·16	1·23	1·26	1·31	1·35	1·39	1·1[illegible]
7	1·04	1·19	1·31	1·41	1·50	1·58	1·65	1·72	1·78	1·84	1·89	1·9[illegible]
8	1·36	1·56	1·72	1·85	1·96	2·07	2·16	2·25	2·33	2·40	2·47	2·6[illegible]
9	1·72	1·97	2·17	2·34	2·49	2·62	2·74	2·84	2·95	3·04	3·13	3·3[illegible]
10	2·13	2·44	2·68	2·89	3·08	3·23	3·38	3·51	3·64	3·76	3·87	4·0[illegible]
11	2·57	2·95	3·24	3·49	3·77	3·91	4·15	4·25	4·40	4·54	4·68	4·9[illegible]
12	3·06	3·51	3·86	4·16	4·42	4·65	4·86	5·06	5·24	5·41	5·57	5·8[illegible]
13	3·60	4·12	4·53	4·88	5·19	5·46	5·64	5·94	6·15	6·35	6·53	6·8[illegible]
14	4·17	4·77	5·25	5·66	6·01	6·33	6·62	6·88	7·13	7·36	7·58	7·9[illegible]
15	4·77	5·48	6·03	6·50	6·90	7·27	7·60	7·90	8·19	8·45	8·70	9·1[illegible]
16	5·45	6·23	6·86	7·39	7·86	8·27	8·65	8·99	9·31	9·61	9·90	10·4[illegible]
17	6·15	7·04	7·75	8·35	8·86	9·34	9·76	10·15	10·52	10·85	11·17	11·76
18	6·89	7·89	8·68	9·36	9·94	10·47	10·94	1[illegible]·38	11·79	12·17	12·53	13·1[illegible]
19	7·68	8·79	9·63	10·42	11·17	11·66	12·19	12·68	13·13	13·56	13·96	14·6[illegible]
20	8·51	9·74	10·72	11·55	12·27	12·92	13·51	14·05	14·55	15·02	15·46	16·28
22	10·30	11·79	12·97	13·98	14·85	15·63	16·62	17·30	17·65	18·18	18·71	19·70
24	12·26	14·03	15·44	16·63	17·67	18·61	19·45	20·23	20·95	21·63	22·27	23·44
26	14·39	16·16	18·12	19·52	20·75	21·84	22·56	23·75	24·68	25·39	26·14	27·51
28	16·68	19·09	21·02	22·64	24·06	25·33	26·48	27·54	28·52	29·44	30·31	31·90
30	19·15	21·92	24·13	25·99	27·62	29·07	30·40	31·61	32·74	33·80	34·80	36·63
32	21·79	24·96	27·51	29·57	31·42	33·08	34·59	35·97	37·26	38·46	39·59	41·68
34	24·60	28·16	30·99	33·39	35·44	37·31	39·04	40·60	42·06	43·41	44·69	47·05
36	27·57	31·56	34·74	37·42	39·77	41·87	43·77	45·52	47·15	48·67	50·11	52·75
38	30·72	35·17	38·71	41·69	44·66	46·64	48·77	50·72	52·54	54·23	55·83	58·79
40	34·04	38·97	42·89	46·20	49·10	51·69	54·04	56·20	58·21	60·09	61·86	65·12
42	37·53	42·96	47·29	50·94	54·13	56·98	59·58	61·96	64·18	66·25	68·21	71·78
44	41·19	47·15	51·90	55·91	59·28	62·54	65·46	68·00	70·44	72·71	74·85	78·79
46	45·02	51·54	56·72	61·10	64·88	68·19	71·43	74·33	76·69	79·47	81·81	86·12
48	49·02	56·11	61·76	66·54	70·70	74·42	77·82	80·94	83·83	86·53	89·08	93·78
50	53·19	60·89	67·02	72·18	76·71	80·76	84·44	87·82	90·96	93·89	96·65	104·7
52	57·55	65·86	72·48	78·08	83·00	87·35	90·25	94·98	98·40	101·55	104·5	110·0
54	62·04	71·02	78·17	84·20	89·48	94·20	98·49	102·4	106·1	109·5	112·7	118·7
56	66·72	76·38	84·07	90·55	96·23	101·30	105·9	110·1	114·1	117·8	121·2	127·6
58	71·53	81·93	90·18	97·14	103·2	108·6	113·6	118·2	122·4	126·3	129·2	136·7
60	76·60	87·68	96·50	103·9	110·4	116·3	121·6	126·4	131·0	135·2	139·2	146·5
62	81·79	93·62	103·04	111·0	117·96	124·18	129·81	135·03	139·86	144·37	148·6	156·7
64	87·15	99·84	110·0	118·3	125·7	132·3	138·3	143·9	149·0	153·82	158·4	166·7
66	92·68	106·1	116·8	125·8	133·6	140·7	147·3	153·0	158·5	163·6	168·4	177·3
68	98·40	112·6	123·9	133·6	141·8	149·4	156·2	162·4	168·2	173·6	178·8	188·2
70	104·26	119·3	131·3	141·5	150·4	158·3	165·5	170·1	178·2	184·0	189·4	199·4
72	110·30	126·2	139·0	149·7	159·1	167·4	175·1	182·1	188·6	194·7	200·4	211·0
74	116·5	133·4	146·8	158·1	167·9	176·7	185·4	192·4	199·2	205·7	211·6	223·4
76	122·9	140·7	154·8	166·8	178·6	186·6	195·0	202·9	210·1	216·9	223·3	235·1
78	129·1	148·2	163·1	175·6	186·7	196·5	205·4	212·1	221·4	228·5	235·2	247·6
80	136·2	155·8	171·6	181·8	196·4	208·7	216·1	224·8	232·8	240·4	247·4	260·5
82	143·0	163·8	180·2	191·2	206·2	217·3	226·9	237·8	244·6	252·5	260·0	273·8
84	150·1	171·8	189·1	203·8	216·5	227·9	238·3	247·8	256·7	265·0	272·8	287·1
86	157·4	180·1	198·2	213·6	227·0	237·8	247·4	258·2	269·1	277·8	286·0	301·0
88	164·8	188·6	207·6	223·6	237·5	250·2	261·0	272·0	281·7	290·8	299·4	315·2
90	172·3	197·3	217·1	233·9	248·6	261·7	273·6	284·5	291·7	301·2	313·2	329·7

TABLE

Of Nominal Horse Power of High Pressure Engines.

Diam. of Cylinder in Inches.	LENGTH OF STROKE IN FEET.											
	1	1½	2	2½	3	3½	4	4½	5	5½	6	7
2	.25	.29	.32	.35	.37	.38	.40	.42	.44	.45	.46	.49
2½	.39	.45	.50	.54	.57	.60	.63	.66	.68	.70	.72	.76
3	.57	.65	.72	.78	.83	.87	.91	.95	.98	1.01	1.04	1.10
3½	.78	.89	.98	1.06	1.13	1.19	1.24	1.29	1.34	1.38	1.42	1.49
4	1.02	1.17	1.29	1.38	1.47	1.56	1.62	1.68	1.74	1.80	1.86	1.95
4½	1.29	1.48	1.63	1.75	1.80	1.96	2.05	2.13	2.21	2.28	2.35	2.47
5	1.59	1.83	2.01	2.16	2.28	2.43	2.52	2.64	2.73	2.82	2.88	3.06
5½	1.93	2.21	2.43	2.62	2.78	2.93	3.12	3.18	3.30	3.42	3.51	3.69
6	2.[illegible]8	2.61	2.88	3.12	3.30	3.48	3.66	3.78	3.93	4.05	4.17	4.41
6½	2.69	3.09	3.39	3.66	3.90	4.08	4.23	4.44	4.62	4.77	4.89	5.16
7	3.12	3.57	3.93	4.23	4.50	4.74	4.95	5.16	5.34	5.52	5.67	5.97
7½	3.60	4.11	4.53	4.86	5.19	5.46	5.70	5.94	6.15	6.33	6.51	6.87
8	4.08	4.68	5.16	5.55	5.88	6.21	6.48	6.75	6.99	7.20	7.41	7.80
8½	4.62	5.28	5.82	6.27	6.63	6.99	7.32	7.62	7.89	8.13	8.37	8.92
9	5.16	5.91	6.51	7.02	7.47	7.86	8.22	8.52	8.85	9.12	9.39	9.90
9½	5.76	6.60	7.26	7.80	8.37	8.76	9.15	9.51	9.84	10.17	10.47	10.01
10	6.39	7.32	8.04	8.67	9.21	9.69	10.14	10.53	10.92	11.28	11.61	12.21
10½	7.05	8.04	8.88	9.54	10.14	10.68	11.16	11.61	12.03	12.42	12.78	13.47
11	7.71	8.85	9.72	10.47	11.31	11.73	12.45	12.75	13.20	13.62	14.04	14.76
11½	8.43	9.66	10.62	11.46	12.15	12.78	13.80	13.92	14.61	14.91	15.39	16.14
12	9.18	10.53	11.58	12.41	13.26	13.95	14.58	15.18	15.72	16.23	16.71	17.58
12½	9.96	11.40	12.87	13.53	14.37	15.15	15.84	16.47	17.04	17.58	18.12	19.08
13	10.80	12.36	13.59	14.64	15.57	16.38	16.92	17.82	18.45	19.05	19.59	21.64
13½	11.64	13.32	14.64	15.78	16.77	17.67	18.48	19.20	19.89	20.52	21.15	22.26
14	12.51	14.31	15.75	16.98	18.03	18.99	19.86	20.64	21.39	22.08	22.74	23.94
14½	13.41	15.36	16.92	18.21	19.35	20.37	21.30	22.14	22.95	23.70	24.39	25.62
15	14.31	16.44	18.09	19.50	20.70	21.81	22.80	23.70	24.57	25.25	26.18	27.48
16	16.35	18.69	20.58	22.17	23.58	24.81	25.95	26.97	27.93	28.83	29.70	31.26
17	18.45	21.12	23.25	22.05	26.58	28.02	29.28	30.45	31.56	32.55	33.57	35.28
18	20.67	23.67	26.04	28.08	29.82	31.41	32.82	34.14	35.37	36.51	37.59	39.57
19	23.04	26.37	29.04	31.26	33.51	34.98	36.57	38.04	39.39	40.68	41.88	44.07
20	25.53	29.22	32.16	34.65	36.81	38.76	40.53	42.15	43.65	45.06	46.38	48.84
22	30.90	35.37	38.91	41.91	44.55	46.[illegible]9	49.86	51.90	52.95	54.54	56.13	59.10
24	36.78	42.09	46.32	49.89	53.01	55.83	58.35	60.69	62.85	64.[illegible]9	66.81	70.32
26	43.17	49.33	54.36	58.56	62.25	65.52	67.68	71.25	73.8	76.17	78.42	82.53
28	50.04	57.27	63.06	67.92	72.18	75.99	79.44	82.62	85.56	88.32	90.93	95.70
30	57.45	65.76	72.39	77.97	82.86	87.21	91.20	94.83	98.22	101.40	104.4	109.9
32	65.37	74.88	82.53	83.71	94.26	99.24	103.7	107.9	111.8	115.4	118.7	125.0
34	73.80	84.48	92.9	100.22	106.3	112.0	117.1	121.8	126.2	130.2	134.0	141.1
36	82.71	94.68	104.2	112.2	119.3	125.6	113.3	136.5	111.4	146.0	150.3	158.2
38	92.16	105.5	116.1	125.0	134.0	136.9	146.3	152.1	157.6	162.7	167.5	176.3
40	102.1	116.9	129.6	128.6	147.3	155.1	162.1	168.6	174.6	180.2	185.6	195.3
42	112.6	128.9	141.8	152.8	162.4	170.9	178.7	185.9	192.5	198.7	204.6	215.3
44	1[illegible]3.5	141.4	153.7	167.7	178.1	187.6	199.4	204.0	211.3	218.1	224.5	236.3
46	1[illegible]5.0	154.6	170.1	183.3	194.6	204.6	214.3	223.0	230.0	238.4	245.4	258.3
48	147.0	168.3	185.3	199.6	212.1	223.2	233.4	242.8	251.5	259.6	267.2	281.3
50	159.6	182.6	201.0	216.5	230.1	242.3	253.3	263.4	27[illegible].9	281.6	289.9	305.1
52	172.6	197.6	217.4	224.2	249.0	262.0	270.7	284.9	295.2	304.6	313.5	330.0
54	186.1	213.0	234.5	252.6	268.4	282.6	295.4	307.2	318.3	328.5	338.1	356.1
56	200.1	299.1	252.2	271.6	288.7	303.9	317.7	330.3	342.3	353.4	363.6	382.8
58	214.7	245.8	270.5	291.4	309.6	325.8	340.8	351.6	367.2	378.9	389.7	410.1
60	229.8	263.0	289.5	311.7	331.2	348.9	364.8	379.2	393.0	405.6	417.6	439.5

Proportions of Condensing Engines.

Horse Power.	Diameter of Cylinder.	Length of Stroke.	Size of Ports.	Diameter of Air-pump.	Stroke of Air-pump.	Size of Delivering Valves.	Size of Foot Valves.	Diam. of crk. journal	Length of crk. journal	Swivels. Dia.	Swivels. Lgth.	Diam. of Piston-rod.	Main Links. Lgth.	Main Links. Wth.	Main Links. Tck.	Back Links. Wth.	Back Links. Tck.	Diam. of Valve-rod.	Dm. of Air-pump Rod.	Dm. of Hot water pump	Dm. of Cold water pump	Beam Ctrs. Dia.	Beam Ctrs. Lgth.	Crank Pin. Dia.	Crank Pin. Lgth.	Diameter of Condenser.
	"	' "	" "	"	' "	" "	" "	"	"	"	"	"	' "	"	"	"	"	"	"	"	"	"	"	"	"	"
4	12½	2 6	4¾ × 1⅛	8	1 3	7½ × 3½	5½ × 3½	2¼	3¼	1¾	2¼	1¼		1¾	⅝	1¼	⅜	¾	1¼	2½	4	2⅝	2¾	2	3	10
6	14½	3 0	6 × 1¼	9	1 6	8¼ × 3½	6¼ × 3½	2½	4	2	2½	1⅝				1¼	⅜	⅞	1⅜	2¾	4½	2⅞	3	2	3½	11
8	17	3 0	6 × 1¾	11	1 6	9 × 3½	7 × 3½	4½	6	2½	2½	1⅞	1 6	2	½	1½	⅜	⅞	1⅜	2¾	5	3¼	3½	2¼	3½	13¼
12	19½	4 0	10 × 1½	12½	2 0	10 × 3½	9 × 3½	5½	7	2¾	2¾	2	1 7	2	⅝	1½	⅜	1	1½	2¾	5½	3⅝	4½	2¾	4¾	15
15	24	3 0	9½ × 2½	13½	1 6	10 × 4	9 × 4	6	8	2¾	2¾	2¼	1 6	2	⅝	1⅝	⅜	1	1½	3¾	6	3¾	4½	2¾	4¾	10 × 20
15	21	4 0	10 × 1½	13½	2 0	10 × 4	9 × 4	6¼	8⅛	2¾	2	2¼	1 7	2	⅝	1⅝	⅜	1	1½	2¾	6	3¾	4½	2¾	4¾	16¼
20	27	3 0	11 × 2½	15	1 6	12 × 5	12 × 4	6½	8⅞	3	3	2⅞	1 6	2¼	⅝	2	⅜	1⅛	1¾	2¾	7	4¼	6	3	5½	12 × 20
20	24½	5 0	10 × 2	15½	2 6	12 × 5	12 × 4	7¼	9	3	3	2⅞	2 0	2	¾	2	⅜	1⅛	1¾	2¾	7	4¼	6	3	5½	18½
25	27	5 0	10 × 3	17½	2 6	14 × 5	12 × 4½	8	10	3¾	3¾	3	2 6	2¼	¾	2¼	⅜	1⅛	2	3⅛	8	5	6	3¼	6	19¼
30	32	3 0	14 × 3	18	1 6	15 × 5½	12 × 5	7¾	9¾	3½	3½	3¼	1 6	2¼	¾	2¼	7/16	1¼	2	3½	8	5½	6	3½	6	24 × 25
30	29	6 0	11 × 3	18	3 0	15 × 5½	12 × 5	9	11	3½	3½	3¼	2 6	2½	¾	2¼	7/16	1¼	2	3½	8	5½	6	3½	6	20
40	36	3 0	15 × 3½	20	1 9	16 × 6½	15 × 6	10	12	4	4	3½	1 9	3	¾	2¼	½	1⅜	2½	4	10	6	8	4	7	22
40	33	6 0	14 × 3	21	3 0	16 × 6½	15 × 6	10	12	4	4	3½	2 6	3	¾	2¼	½	1⅜	2½	4	10	6	8	4	7	24
50	40	4 0	16 × 3¾	22	2 0	17 × 6½	16 × 6	10	12	4¼	4¼	4	2 0	3½	¾	2¼	9/16	1½	2¾	4	11	6½	9	4¾	7	24
60	43	4 0	18 × 4	24	2 0	21½ × 6½	18 × 6	11	14	4½	4½	4	2 0	3½	⅞	2¼	⅝	1½	2¾	4½	12	6	8	4	7	24 × 38½
70	40	7 0	16 × 3¾	25	3 6	21 × 6½	18 × 6	12½	16	4½	4½	4	3 0	3½	⅞	2¼	⅝	1½	2¾	4½	12	7	9	5	8	28
70	43	7 0	17 × 3¾	27	3 6	24 × 6½	19 × 6½	12¾	16	4¾	4¾	4¼	3 0	3¾	⅞	2½	½	1⅝	2⅞	5½	12	7½	9	5½	8½	29½
80	46	7 0	18 × 4	29	3 6	24 × 7½	20 × 7	13	16	5	5	4½	3 0	4	⅞	2½	9/16	1⅝	3	6	13½	8	10	6	8½	31

Dia. of piston-rod $=\frac{1}{10}$ the dia. of cyl.
" cross-head ends $=\frac{1}{10}$ " "
Depth in centre $=\frac{1}{8}$ " "
Thickness in centre $=\frac{1}{4}$ " "

Air-pump rod iron $\frac{1}{10}$ the dia. of air-p.
" " brass $\frac{1}{9}$ " "
" " copper $\frac{1}{8}$ " "

Side-rods full $\frac{3}{4}$ of piston-rods.
Connecting-rod at thinnest part $\frac{1}{7}$ diameter of cylinder. Crank-pin $\frac{1}{8}$ diameter of cylinder.

TABLE

Of the Revolutions per Mile of Driving Wheels, and Consumption of Steam and Water for each sized Wheel; taking the steam admitted to each cylinder as exactly one cube foot, at a gross pressure of 98lbs. or 83lbs. on the spring balance.

WHEELS.			Cylinder of Steam per Mile, and Consumption, taking Cylinder at one Cube Foot.	Water per Mile, taking Steam at 94lbs. above atmosphere.
Diameter.	Circumference.	Revolutions per Mile.		
feet.	ft. in.	No.	cube feet.	gallons.
10	31 5	168	672	14·0
9½	29 10⅛	176·9	707·6	14·74
9	28 3¼	186·7	746·8	15 55
8½	26 8⅜	197·4	789·6	16·44
8	25 1½	210·1	840·4	17·5
7½	23 6¾	224	897·6	18·69
7	21 11⅞	240	960	20·0
6½	20 5	258·6	1034	21·5
6	18 9·18	280·5	1122	23·87
5½	17 3·33	305·6	1222·4	25·45
5	15 8·48	336·3	1344·4	28·0
4½	13 11·1	379·0	1493·6	31·11
4	12 6·92	420·3	1680·4	35·0
3¾	11 9·37	441·1	1792·2	37·33
3½	10 11·94	480·1	1920·8	40·0
3	9 5·08	560·2	2240	46·67

Note.—As there are two cylinders at work in a locomotive, consequently there are four cylinders of steam for each revolution.

MODELLING WAX.—This is made of white wax, which is melted and mixed with lard to make it malleable. In working it, the tools and the board or stone are moistened with water, to prevent its adhering; it may be colored to any desirable tint with dry color.

TABLE
Of Pressure of Steam, exclusive of that of the Atmosphere.

PRESSURE.			Temperature in degrees of Fahrenheit	PRESSURE.			Temperature in degrees of Fahrenheit
lbs. on the sq. inch	In inches of mercury.	In atmospheres.		lbs. on the sq. inch.	In inches of mercury.	In atmospheres.	
1	2·04	·068	213°	51	104·04	3·468	301°
2	4·08	·136	216	52	106·08	3·536	302½
3	6·12	·204	219½	53	108·12	3·604	303½
4	8·16	·272	222	54	110·16	3·672	304½
5	10·20	·340	225½	55	112·20	3·740	305½
6	12·24	·408	228½	56	114·24	3·808	306½
7	14·28	·476	231	57	116·28	3·876	307½
8	16·32	·544	234	58	118·32	3·944	308½
9	18·36	·612	236	59	120·36	4·012	309
10	20·40	·680	239	60	122·40	4·080	310
11	22·44	·748	241	61	124·44	4·148	311
12	24·48	·816	243	62	126·48	4·216	312
13	26·52	·884	245½	63	128·52	4·284	313
14	28·56	·952	247½	64	130·56	4·352	314
15	30·60	1·020	249½	65	132·60	4·420	315
16	32·64	1·088	251½	66	134·64	4·488	316
17	34·68	1·156	253½	67	136·68	4·556	317
18	36·72	1·224	255½	68	138·72	4·624	317½
19	38·76	1·292	257	69	140·76	4·692	318½
20	40·80	1·360	259	70	142·80	4·760	319
21	42·84	1·428	261	71	144·84	4·828	320
22	44·88	1·496	262½	72	146·88	4·896	321
23	46·92	1·564	264	73	148·92	4·964	322
24	48·96	1·632	266	74	150·96	5·032	322½
25	51·00	1·700	267½	75	153·00	5·100	323½
26	53·04	1·768	269	76	155·04	5·168	324
27	55·08	1·836	270½	77	157·08	5·236	325
28	57·12	1·904	272	78	159·12	5·304	326
29	59·16	1·972	273½	79	161·16	5·372	327
30	61·20	2·040	275	80	163·20	5·440	327½
31	63·24	2·108	276½	81	165·24	5·508	328
32	65·28	2·176	278	82	167·28	5·576	329
33	67·32	2·244	279	83	169·32	5·644	330
34	69·36	2·312	280½	84	171·36	5·712	330½
35	71·40	2·380	282	85	173·40	5·780	331
36	73·44	2·448	283	86	175·44	5·848	332
37	75·48	2·516	284½	87	177·48	5·916	333
38	77·52	2·584	286	88	179·52	5·984	333½
39	79·56	2·652	287	89	181·56	6·052	334
40	81·60	2·720	288	90	183·60	6·120	335
41	83·64	2·788	289	91	185·64	6·188	335½
42	85·68	2·856	290½	92	187·68	6·256	336
43	87·72	2·924	292	93	189·72	6·324	337
44	89·76	2·992	293	94	191·76	6·392	338
45	91·80	3·060	294	95	193·80	6·460	338½
46	93·84	3·128	295½	96	195·84	6·528	339
47	95·88	3·196	297	97	197·88	6·596	340
48	97·92	3·264	298	98	199·92	6·664	340½
49	99·96	3·332	299	99	201·96	6·732	341
50	102·00	3·400	300	100	204·00	6·800	342

LINSEED OIL, CLARIFIED, FOR VARNISHES.—Heat in a copper boiler 50 gallons of linseed oil to 280° Fah.; add 2½ lbs. of calcined white vitriol, and keep the oil at the above temperature for half an hour; then remove it from the fire, and in twenty-four hours decant the clear oil, which should stand for a few weeks before it is used for varnish.

TABLE

Of the Pressure on a square and circular Inch, respectively, exerted by the elastic force of Steam at various degrees of Temperature, with the Height of the column of Mercury it will support.

I. PRESSURE ON A SQUARE INCH.				II. PRESSURE ON A CIRCULAR INCH.			
Temperature, Fahrenheit.	Pressure on a square inch in lbs.	Proportional pressure on a circular inch in lbs.	Inches of Mercury supported.	Temperature, Fahrenheit.	Pressure on a circular inch in lbs.	Proportional pressure on a square inch in lbs.	Inches of Mercury supported.
°				°			
220	2½	1·963	5·15	222	2½	3·183	6·56
222	3	2·356	6·18	224	3	3·819	7·87
223	3½	2·749	7·21	226	3½	4·456	9·18
225	4	3·141	8·24	228	4	5·093	10·5
227	4½	3·534	9·27	230	4½	5·729	11·8
228	5	3·927	10·3	232	5	6·366	13·1
230	5½	4·320	11·3	234	5½	7·002	14·4
231	6	4·712	12·3	235	6	7·639	15·7
233	6½	5·105	13·4	236	6½	8·276	17·0
234	7	5·498	14·4	238	7	8·912	18·3
235	7½	5·890	15·4	239	7½	9·549	19·7
236	8	6·283	16·5	241	8	10·18	21·0
237	8½	6·676	17·5	242	8½	10·82	22·3
239	9	7·068	18·5	244	9	11·45	23·6
240	9½	7·461	19·6	245	9½	12·09	24·9
241	10	7·854	20·6	247	10	12·73	26·2
242	10½	8·247	21·6	248	10½	13·36	27·5
243	11	8·639	22·6	250	11	14·00	28·9
244	11½	9·032	23·7	251	11½	14·64	30·1
245	12	9·424	24·7	252	12	15·27	31·5
252	15	11·78	30·9	259	15	19·09	39·3
261	20	15·71	41·2	270	20	25·46	52·5
269	25	19·63	51·5	278	25	31·83	65·6
276	30	23·56	61·8	287	30	38·19	78·7
283	35	27·49	72·1	294	35	44·56	91·8
289	40	31·41	82·4	300	40	50·92	105
294	45	35·34	92·7	305	45	57·20	118
300	50	39·27	103	309	50	63·66	131

AMALGAMS.

When mercury is alloyed with any metal the compound is called an amalgam of that metal; as, for example, an amalgam of tin, bismuth, &c.

Amalgam for Electrical Machines.

1. Fuse 1 oz. of zinc with ½ oz. of tin, at as low a temperature as possible; then add 1½ oz. of quicksilver, previously made hot; mix, pour out, and when cold reduce it to powder, and triturate it with sufficient quicksilver to bring it to a proper consistence.

2. Zinc 1 part; tin 1; quicksilver 2. Melt together.

3. Zinc 2 parts; tin 1; mercury 5.

4. *La Beaume's.* Pour into a chalked wooden box 6 oz. of quicksilver; put into an iron ladle ½ oz. of beeswax, with 2 oz. of purified zinc, and 1 oz. of grain tin; set it over a brisk fire, and when the metals are melted pour them into the box, avoiding the dross. When cold reduce it to powder, and mix it with lard. Keep it in a box covered with tallow, and spread it on leather for use.

Liquid Amalgam for Silvering Globes, &c.

Pure lead 1 oz; grain tin 1 oz.; melt in a clean ladle, and immediately add 1 oz. of bismuth. Skim off the dross, remove the ladle from the fire, and before the metal sets add 10 oz. of quicksilver. Stir together, avoiding the fumes.

Amalgam for Varnishing Plastic Figures.

Melt 2 oz. of tin with ½ oz. of bismuth, and add ½ oz. of quicksilver. When cold grind it with white of egg, and apply to the figure.

VARNISHES.

Preparations of Lac.

Stick-lac consists of twigs of several kinds of trees encrusted with a resinous matter, produced by the puncture of an insect called the cocus lacca. This, triturated with water, and dried, forms seed-lac. The seed-lac, when heated and pressed in cotton bags, forms shell-lac. Lac dye is the coloring matter extracted from stick-lac by water, and evaporated to dryness, with the addition of earthy matters, and formed into square cakes. Seed-lac and shell-lac are chiefly used in varnishes, dissolved in rectified spirits, or rectified wood naphtha. The alcoholic solution is rendered paler, so that it may be used for polishing light colored woods, by digesting it in the sun, or near a fire, for two or three weeks, with good animal charcoal, and then filtering it through paper in a funnel heated with hot water. Shell-lac may be bleached by dissolving it in a solution of potash, or soda, and passing chlorine into the solution,

The precipitated lac is collected, and well washed. Kastner directs 3 parts of carbonate of potash to be dissolved in 24 of water, and 3 of lime added, and the whole digested in a close vessel for twenty-four hours. The clear liquor is poured off, and boiled with 4 parts of shell-lac. When cold, dilute with 4 times its bulk of water, and filter; then add chloride of lime, and afterwards diluted muriatic acid. With these preliminary remarks we come now to the lacquers, or varnishes.

The Famous Brilliant French Varnish for Boots and Shoes.

Take ¾ of a pint of spirits of wine; 5 pints white wine; ½ pound of powdered gum senegal; 6 oz. loaf sugar; 2 oz. powdered galls; 4 oz. green copperas. Dissolve the sugar and gum in the wine. When dissolved, strain; then put it on a slow fire, being careful not to let it boil. In this state put in the galls, copperas, and the alcohol, stirring it well for five minutes. Then set off, and when nearly cool strain through flannel, and bottle for use. It is applied with a pencil brush. If not sufficiently black a little sulphate of iron, and half a pint of a strong decoction of logwood, may be added, with $\frac{1}{16}$ oz. pearlash.

Black Varnish.

Take any varnish, of the class you wish, 16 parts; lampblack 2 parts. Grind the black in a small quantity of the varnish, then mix it with the remainder.

Cabinet-makers' Varnish.

Pale shell-lac 700 parts; mastic 65 parts; strongest alcohol 1000 parts. Dissolve. Dilute with alcohol.

Callott's Soft Etching Varnish.

Linseed oil 8 parts; benzoin 1 part; white wax 1 part. Melt and keep it heated until reduced to two thirds.

Pale Carriage Varnish.

Copal 32 parts; pale oil 80 parts. Fuse and boil until stringy; then add dried white copperas 1 part; litharge 1 part. Boil again, then cool a little, and mix in spirits of turpentine 150 parts. Strain. While making the foregoing, take of gum animé 32 parts; pale oil 80 parts; dried sugar of lead 1 part; litharge 1 part; spirits of turpentine 170 parts. Pursue the same treatment as before, and mix the two compositions while hot.

Second Quality of Carriage Varnish.

Take of gum animé 32 parts; oil 100 parts; spirits of turpentine 150 parts; litharge 1 part; dried sugar of lead 1 part; dried copperas 1 part. Proceed as above.

Copal Varnish.

Copal 30 parts; drying oil 25 parts; spirits of turpentine 50 parts. Put the copal into a vessel capable of holding 200 parts,

and fuse it as quickly as possible, then add the oil, previously heated to nearly the boiling point. Mix well, then cool a little, and add the spirit of turpentine; again mix well, and cover up until the temperature has fallen to 140° Fah.; then strain.

To Dissolve Copal in Spirit.

Take the copal and expose it in a vessel formed like a colander to the front of a fire, and receive the drops of melted gum in a basin of cold water; then well dry them, in a temperature of about 95° Fah. By treating copal in this way it acquires the property of dissolving in alcohol.

Black Copal Varnish.

Take lamp-black, or ivory-black, in fine powder, and mix it with the varnish.

Blue Copal Varnish.

Indigo, Prussian blue, blue verditer, or ultra-marine. These substances must be powdered fine. Proceed as before.

Fine Pale Copal Varnish.

Pale African copal 1 part. Fuse, then add hot pale oil 2 parts Boil until the mixture is stringy, then cool a little, and add 3 parts of pale spirits of turpentine. Mix well.

Flaxen Grey Copal Varnish.

Ceruse, which forms the ground of the paste, mixed with a small quantity of Cologne earth, as much English red, or carminated lake, and a particle of Prussian blue, and color the varnish therewith.

Green Copal Varnish.

Verdigris, crystallized verdigris, compound green (a mixture of yellow and blue). The first two require a mixture of white in proper proportions, from a fourth to two-thirds, according to the tint intended to be given. The white used for this purpose is ceruse, or the white oxide of lead, or Spanish white. Proceed as before.

Improved Copal Varnish.

Caoutchoucine (white and scentless), strong alcohol, equal parts; copal in the proportion of two pounds to a gallon. Digest in a close vessel, without heat, for one week.

Pearl Grey Copal Varnish.

White and black; white and blue; for example, ceruse and lamp-black; ceruse and indigo. Mix them with the varnish, according to the tint required.

Purple Copal Varnish.

Prussian blue and vermilion, or any other blue and red; then proceed as before.

Red Copal Varnish.

1. Vermilion, red oxide of lead (minium), red ochre, or Prussian red, &c., and proceed as before.
2. Dragon's blood, brick red, or Venetian red, &c., and proceed as before.

Violet Copal Varnish.

Vermilion, blue, white, in proportions required to color the varnish.

White Copal Varnish.

Copal 16 parts; melt, and add hot linseed oil 8 parts; spirits of turpentine 15 parts; finest white lead to color.

Yellow Copal Varnish.

Yellow oxide of lead, or Naples and Montpelier, both reduced to impalpable powder. These yellows are hurt by contact with iron or steel. In mixing them, therefore, a horn spatula, with a glass mortar and pestle, must be employed. Or gum guttæ, yellow ochre, or Dutch pink, according to the nature and tone of the color to be imitated, and proceed as before.

Mastic Varnish.

Gum mastic 5 pounds; spirits of turpentine 2 gallons. Mix with a moderate heat (carefully applied), in a close vessel, then add pale turpentine varnish 3 pints. Mix well.

Another.

Mastic 1 pound; white wax 1 ounce; oil of turpentine 1 gallon. Reduce the wax and mastic small, then digest in a close vessel, with heat, until dissolved.

Common Oil Varnish.

Resin 4 pounds; genuine beeswax ½ pound; boiled oil 1 gallon. Mix with heat, then add spirits of turpentine 2 quarts.

Turpentine Varnish.

Resin 1 part; boiled oil 1 part. Melt, then add turpentine 2 parts. Mix well.

White Hard Spirit Varnish.

Gum sandarach 2½ pounds; alcohol (65 op.) 1 gallon. Place them in a strong, well closed vessel, and apply the heat of warm water, with occasional agitation, until dissolved; then add pale turpentine varnish 1 pint. Mix well, and let the whole rest for twenty-four hours, when it will be ready for use.

White Spirit Varnish.

Strongest alcohol 100 parts; sandarach 25 parts; tears mastic 6 parts; elemi 3 parts; Venice turpentine 3 parts. Dissolve in a closely corked vessel.

Varnish for Toys.

Copal 7 parts; mastic 1 part; Venice turpentine ½ part; strongest alcohol 11 parts. Dissolve the copal first, with the aid of a little camphor, then add the mastic, &c., and thin with alcohol, as required.

To Clean Varnish.

Use a ley of potash, or soda, mixed with a little powdered chalk. Do not make the liquor too strong of the alkali.

To Polish Varnish.

Take 2 oz. powdered tripoli, put it in an earthen pot, with water to cover it; then take a piece of white flannel, lay it over a piece of cork or rubber, and proceed to polish the varnish, always wetting it with the tripoli and water. It will be known when the process is finished by wiping a part of the work with a sponge, and observing whether there is a fair even gloss. When this is the case, take a bit of mutton suet and fine flour, and clean the work.

Varnish for Harness.

Take ½ pound of India-rubber; one gallon of spirit of turpentine; dissolve enough to make it into a jelly; then take equal quantities of good hot linseed oil, and the above mixture. Incorporate them well on a slow fire, and it is fit for use.

A Varnish for Fastening the Leather on Top Rollers in Factories.

Dissolve 2½ oz. of gum arabic in water; and a like amount of isinglass dissolved in brandy, and it is fit for use.

A Varnish to Preserve Glass from the Rays of the Sun.

Reduce a quantity of gum tragacanth to fine powder, and let it dissolve for twenty-four hours in white of eggs well beat up; then rub it gently on the glass with a brush.

A fine Black Varnish for Coaches and Iron Work.

Bitumen of Palestine 2 oz.; resin 2 oz.; umber 12 oz. Melt them separately, and then mix together over a moderate fire. Then pour upon them, while on the fire, 6 oz. clear boiled linseed oil, stirring the whole from time to time. Take it off the fire, and when moderately cool pour in 12 oz. of essence of turpentine.

Varnish for Clock Faces.

Spirits of wine 1 pint; divide it into four parts; mix one part with ½ an oz. of gum mastic in a bottle by itself; one part of spirit and ½ oz. gum sandarach in another bottle; and one part spirit and ½ oz. whitest part of gum benzoin. Mix and temper them to suit; if too thick add spirit; if too thin a little mastic; if too soft some sandarach or benzoin. When about to use it warm the silvered plate before the fire, and with a flat camel-hair pencil stroke it over till no white streaks appear; this will preserve it for many years.

Brown Varnish.

Rectified spirit 2 gallons; sandarach 3 pounds; shell-lac 2 pounds; pale turpentine varnish 1 quart. Put them into a tin bottle, cork securely, and agitate frequently, placing the tin occasionally in hot water till the gum is dissolved, then add a quart of pale turpentine varnish.

Brilliant Amber Spirit Varnish.

Fused amber 4 oz.; sandarach 4 oz.; mastic 4 oz.; highly rectified spirit 1 quart. Expose to the heat of a sand bath, with occasional agitation, till dissolved. The amber is fused in a close copper vessel, having a funnel-shaped projection, which passes through the bottom of the furnace by which the vessel is heated.

Chinese Varnish.

Mastic 2 oz.; sandarach 2 oz.; rectified spirit 1 pint. Close the matrass with bladder, with a pin hole for the escape of vapor; heat to boiling in a sand or water bath, and when dissolved strain through linen.

Crystal Varnish.

Picked mastic 4 oz.; rectified spirit 1 pint; animal charcoal 1 oz. Digest, and filter.

Picture Varnish.

Chio turpentine 2 oz.; mastic 12 oz.; camphor ½ drachm; pounded glass 4 oz.; rectified oil of turpentine 3 pints. This is for oil paintings.

Canada Varnish.

Clear balsam of Canada 4 oz.; camphene 8 oz. Warm gently, and shake together till dissolved. This varnish is for maps, drawings, &c., which must be first sized over with a solution of isinglass, taking care that every part is covered. When dry, the varnish is brushed over it.

Tingry's Essence Varnish.

Powdered mastic 12 oz.; pure turpentine 1½ oz.; camphor ½ oz.; powdered glass 5 oz.; rectified oil of turpentine 1 quart.

Common Turpentine Varnish.

This is merely clear pale resin, dissolved in oil of turpentine; usually 5 pounds of resin to 7 pounds of turpentine.

Amber Varnish.

Amber 16 oz.; melt in an iron pot, and add ½ pint of drying linseed oil, boiling hot, and add 3 oz. resin, and 3 oz. asphalte, each in fine powder. Stir till they are thoroughly incorporated; remove from the fire, and add a pint of warm oil of turpentine.

Balloon Varnish.

Melt india-rubber in small pieces with its weight of boiled linseed oil, and thin it with oil of turpentine.

Varnish for Engraving on Copper.

Yellow wax 1 oz.; mastic 1 oz.; asphaltum ½ oz. Melt, pou into water, and form into balls for use. A softer varnish fo engravers is made thus: Tallow 1 part, and 2 of yellow wax; o with 2 oz. wax, 1 drachm common turpentine, and 1 drachm oliv oil.

Etching Varnishes.

White wax 2 oz.; asphaltum 2 oz. Melt the wax in a clea pipkin, add the asphaltum in powder, and boil to a proper con sistence. Pour it into warm water, and form it into balls, whicl must be kneaded, and put into taffeta for use.

Another.

White wax 2 oz.; Burgundy pitch ½ oz; black pitch ½ oz.; mel together, and add by degrees 2 oz. powdered asphaltum, and boi it till a drop cooled on a plate becomes brittle.

Another.

Equal quantities of linseed oil and mastic, melted together.

Engraving Mixture for Writing on Steel.

Sulphate of copper 1 oz.; sal ammoniac ½ oz. Pulverize sepa rately, adding a little vermilion to color it, and mix with 1½ oz vinegar. Rub the steel with soft soap, and write with a hard clean pen, without a slit, dipped in the mixture.

Etching Fluids.

For COPPER.—1. Aquafortis 2 oz.; water 5 oz. Mix.

2. *Callot's Eau Forte for Fine Touches.*—Dissolve 4 parts each o verdigris, alum, sea salt, and sal ammoniac, in 8 parts vinegar; add 16 parts water, boil for a minute, and let it cool.

For STEEL.—1. Iodine 1 oz.; iron filings ½ drachm; water 4 oz. Digest till the iron is dissolved.

2. Pyroligneous acid 4 parts by measure; alcohol 1 part. Mix, and add 1 part double aquafortis (sp. gr. 1·28). Apply it from 1½ to 15 minutes.

Varnish for Engraving on Glass.

Wax 1 oz.; mastic ½ oz.; asphaltum ½ oz.; turpentine ½ drachm.

Another.

Mastic 15 parts; turpentine 7; oil of spike 4.

Le Blond's Varnish.

Keep 4 pounds balsam of copaiva warm in a sand or water bath, and add 16 oz. of copal, previously fused and coarsely powdered, by single ounces, daily, and stir it frequently. When dissolved add a little Chio turpentine.

Sealing Wax Varnish.

Black or colored sealing wax, broken small, and sufficient rectified spirit to cover it; digest till dissolved.

Black Japan.

Boil together a gallon of boiled linseed oil, 8 oz. umber, and 3 oz. asphaltum. When sufficiently cool thin it with oil of turpentine.

Brunswick Black.

Melt 4 pounds asphaltum, add 2 pounds hot linseed oil, and when sufficiently cool add 1 gallon oil of turpentine.

Varnish for Gun Barrels, after browning them.

Shell-lac 1 oz; dragon's blood ½ oz.; rectified spirit 1 quart. Dissolve and filter.

Transfer Varnish.

Alcohol 5 oz.; pure Venice turpentine 4 oz.; mastic 1 oz.

Hair Varnish.

Dissolve 1 part of clippings of pigs' bristles, or horsehair, in 10 parts of drying linseed oil, by heat. Fibrous materials (cotton, flax, silk, &c.), imbued with the varnish and dried, are used as a substitute for hair cloth.

Glass Varnish.

This is a solution of soluble glass, and is thus made: Fuse together 15 parts powdered quartz (or fine sand), 10 parts potash, and 1 charcoal. Pulverize the mass, and expose it for some days to the air; treat the whole with cold water, which removes the foreign salts, &c.; boil the residue in 5 parts of water until it dissolves. It is permanent in the air, and not dissolved by water. This varnish is used to protect wood, &c., from fire.

Varnish for Gilded Articles.

Gum-lac 4 parts; dragon's blood 4; annatto 4; gamboge 4; saffron 1. Dissolve each resin separately in 8 parts alcohol, and make a separate tincture with the dragon's blood and annatto, also in 8 parts alcohol each; then mix the former together, and add a sufficient quantity of the tinctures to give the required shade and color to the varnish.

Gold Varnishes.

Turmeric 1 drachm; gamboge 1 drachm; oil of turpentine 2 pints; shell-lac 5 ounces; sandarach 5 oz.; dragon's blood 7 drachms; thin mastic varnish 8 oz. Digest, with occasional agitation, for fourteen days, in a warm place; then set it aside to fine, and pour off the clear.

Another.

Dutch leaf 1 part; gamboge 4; gum dragon 4; proof spirit 18. Macerate for twelve hours, then grind on a stone slab.

Varnish for Water Color Drawings.

Canada balsam 1 pint; oil of turpentine 2 parts, mixed. Size the drawing before applying the varnish.

Earthenware Varnish.

Flint glass 1 part; soda 1. Mix.

Magilp.

Mastic varnish 1 part; drying oil 2. Mix.

Another.

Mastic varnish 1 part; drying oil 1. Mix.

Another.

Equal parts of mastic varnish, drying oil, and turpentine. Mix.

Metallic Varnish for Coach Work, &c.

Asphaltum 56 pounds. Melt, then add litharge 9 pounds; red lead 7 pounds; boil, then add boiled oil 12 gallons; yellow resin 12 pounds. Again boil, until in cooling the mixture may be rolled into pills; then add spirit of turpentine 30 gallons; lampblack 7 pounds. Mix well.

Impermeable Varnish.

Boiled oil 100 parts; finely powdered litharge 6 parts; genuine beeswax 5 parts. Boil until sufficiently thick and stringy, then pour off the clear.

Engravers' Stopping-out Varnish.

Take lampblack and turpentine to make a paste.

PRACTICAL TABLES.

WEIGHT OF METALS—WROUGHT IRON; SQUARE, ROUND, AND FLAT.

Table I. contains the weight of Square Iron in sizes, from $\frac{1}{4}$ inch to six inches square, advancing by $\frac{1}{8}$ inch; and from 6 to 12 inches square, advancing by $\frac{1}{4}$ inch; and in lengths, from 1 foot to 18 feet. The sizes are arranged in the first column of each page, and the lengths along the top; the weight in lbs. immediately under the lengths, and in a line with the sizes.

Table II. contains the weight of Round Iron in sizes from $\frac{1}{4}$ inch to 6 inches diameter, advancing by $\frac{1}{8}$ inch; and from 6 to 12 inches diameter, advancing by $\frac{1}{4}$ inch; and in lengths, from 1 foot to 18 feet. The sizes, lengths, and weights are arranged as in Table I.

Table III. contains the weight of Flat Iron in widths, from $\frac{1}{4}$ inch to 6 inches diameter, advancing by $\frac{1}{4}$ inch; in thicknesses, from $\frac{1}{4}$ inch to 1 inch, advancing by $\frac{1}{8}$ inch; and in lengths, from 1 to 18 feet. The widths, lengths, and weights are arranged as in the preceding tables, and the thicknesses alongside of the widths.

The tables are all calculated to the nearest tenth of a pound. To the weights of bars of Wrought Iron add $\frac{1}{160}$th part for bars of soft steel; and from the same weights subtract $\frac{1}{14}$th part for bars of Cast Iron.

TABLE I.

SQUARE IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	0·2	0·4	0·6	0·8	1·1	1·3	1·5	1·7	1·9
⅜	0·5	1·0	1·4	1·9	2·4	2·9	3·3	3·8	4·3
½	0·8	1·7	2·5	3·4	4·2	5·1	5·9	6·8	7·6
⅝	1·3	2·6	4·0	5·3	6·6	7·9	9·2	10·6	11·9
¾	1·9	3·8	5·7	7·6	9·5	11·4	13·3	15·2	17·1
⅞	2·6	5·2	7·8	10·4	12·9	15·5	18·1	20·7	23·3
1	3·4	6·8	10·1	13·5	16·9	20·3	23·7	27·0	30·4
1⅛	4·3	8·6	12·8	17·1	21·4	25·7	29·9	34·2	38·5
1¼	5·3	10·6	15·8	21·1	26·4	31·7	37·0	42·2	47·5
1⅜	6·4	12·8	19·2	25·6	32·0	38·3	44·7	51·1	57·5
1½	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·8	68·4
1⅝	8·9	17·9	26·8	35·7	44·6	53·6	62·5	71·4	80·3
1¾	10·4	20·7	31·1	41·4	51·8	62·1	72·5	82·8	93·2
1⅞	11·9	23·8	35·6	47·5	59·4	71·3	83·2	95·1	106·9
2	13·5	27·0	40·6	54·1	67·6	81·1	94·6	108·2	121·7
2⅛	15·3	30·5	45·8	61·1	76·3	91·6	106·8	122·1	137·4
2¼	17·1	34·2	51·3	68·4	85·6	102·7	119·8	136·9	154·0
2⅜	19·1	38·1	57·2	76·3	95·3	114·4	133·5	152·5	171·6
2½	21·1	42·2	63·4	84·5	105·6	126·7	147·8	169·0	190·1
2⅝	23·3	46·6	69·9	93·2	116·5	139·8	163·0	186·3	209·6
2¾	25·6	51·1	76·7	102·2	127·8	153·4	178·9	204·5	230·0
2⅞	27·9	55·9	83·8	111·8	139·7	167·6	195·7	223·5	251·5
3	30·4	60·8	91·2	121·7	152·1	182·5	212·9	243·3	273·7
3⅛	33·0	66·0	99·0	132·0	165·1	198·1	231·1	264·1	297·1
3¼	35·7	71·4	107·1	142·8	178·5	214·2	249·9	285·6	321·3
3⅜	38·5	77·0	115·5	154·0	192·5	231·0	269·5	308·0	346·5
3½	41·4	82·8	124·2	165·6	207·0	248·4	289·8	331·3	372·7
3⅝	44·4	88·8	133·3	177·7	222·1	266·5	310·9	355·3	399·8
3¾	47·5	95·1	142·6	190·1	237·7	285·2	332·7	380·3	427·8
3⅞	50·8	101·5	152·3	203·0	253·8	304·5	355·3	406·0	456·8

TABLE I.

SQUARE IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	2·1	2·3	2·5	2·7	3·0	3·2	3·4	3·6	3·8
⅜	4·8	5·2	5·7	6·2	6·7	7·1	7·6	8·1	8·6
½	8·5	9·3	10·1	11·0	11·8	12·0	13·5	14·4	15·2
⅝	13·2	14·5	15·8	17·2	18·5	19·8	21·1	22·4	23·8
¾	19·0	20·9	22·8	24·7	26·6	28·5	30·4	32·3	34·2
⅞	25·9	28·5	31·1	33·6	36·2	38·8	41·4	44·0	46·6
1	33·8	37·2	40·6	43·9	47·3	50·7	54·1	57·5	60·8
1⅛	42·8	47·1	51·3	55·6	59·9	64·2	68·4	72·7	77·0
1¼	52·8	58·1	63·4	68·6	73·9	79·2	84·5	89·8	95·0
1⅜	63·9	70·8	76·7	83·1	89·5	95·9	102·2	108·6	115·0
1½	76·0	83·6	91·2	98·9	106·5	114·1	121·7	129·3	136·9
1⅝	89·3	98·2	107·1	116·0	125·0	133·9	142·8	151·7	160·7
1¾	103·5	133·9	124·2	134·6	144·9	155·3	165·6	176·0	186·3
1⅞	118·8	130·7	142·6	154·5	166·4	178·2	190·1	202·0	213·9
2	135·2	148·7	162·2	175·8	189·3	202·8	216·3	229·8	243·4
2⅛	152·6	167·9	183·2	198·4	213·7	228·9	244·2	259·5	274·7
2¼	171·1	188·2	205·3	222·5	239·6	256·7	273·8	290·9	308·0
2⅜	190·7	209·7	228·8	247·9	266·9	286·0	305·1	324·1	343·2
2½	211·2	232·3	253·4	274·6	295·7	316·8	337·9	359·0	380·2
2⅝	232·9	256·2	279·5	302·8	326·1	349·4	372·7	396·0	419·3
2¾	255·6	281·2	306·7	332·3	357·8	383·4	409·0	434·5	460·1
2⅞	279·4	307·3	335·3	363·2	391·1	419·1	447·0	475·0	502·9
3	304·2	334·6	365·0	395·4	425·8	456·2	486·7	517·1	547·5
3⅛	330·1	363·1	396·1	429·1	462·1	495·2	528·2	561·2	594·2
3¼	357·0	392·7	428·4	464·2	499·9	535·6	571·3	607·0	642·7
3⅜	385·0	423·5	462·0	500·5	539·0	577·5	616·0	654·6	693·1
3½	414·1	455·5	496·9	538·3	579·7	621·1	662·5	703·9	745·3
3⅝	444·2	488·6	533·0	577·4	621·9	666·3	710·7	755·1	799·5
3¾	475·3	522·9	570·4	617·9	665·5	713·0	760·5	808·1	855·6
3⅞	507·6	558·3	609·1	659·8	710·6	761·3	812·1	862·9	913·6

TABLE I.

SQUARE IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
4	54·1	108·2	162·3	216·3	270·4	324·5	378·6	432·7	486·8
4⅛	57·5	115·0	172·6	230·1	287·6	345·1	402·6	460·1	517·7
4¼	61·1	122·1	183·2	244·2	305·3	366·3	427·4	488·4	549·5
4⅜	64·7	129·4	194·1	258·8	323·5	388·2	452·9	517·6	582·3
4½	68·4	136·9	205·3	273·8	342·2	410·7	479·1	547·6	616·0
4⅝	72·3	144·6	216·9	289·2	361·5	433·8	506·1	578·4	650·7
4¾	76·3	152·5	228·8	305·1	381·3	457·6	533·8	610·1	686·4
4⅞	80·3	160·7	241·0	321·3	401·7	482·0	562·3	642·7	723·0
5	84·5	169·0	253·4	337·9	422·4	506·9	591·4	675·8	760·3
5⅛	88·8	177·6	266·4	355·1	443·9	532·7	621·5	710·3	799·1
5¼	93·2	186·3	279·5	372·7	465·8	559·0	652·2	745·3	838·5
5⅜	97·7	195·3	293·0	390·6	488·3	585·9	683·6	781·3	878·9
5½	102·2	204·5	306·7	409·0	511·2	613·4	715·7	817·9	920·2
5⅝	107·0	213·5	320·9	427·8	534·8	641·7	748·7	855·6	962·6
5¾	111·8	223·5	335·3	447·0	558·8	670·5	782·3	894·0	1005·8
5⅞	116·7	233·3	350·0	466·7	583·4	700·0	816·7	933·4	1050·0
6	121·7	243·3	365·0	486·7	608·3	730·0	841·6	973·3	1095·0
6¼	132·0	264·1	396·1	528·2	660·2	792·2	924·3	1056·0	1188·4
6½	142·8	285·6	428·4	571·3	714·1	856·9	999·7	1142·5	1285·3
6¾	154·0	308·0	462·0	616·0	770·1	924·1	1078·1	1232·1	1386·1
7	165·6	331·2	496·9	662·5	828·2	993·8	1159·4	1325·1	1490·7
7¼	177·7	355·3	533·0	710·7	888·4	1066·0	1243·7	1421·4	1599·0
7½	190·1	380·2	570·4	760·5	950·7	1140·8	1331·0	1521·1	1711·2
7¾	203·0	406·0	609·1	812·1	1015·1	1218·1	1421·2	1624·2	1827·2
8	216·3	432·7	649·0	865·3	1081·7	1298·0	1514·4	1730·7	1947·0
8¼	230·1	460·1	690·2	920·3	1150·3	1380·4	1610·5	1840·5	2070·6
8½	244·2	488·4	732·7	976·9	1221·1	1465·3	1709·5	1953·8	2198·0
8¾	258·8	517·6	776·4	1035·2	1294·0	1552·8	1811·6	2070·4	2329·2
9	273·8	547·6	821·4	1095·2	1369·0	1642·8	1916·5	2190·3	2464·1

TABLE I.

SQUARE IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
4	540·8	594·9	649·0	703·1	757·2	811·3	865·3	919·4	973·5
4⅛	575·2	632·7	690·2	747·7	805·2	862·8	920·3	977·8	1035·3
4¼	610·6	671·6	732·7	793·7	854·8	915·8	976·9	1037·9	1099·0
4⅜	646·0	711·7	776·4	841·1	905·8	970·5	1035·2	1099·9	1164·6
4½	684·5	752·9	821·4	889·8	958·3	1026·7	1095·2	1163·6	1232·1
4⅝	723·1	795·4	867·7	940·0	1012·3	1084·6	1156·9	1229·2	1301·5
4¾	762·6	838·9	915·2	991·4	1067·7	1144·0	1220·2	1296·5	1372·8
4⅞	803·3	883·7	964·0	1044·3	1124·7	1205·0	1285·3	1365·7	1446·0
5	844·8	929·3	1013·8	1098·2	1182·7	1267·2	1351·7	1436·2	1520·6
5⅛	887·8	976·6	1065·4	1154·2	1243·0	1331·8	1420·5	1509·3	1598·1
5¼	931·7	1024·8	1118·0	1211·2	1304·4	1397·5	1490·7	1583·9	1677·0
5⅜	976·6	1074·2	1171·9	1269·5	1367·2	1464·9	1562·5	1660·2	1757·8
5½	1022·4	1124·6	1226·9	1329·1	1431·4	1533·6	1635·8	1738·1	1840·3
5⅝	1069·5	1176·5	1283·4	1390·4	1497·3	1604·3	1711·2	1818·2	1925·2
5¾	1117·6	1229·3	1341·1	1452·8	1564·6	1676·3	1788·1	1899·9	2011·6
5⅞	1160·6	1283·4	1400·1	1516·7	1633·4	1750·1	1866·7	1983·4	2100·1
6	1229·6	1338·3	1460·0	1581·6	1703·3	1825·0	1946·6	2068·3	2190·0
6¼	1320·4	1452·4	1584·4	1716·5	1848·6	1980·6	2112·6	2244·7	2376·7
6½	1428·2	1571·0	1713·8	1856·6	1999·4	2142·2	2285·1	2427·9	2570·7
6¾	1540·1	1694·1	1848·1	2002·2	2056·2	2310·2	2464·2	2618·2	2772·2
7	1656·3	1822·0	1987·6	2153·2	2318·8	2484·5	2650·1	2815·7	2981·4
7¼	1776·7	1954·4	2132·1	2309·7	2487·4	2665·1	2842·8	3020·4	3198·1
7½	1901·4	2091·5	2281·6	2471·8	2661·9	2852·0	3042·2	3232·3	3422·4
7¾	2030·2	2233·3	2436·3	2639·3	2842·3	3045·4	3248·4	3451·4	3654·4
8	2163·4	2379·7	2596·0	2812·4	3028·7	3245·0	3461·4	3677·7	3894·0
8¼	2300·7	2530·7	2760·8	2990·9	3220·9	3451·0	3681·1	3911·1	4141·2
8½	2442·2	2686·4	2930·6	3174·9	3419·1	3663·3	3907·5	4151·7	4396·0
8¾	2588·0	2846·8	3105·6	3364·4	3623·2	3882·0	4140·8	4399·6	4658·4
9	2737·9	3011·7	3285·5	3559·3	3833·1	4106·9	4380·7	4654·5	4928·3

TABLE I.

SQUARE IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
9¼	289·2	578·4	867·7	1156·9	1446·1	1735·3	2024·5	2313·8	2603·0
9½	305·1	610·1	915·2	1220·2	1525·3	1830·3	2135·4	2440·4	2745·5
9¾	321·3	642·7	964·0	1285·3	1606·7	1928·0	2249·3	2570·7	2892·3
10	337·9	675·8	1013·8	1351·7	1689·6	2027·5	2365·4	2703·4	3041·0
10¼	355·1	710·2	1065·4	1420·5	1775·7	2130·8	2486·0	2841·1	3196·2
10½	372·7	745·3	1118·0	1490·7	1863·4	2236·0	2608·7	2981·4	3354·0
10¾	390·6	781·3	1171·9	1562·5	1953·1	2343·8	2734·4	3125·0	3515·7
11	409·0	817·9	1226·9	1635·8	2044·8	2453·8	2862·7	3271·7	3680·6
11¼	427·8	855·6	1283·4	1711·2	2139·1	2566·9	2994·7	3422·5	3850·3
11½	447·0	894·0	1341·1	1788·1	2235·1	2682·1	3129·2	3576·2	4023·2
11¾	466·7	933·4	1400·1	1866·7	2333·4	2800·1	3266·8	3733·5	4200·2
12	486·7	973·3	1460·0	1946·6	2433·3	2919·9	3406·6	3893·2	4379·9

GLAZES.—Common earthenware is glazed with a composition containing lead, on which account it is unfit for many pharmaceutical purposes. The following glaze has been proposed, among others, as a substitute: 100 parts of washed sand, 80 of purified potash, 10 of nitre, and 20 of slaked lime; all well mixed, and heated in a blacklead crucible, in a reverberatory furnace, till the mass flows into a clear glass. It is then to be reduced to powder. The goods to be slightly burnt, placed under water, and sprinkled with the powder.

GLAZE FOR PORCELAIN.—Feldspar 27 parts, borax 18, Lynn sand 4, nitre 3, soda 3, Cornwall china clay 3 parts. Melt together to form a frit, and reduce it to a powder, with 3 parts of calcined borax.

SOLVENT FOR OLD PUTTY AND PAINT.—Soft soap mixed with solution of potash or caustic soda; or pearlash and slaked lime mixed with sufficient water to form a paste. Either of these laid on with an old brush or rag, and left for some hours, will render it easily removable.

TABLE I.

SQUARE IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
9¼	2892·2	3181·4	3470·6	3759·9	4049·1	4338·3	4627·5	4916·7	5206·0
9½	3050·6	3355·6	3660·7	3965·7	4270·8	4575·8	4880·9	5186·0	5491·0
9¾	3213·3	3534·7	3856·4	4177·3	4498·6	4820·0	5141·3	5462·6	5784·0
10	3879·2	3717·1	4055·0	4393·0	4730·9	5068·8	5406·7	5744·6	6082·6
10¼	3551·4	3906·5	4261·6	4616·8	4971·9	5327·0	5682·2	6037·3	6392·4
10½	3726·7	4099·4	4472·1	4844·7	5217·4	5590·1	5962·8	6335·4	6708·1
10¾	3906·3	4297·0	4687·5	5078·2	5468·8	5859·4	6250·0	6644·7	7031·3
11	4089·6	4498·6	4907·4	5316·5	5725·4	6134·4	6543·4	6952·3	7361·3
11¼	4278·1	4705·9	5133·7	5561·6	5989·4	6417·2	6845·0	7272·8	7700·6
11½	4470·2	4917·3	5364·3	5811·3	6258·3	6705·4	7152·4	7599·4	8046·4
11¾	4666·8	5133·5	5600·2	6066·9	6533·6	7000·3	7466·9	7933·6	8400·3
12	4866·6	5353·2	5839·9	6326·5	6813·2	7299·8	7786·5	8273·2	8759·8

SCOURING DROPS FOR REMOVING GREASE.—1. Alcohol (pure) 6 oz., camphor 2 oz., rectified essence of lemon 8 oz.

2. Camphene 3 oz., essence of lemon 1 oz. Mix. Some direct them to be distilled together.

3. *French.* Camphene 8 oz., pure alcohol 1 oz., sulphuric ether 1 oz., essence of lemon 1 dr.

4. Spirits of wine 1 pint, white soap 3 oz., ox-gall 3 oz., essence of lemon ½ oz.

BALLS, HEEL.—1. Melt together 4 oz. of mutton suet, 1 oz. of beeswax, 1 oz. of sweet oil, ½ oz. oil of turpentine, and stir in 1 oz. of powdered gum arabic, and ½ oz. of fine lampblack.

2. Beeswax 8 oz., tallow 1 oz., powdered gum 1 oz., lampblack q. s. These are used not merely by the shoemaker, but to copy inscriptions, raised patterns, &c., by rubbing the ball on paper laid over the article to be copied. For copying ancient monumental brasses, a similar compound, colored with bronze powder instead of lampblack, is sometimes employed.

TABLE II.

ROUND IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1/4	0·2	0·3	0·5	0·7	0·8	1·0	1·2	1·3	1·5
3/8	0·4	0·7	1·1	1·5	1·9	2·2	2·6	3·0	3·4
1/2	0·7	1·3	2·0	2·7	3·3	4·0	4·6	5·3	6·0
5/8	1·0	2·1	3·1	4·2	5·2	6·3	7·3	8·3	9·4
3/4	1·5	3·0	4·5	6·0	7·5	9·0	10·5	11·9	13·4
7/8	2·0	4·1	6·1	8·1	10·2	12·2	14·2	16·3	18·3
1	2·7	5·3	8·0	10·6	13·3	15·9	18·6	21·2	23·9
1 1/8	3·4	6·7	10·1	13·4	16·8	20·2	23·5	26·9	30·2
1 1/4	4·2	8·3	12·5	16·7	20·9	25·0	29·2	33·4	37·5
1 3/8	5·0	10·0	15·1	20·1	25·1	30·1	35·1	40·2	45·2
1 1/2	6·0	11·9	17·9	23·9	29·9	35·8	41·8	47·8	53·7
1 5/8	7·0	14·0	21·0	28·0	35·1	42·1	49·1	56·1	63·1
1 3/4	8·1	16·3	24·4	32·5	40·6	48·8	56·9	65·0	73·2
1 7/8	9·3	18·7	28·0	37·3	46·7	56·0	65·3	74·7	84·0
2	10·6	21·2	31·8	42·5	53·1	63·7	74·3	84·9	95·5
2 1/8	12·0	24·0	36·0	48·0	59·9	71·9	83·9	95·9	107·9
2 1/4	13·5	26·9	40·3	53·8	67·2	80·6	94·1	107·5	121·0
2 3/8	15·0	30·0	44·9	60·0	74·9	89·9	104·8	119·8	134·8
2 1/2	16·7	33·4	50·1	66·8	83·4	100·1	116·8	133·5	150·2
2 5/8	18·3	36·6	54·9	73·2	91·5	109·8	128·1	146·3	164·6
2 3/4	20·1	40·2	60·2	80·3	100·4	120·5	140·5	160·6	180·7
2 7/8	21·9	43·9	65·8	87·8	109·7	131·7	153·6	175·6	197·5
3	23·9	47·8	71·7	95·6	119·4	143·3	167·2	191·1	215·0
3 1/8	25·9	51·9	77·8	103·7	129·6	155·6	181·5	207·4	233·3
3 1/4	28·0	56·1	84·1	112·2	140·2	168·2	196·3	224·3	253·4
3 3/8	30·2	60·5	90·7	121·0	151·2	181·4	211·7	241·9	272·2
3 1/2	32·5	65·0	97·5	130·0	162·6	195·1	227·6	260·1	292·6
3 5/8	34·9	69·8	104·7	139·5	174·4	209·3	244·2	279·1	314·0
3 3/4	37·3	74·7	112·0	149·3	186·7	224·0	261·3	298·7	336·0
3 7/8	39·9	79·7	119·6	159·5	199·3	239·2	279·0	318·9	358·8

TABLE II.

ROUND IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	1·7	1·8	2·0	2·1	2·3	2·5	2·6	2·8	3·0
⅜	3·7	4·1	4·5	4·8	5·2	5·6	6·0	6·3	6·7
½	6·6	7·3	8·0	8·6	9·3	9·9	10·6	11·3	11·9
⅝	10·4	11·5	12·5	13·6	14·6	15·6	16·7	17·3	18·8
¾	14·9	16·4	17·9	19·4	20·9	22·4	23·9	25·4	26·9
⅞	20·3	22·4	24·4	26·4	28·4	30·5	32·5	34·5	36·6
1	26·5	29·2	31·8	34·5	37·2	39·8	42·5	45·1	47·8
1⅛	33·6	37·0	40·3	43·7	47·0	50·4	53·8	57·1	60·5
1¼	41·7	45·9	50·1	54·2	58·4	62·6	66·8	70·9	75·1
1⅜	50·2	55·2	60·2	65·2	70·3	75·3	80·3	85·3	90·3
1½	59·7	65·7	71·7	77·6	83·6	89·6	95·6	101·5	107·5
1⅝	70·1	77·1	84·1	91·1	98·1	105·2	112·2	119·2	126·2
1¾	81·3	89·4	97·5	105·7	113·8	121·9	130·0	138·2	146·3
1⅞	93·3	102·7	112·0	121·3	130·7	140·0	149·3	158·7	168·0
2	106·2	116·8	127·4	138·0	148·6	159·2	169·9	180·5	192·1
2⅛	119·9	131·9	143·9	155·8	167·8	179·8	181·8	193·8	205·8
2¼	134·4	147·8	161·3	174·7	188·2	201·6	215·0	228·5	241·9
2⅜	149·8	164·7	179·7	194·7	209·7	224·6	239·6	254·6	269·6
2½	166·9	183·6	200·3	216·9	233·6	250·3	267·0	283·7	300·4
2⅝	182·9	201·2	219·5	237·8	256·1	274·4	292·7	311·0	329·3
2¾	200·8	220·8	240·9	261·2	281·1	301·1	321·2	341·3	361·4
2⅞	219·4	241·4	263·4	285·3	307·2	329·2	351·1	373·0	395·0
3	238·9	262·8	286·7	310·5	334·4	358·3	382·2	406·1	430·0
3⅛	259·3	285·2	311·1	337·0	363·0	388·9	414·8	440·7	466·7
3¼	280·4	308·4	336·5	364·5	392·6	420·6	448·6	476·7	504·7
3⅜	302·4	332·6	362·9	393·1	423·4	453·6	483·8	514·1	544·3
3½	325·1	357·6	390·1	422·7	455·2	487·7	520·2	552·7	585·2
3⅝	348·9	383·7	418·6	455·5	488·4	523·3	558·2	593·1	627·9
3¾	373·3	410·7	448·0	486·3	522·6	560·0	597·3	634·6	672·0
3⅞	398·6	438·5	478·3	518·2	558·1	598·0	637·8	677·7	717·6

TABLE II.

ROUND IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
4	42·5	84·9	127·4	169·9	212·3	254·8	297·2	339·7	382·2
4⅛	45·2	90·3	135·5	180·7	225·9	271·0	316·2	361·4	406·6
4¼	48·0	95·9	143·9	191·8	239·8	287·7	335·7	383·6	431·6
4⅜	50·8	101·6	152·4	203·3	254·1	304·9	355·7	406·5	457·3
4½	53·8	107·5	161·3	215·0	268·8	322·6	376·3	430·1	483·8
4⅝	56·8	113·6	170·4	227·2	283·9	340·7	397·5	454·3	511·1
4¾	60·0	119·8	179·7	239·6	299·5	359·4	419·3	479·2	539·1
4⅞	63·1	126·2	189·3	252·4	315·5	378·6	441·7	504·8	567·8
5	66·8	133·5	200·3	267·0	333·8	400·5	467·3	534·0	600·8
5⅛	69·7	139·5	209·2	278·9	348·7	418·4	488·1	557·8	627·6
5¼	73·2	146·3	219·5	292·7	365·9	439·0	512·2	585·4	658·5
5⅜	76·7	153·4	230·1	306·8	383·5	460·2	536·9	613·6	690·3
5½	80·3	160·6	240·9	321·2	401·5	481·8	562·1	642·4	722·7
5⅝	84·0	168·0	252·0	336·0	420·0	504·0	588·0	672·0	756·0
5¾	87·8	175·6	263·3	351·1	438·9	526·7	614·4	702·2	790·0
5⅞	91·6	183·3	274·9	366·5	458·2	549·8	641·4	733·1	824·7
6	95·6	191·1	286·7	382·2	477·8	573·3	668·9	764·4	860·0
6¼	103·7	207·4	311·1	414·8	518·5	622·2	725·9	829·6	933·3
6½	112·2	224·3	336·5	448·6	560·8	673·0	785·1	897·3	1009·4
6¾	121·0	241·9	362·9	483·8	604·8	725·8	846·7	967·6	1088·6
7	130·0	260·1	390·1	520·2	650·2	780·3	910·3	1040·4	1170·4
7¼	139·5	279·1	418·6	558·2	697·7	837·3	976·8	1116·4	1255·9
7½	149·3	298·7	448·0	597·3	741·6	896·0	1045·3	1194·6	1344·0
7¾	159·5	318·9	478·4	637·8	797·3	956·7	1116·2	1275·6	1435·1
8	169·9	339·7	509·6	679·4	849·3	1019·1	1189·0	1358·8	1528·7
8¼	180·7	361·4	542·1	722·8	903·5	1084·2	1264·9	1445·6	1626·3
8½	191·8	383·6	595·4	767·2	959·0	1150·8	1342·6	1534·5	1726·3
8¾	203·3	406·5	609·8	813·0	1016·3	1219·6	1422·8	1626·1	1829·3
9	215·0	430·1	645·1	860·2	1075·2	1290·2	1505·3	1720·3	1935·4

TABLE II.

ROUND IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
4	424·6	467·1	509·6	552·0	594·5	637·0	676·4	721·9	764·[illegible]
4⅛	451·7	496·9	542·1	587·3	632·4	677·6	722·8	761·0	813·[illegible]
4¼	479·5	527·5	575·4	623·4	671·8	719·8	767·2	815·2	863·[illegible]
4⅜	508·2	559·0	609·8	660·6	711·4	762·2	813·0	863·9	914·[illegible]
4½	537·6	591·4	645·1	698·9	752·6	806·4	860·2	913·9	967·[illegible]
4⅝	567·9	624·7	681·[illegible]	738·2	795·0	851·8	908·6	965·4	1022·[illegible]
4¾	599·0	658·9	718·[illegible]	778·7	838·6	898·5	958·4	1018·3	1078·[illegible]
4⅞	630·9	694·0	757·1	820·2	883·3	946·4	1009·5	1072·6	1135·[illegible]
5	667·5	734·3	801·0	867·8	934·5	1001·3	1068·0	1134·8	1201·[illegible]
5⅛	697·3	767·0	836·8	906·5	976·2	1046·0	1115·7	1185·4	1255·[illegible]
5¼	731·7	804·9	878·1	951·2	1024·4	1097·6	1170·8	1243·9	1317·1
5⅜	767·0	813·7	920·4	997·1	1073·8	1150·5	1227·2	1303·9	1380·6
5½	803·0	883·3	963·6	1044·0	1124·3	1204·6	1284·9	1365·2	1445·5
5⅝	840·0	924·0	1008·0	1092·0	1176·0	1260·0	1344·0	1428·0	1512·0
5¾	877·8	965·5	1053·3	1141·1	1228·9	1316·6	1404·4	1492·2	1580·0
5⅞	916·3	1008·0	1099·6	1191·2	1282·9	1374·5	1466·1	1557·8	1649·4
6	955·5	1051·1	1146·6	1242·2	1337·7	1433·3	1528·8	1624·4	1719·9
6¼	1037·0	1140·7	1244·4	1348·2	1451·9	1555·6	1659·3	1763·0	1866·7
6½	1121·6	1233·8	1345·9	1458·1	1570·2	1682·4	1794·6	1906·7	2018·9
6¾	1209·6	1330·6	1451·5	1572·5	1693·4	1814·4	1935·4	2056·3	2177·3
7	1300·5	1430·5	1560·6	1690·6	1820·7	1950·7	2088·8	2210·8	2340·9
7¼	1395·4	1535·0	1674·5	1814·1	1953·6	2093·2	2232·7	2372·2	2511·8
7½	1493·3	1642·6	1791·9	1941·3	2090·6	2239·9	2389·2	2538·6	2687·9
7¾	1594·6	1754·0	1913·5	2072·9	2232·4	2391·8	2551·3	2710·8	2870·2
8	1698·6	1868·4	2038·3	2208·1	2378·0	2547·8	2717·7	2887·6	3057·4
8¼	1809·0	1987·7	2168·4	2349·0	2529·7	2740·4	2891·1	3071·8	3252·5
8½	1918·1	2109·9	2301·7	2493·5	2685·3	2879·1	3068·9	3260·7	3452·5
8¾	2032·6	2235·9	2439·1	2642·4	2845·6	3048·9	3252·2	3455·4	3658·7
9	2150·4	2365·4	2580·5	2795·5	3010·6	3225·6	3440·6	3655·7	3870·7

TABLE II.

ROUND IRON.

Size.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
9¼	227·2	454·3	681·5	908·6	1135·8	1362·9	1590·1	1817·2	2044 4
9½	239·6	479·2	718·8	958·4	1198·0	1437·6	1677·2	1916·8	2156·4
9¾	252·4	505·8	757·1	1009·5	1261·9	1514·3	1766·6	2019·0	2291·4
10	266·3	532·6	798·9	1065·2	1331·4	1597·7	1864·0	2130·3	2396·6
10¼	278·9	557·8	836·8	1115·7	1394·6	1673·5	1952·5	2231·4	2510·3
10½	292·7	585·4	878·1	1170·8	1463·4	1756·1	2048·8	2341·5	2634·2
10¾	306·8	603·6	920·4	1227·2	1534·0	1840·8	2147·6	2454·4	2761·2
11	321·2	642·4	963·6	1284·9	1606·1	1927·3	2248·5	2569·7	2890·9
11¼	336·0	672·0	1008·0	1344·0	1680·0	2016·0	2352·0	2688 0	3024·0
11½	351·1	702·2	1053·3	1404·4	1755·5	2106·6	2457·7	2808·8	3159·9
11¾	366·5	733·1	1099·6	1466·1	1832·7	2199·2	2565·8	2932·3	3298 8
12	382·2	764·4	1146·5	1528·8	1911·0	2293·2	2675·5	3057·7	3439·9

BRONZING LIQUIDS, FOR BRONZING COPPER MEDALS, FIGURES, INSTRUMENTS, &c.—1. Sal ammoniac 1 dr., oxalic acid 15 gr., vinegar 1 pint. After well cleaning the article to be bronzed, warm it gently, and brush it over with the liquid, using only a small quantity at a time. When rubbed dry, repeat the application till the desired tint is obtained. [For copper medals, electrotype casts, &c.]

2. Sal ammoniac 1 oz., cream of tartar 3 oz., salt 6 oz. Dissolve in a pint of hot water, add 2 oz. of nitre, and 2 oz. of nitrate of copper dissolved in ½ pint of water.

3. Salt of sorrel 1 oz., sal ammoniac 2 oz., white vinegar 14 oz. [To give an antique appearance to bronze figures, &c.]

4. A diluted solution of muriate of platina. [For copper binding screws, and other small articles.]

5. A weak solution of hydro-sulphuret of ammonia, or of sulphuret of potassium. [For electrotype medals. Another method is the following: Immediately on removing the electrotype cast from the solution, brush it over with good black lead; then heat it moderately, and brush it over with a painting brush, the slightest moisture being used.]

TABLE II.

ROUND IRON.

Size.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
Inch.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
9¼	2271·5	2498·7	2725·8	2953·0	3180·1	3407·3	3634·4	3861·6	4088·7
9½	2396·0	2635·6	2875·2	3114·8	3354·4	3594·0	3833·6	4073·2	4312·8
9¾	2523·8	2776·1	3028·5	3280·9	3533·3	3785·6	4038·0	4290·4	4542·8
10	2662·9	2929·2	3195·5	3461·7	3728·0	3994·3	4260·6	4526·9	4793·2
10¼	2789·2	3068·2	3347·1	3626·0	3904·9	4188·8	4462·8	4741·7	5020·6
10½	2926·9	3219·6	3512·3	3804·9	4097·6	4390·3	4688·0	4975·7	5268·4
10¾	3068·0	3374·8	3681·6	3988·4	4295·2	4602·0	4908·8	5215·6	5522·4
11	3212·2	3533·4	3854·6	4175·8	4497·0	4818·2	5139·5	5460·7	5781·9
11¼	3360·0	3696·0	4032·0	4368·1	4704·1	5040·1	5376·1	5712·1	6048·1
11½	3511·0	3862·1	4213·2	4564·4	4915·5	5266·6	5619·7	5968·8	6319·9
11¾	3665·4	4031·9	4398·4	4765·0	5131·5	5498·0	5864·6	6231·1	6597·6
12	3822·1	4204·3	4586·5	4968·7	5350·9	5733·1	6115·3	6497·5	6879·7

SOLUTIONS USED IN ELECTROTYPE MANIPULATIONS, &c.

1. *Acid Solution of Copper for the Decomposing Cell.* Saturated solution of sulphate of copper 2 parts, sulphuric acid 2 parts, water 6 or 8 parts.

2. *Gold Solution.* Dissolve 2 oz of cyanide of potassium (by Liebig's method) in a pint of warm distilled water, add ¼ oz. of oxide of gold, and agitate together.

3. *Silver Solution.* Dissolve 2 oz. of Liebig's cyanide of potassium in a pint of distilled water; add ¼ oz. of moist oxide of silver (precipitated by lime water from a solution of the crystallized nitre), and agitate together till the oxide is dissolved.

4. *Solution in which Steel Articles are dipped before Electroplating them.* Nitrate of silver 1 part, nitrate of mercury 1 part, nitric acid (sp. gr. 1·384) 4 parts, water 120 parts.

5. *Solution, or Pickle, for immersing Copper Articles in before Electroplating.* Sulphuric acid 64 parts, water 64, nitric acid 32, muriatic acid 1. Mix. The article, free from grease, is dipped in the pickle for a second or two.

TABLE III.

FLAT IRON.

Thick.	Width	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	1	0·8	1·7	2·5	3·4	4·2	5·1	5·9	6·8	7·6
¼	1¼	1·1	2·1	3·2	4·2	5·3	6·3	7·4	8·4	9·5
¼	1½	1·3	2·5	3·8	5·1	6·3	7·6	8·9	10·1	11·4
¼	1¾	1·5	3·0	4·4	5·9	7·4	8·9	10·4	11·8	13·3
¼	2	1·7	3·4	5·1	6·8	8·5	10·1	11·8	13·5	15·2
¼	2¼	1·9	3·8	5·7	7·6	9·5	11·4	13·3	15·2	17·1
¼	2½	2·1	4·2	6·3	8·4	10·6	12·7	14·8	16·9	19·0
¼	2¾	2·3	4·6	7·0	9·3	11·6	13·9	16·3	18·6	20·9
¼	3	2·5	5·1	7·6	10·1	12·7	15·2	17·7	20·3	22·8
¼	3¼	2·7	5·5	8·2	11·0	13·7	16·5	19·2	22·0	24·7
¼	3½	3·0	5·9	8·9	11·8	14·8	17·7	20·7	23·7	26·6
¼	3¾	3·2	6·3	9·5	12·7	15·8	19·0	22·2	25·4	28·5
¼	4	3·4	6·8	10·1	13·5	16·9	20·3	23·7	27·0	30·4
¼	4¼	3·6	7·2	10·8	14·4	18·0	21·5	25·1	28·7	32·3
¼	4½	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	34·2
¼	4¾	4·0	8·0	12·0	16·1	20·1	24·1	28·1	32·1	36·1
¼	5	4·2	8·4	12·7	16·9	21·1	25·3	29·6	33·8	38·0
¼	5¼	4·4	8·9	13·3	17·7	22·2	26·6	31·1	35·5	39·9
¼	5½	4·6	9·3	13·9	18·6	23·2	27·9	32·5	37·2	41·8
¼	5¾	4·9	9·7	14·6	19·4	24·3	29·2	34·0	38·9	43·7
¼	6	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
⅜	1	1·3	2·5	3·8	5·1	6·3	7·6	8·9	10·1	11·4
⅜	1¼	1·6	3·2	4·8	6·3	7·9	9·5	11·1	12·7	14·3
⅜	1½	1·9	3·8	5·7	7·6	9·5	11·4	13·3	15·2	17·1
⅜	1¾	2·2	4·4	6·7	8·9	11·1	13·3	15·5	17·7	20·0
⅜	2	2·5	5·1	7·6	10·1	12·7	15·2	17·7	20·3	22·8
⅜	2¼	2·9	5·7	8·3	11·4	14·3	17·1	20·0	22·8	25·7
⅜	2½	3·2	6·3	9·5	12·7	15·8	19·0	22·2	25·4	28·5

TABLE III.

FLAT IRON.

Thick.	Width	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
¼	1	8·5	9·3	10·1	11·0	11·8	12·7	13·5	14·4	15·2
¼	1¼	10·6	11·6	12·7	13·7	14·8	15·8	16·9	17·9	19·0
¼	1½	12·7	13·9	15·2	16·5	17·7	19·0	20·3	21·5	22·8
¼	1¾	14·8	16·3	17·7	19·2	20·7	22·2	23·7	25·1	26·6
¼	2	16·9	18·6	20·3	22·0	23·7	25·4	27·0	28·7	30·4
¼	2¼	19·0	20·9	22·8	24·7	26·6	28·5	30·4	32·3	34·2
¼	2½	21·1	23·2	25·3	27·5	29·6	31·7	33·8	35·9	38·0
¼	2¾	23·2	25·6	27 9	30·2	32·5	34·9	37·2	39·5	41·8
¼	3	25·3	27·9	30·4	33·0	35·5	38·0	40·6	43·1	45·6
¼	3¼	27·5	30·2	33·0	35·7	38·5	41·3	43·9	46·7	49·4
¼	3½	29·6	32·5	35·5	38·5	41·4	44·4	47·3	50·3	53·2
¼	3¾	31·7	34·9	38·0	41·2	44·4	47·5	50·7	53·9	57·0
¼	4	33·8	37·2	40·6	43·9	47 3	50·7	54·1	57·5	60·8
¼	4¼	35·9	39·5	43·1	46·7	50·3	53·9	57·5	61·0	64·6
¼	4½	38·0	41·8	45·6	49·4	53·2	57·0	60·8	64·6	68·4
¼	4¾	40·1	44·1	48·2	52·2	56·2	60·2	64·2	68·2	72·2
¼	5	42·2	46·5	50·7	54·9	59·1	63·4	65·6	71·8	76·0
¼	5¼	44·4	48·8	53·2	57·7	62·1	66·5	71·0	75·4	79·9
¼	5½	46·5	51·1	55·8	60·4	65·1	69·7	74 4	79·0	83·6
¼	5¾	48·6	53·4	58·3	63·2	68·0	72·9	77·7	82·6	87·5
¼	6	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·2
⅜	1	12·7	13·9	15·2	16·5	17·7	19·0	20·3	21·5	22 8
⅜	1¼	15·8	17·4	19·0	20·6	22 2	23·8	25·3	28 9	28·5
⅜	1½	19 0	20·9	22·8	24·7	26·6	28·5	30·4	32·3	34·2
⅜	1¾	22 2	24·4	26·6	28·8	31·1	33·3	35·5	37·7	39·9
⅜	2	25·3	27·9	30·4	33 0	35·5	38·0	40·6	43·1	45·6
⅜	2¼	28·5	31·4	34 2	37·1	39·9	42 8	45·6	48·5	51·3
⅜	2½	31·7	34·9	38·0	41·2	44·4	47·5	50·7	53·9	57·0

TABLE III.

FLAT IRON.

Thick.	Width.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
	2¾	3·5	7·0	10·5	13·9	17·4	20·9	24·4	27·9	31·4
	3	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	34·2
	3¼	4·1	8·2	12·4	16·5	20·6	24·7	28·8	33 0	37·1
	3½	4·4	8·9	13·3	17·7	22·2	26·6	31·1	35·5	39·9
	3¾	4·8	9·5	14·3	19·0	23·8	28·5	33·3	38·0	42·8
	4	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
	4¼	5·4	10·8	16·1	21·5	26·9	32·3	37·7	43·1	48·5
	4½	5·7	11·4	17·1	22·8	28·5	34·2	39·9	45·6	51·3
	4¾	6·0	12·0	18·1	24·1	30·1	36 1	42·1	48·2	54·2
	5	6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·0
	5¼	6·7	13 3	20·0	26·6	33·3	39·9	46·6	53·2	59·9
	5½	7·0	13·9	20·9	27·9	34·9	41·8	48·8	55·8	62·7
	5¾	7·3	14·6	21·9	29 2	36·4	43·7	51·0	58·3	65·6
	6	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·8	68·4
	1	1·7	3·4	5·1	6·8	8·5	10·1	11·8	13·5	15·2
	1¼	2·1	4·2	6·3	8·4	10·6	12·7	14·8	16·9	19·0
	½	2·5	5·1	7·6	10·1	12·7	15·2	17·7	20·3	22·8
	¾	3·0	5·9	8·9	11·8	14·8	17·7	20·7	23·7	26·6
		3·4	6·8	10·1	13·5	16·9	20·3	23·7	27·0	30 4
	¼	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	34·2
	½	4·2	8·4	12·7	16·9	21·1	25·3	29·6	33·8	38·0
	¾	4 6	9·3	13·9	18·6	23·2	27·9	32·5	37·2	41·8
		5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
		5·5	11·0	16·5	22·0	27·5	32·9	38·4	43·9	49·4
		5·9	11·8	17·7	23·7	29 6	35·5	41·4	47·3	53·2
		6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·0
		6·8	13·5	20·3	27·0	33·8	40·6	47·3	54·1	60·8

TABLE III.

FLAT IRON.

Thick.	Width.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅝	2¾	34·9	36·5	41·8	45·3	48·8	52·3	55·8	59·3	62·7
⅝	3	38·0	41·8	45·6	49·4	53·2	57·0	60·8	64·6	68·4
⅝	3¼	41·2	45·[illegible]	49·4	53·6	57·7	61·8	65·9	70·0	74·2
⅝	3½	44·4	48·8	53·2	57·7	62·1	66·5	71·[illegible]	75·4	79·9
⅝	3¾	47·5	52·5	57·0	61·8	66·5	71·3	76·[illegible]	80·8	85·5
⅝	4	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·2
⅝	4¼	53·9	59·3	64·7	70·0	75·4	80·8	86·[illegible]	91·6	97·0
⅝	4½	57·0	62·7	68·4	74·2	79·9	85·6	91·3	97·0	102·7
⅝	4¾	60·2	66·[illegible]	72·2	78·3	84·3	90·3	96·3	102·3	108·4
⅝	5	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·0
⅝	5¼	66·5	73·2	79·8	86·5	93·1	99·8	106·5	113·1	119·8
⅝	5½	69·7	76·7	83·7	90·6	97·6	104·5	111·5	118·5	125·5
⅝	5¾	72·[illegible]	80·2	87·5	94·7	102·0	109·3	116·6	123·9	131·2
⅝	6	76·0	83·6	91·2	98·9	106·5	114·1	121·7	129·3	136·9
½	1	16·9	18·6	20·3	22·0	23·7	25·4	27·0	28·7	30·4
½	1¼	21·1	23·2	25·3	27·5	29·6	31·7	33·8	35·9	38·0
½	1½	25·3	27·9	30·4	33·0	35·5	38·0	40·6	43·1	45·6
½	1¾	29·6	32·5	35·5	38·5	41·4	44·4	47·3	50·3	53·2
½	2	33·8	37·2	40·6	43·9	47·3	50·7	54·1	57·5	60·8
½	2¼	38·0	41·8	45·6	49·4	53·2	57·0	60·8	64·6	68·4
½	2½	42·2	46·5	50·7	54·9	59·1	63·4	65·6	71·8	76·0
½	2¾	46·5	51·1	55·8	60·4	65·1	69·7	74·4	79·0	83·6
½	3	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·2
½	3¼	54·[illegible]	60·4	65·9	71·4	76·9	82·4	87·9	93·3	98·8
½	3½	59·2	65·1	71·0	76·9	82·8	88·7	94·6	100·6	106·5
½	3¾	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·0
½	4	67·6	74·4	84·1	87·9	94·6	101·4	108·2	114·9	121·7

TABLE III.

FLAT IRON.

Thick.	Width.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅜	2¾	3·5	7·0	10·5	13·9	17·4	20·9	24·4	27·9	31·4
⅜	3	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	34·2
⅜	3¼	4·1	8·2	12·4	16·5	20·6	24·7	28·8	33·0	37·1
⅜	3½	4·4	8·9	13·3	17·7	22·2	26·6	31·1	35·5	39·9
⅜	3¾	4·8	9·5	14·3	19·0	23·8	28·5	33·3	38·0	42·8
⅜	4	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
⅜	4¼	5·4	10·8	16·1	21·5	26·9	32·3	37·7	43·1	48·5
⅜	4½	5·7	11·4	17·1	22·8	28·5	34·2	39·9	45·6	51·3
⅜	4¾	6·0	12·0	18·1	24·1	30·1	36·1	42·1	48·2	54·2
⅜	5	6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·0
⅜	5¼	6·7	13·3	20·0	26·6	33·3	39·9	46·6	53·2	59·9
⅜	5½	7·0	13·9	20·9	27·9	34·9	41·8	48·8	55·8	62·7
⅜	5¾	7·3	14·6	21·9	29·2	36·4	43·7	51·0	58·3	65·6
⅜	6	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·8	68·4
½	1	1·7	3·4	5·1	6·8	8·5	10·1	11·8	13·5	15·2
½	1¼	2·1	4·2	6·3	8·4	10·6	12·7	14·8	16·9	19·0
½	1½	2·5	5·1	7·6	10·1	12·7	15·2	17·7	20·3	22·8
½	1¾	3·0	5·9	8·9	11·8	14·8	17·7	20·7	23·7	26·6
½	2	3·4	6·8	10·1	13·5	16·9	20·3	23·7	27·0	30·4
½	2¼	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	34·2
½	2½	4·2	8·4	12·7	16·9	21·1	25·3	29·6	33·8	38·0
½	2¾	4·6	9·3	13·9	18·6	23·2	27·9	32·5	37·2	41·8
½	3	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
½	3¼	5·5	11·0	16·5	22·0	27·5	32·9	38·4	43·9	49·4
½	3½	5·9	11·8	17·7	23·7	29·6	35·5	41·4	47·3	53·2
½	3¾	6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·0
½	4	6·8	13·5	20·3	27·0	33·8	40·6	47·3	54·1	60·8

TABLE III.

FLAT IRON.

Thick.	Width.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
3/8	2¾	34·9	36·5	41·8	45·3	48·8	52·3	55·8	59·3	62·
3/8	3	38·0	41·8	45·6	49·4	53·2	57·0	60·8	64·6	68·
3/8	3¼	41·2	45·	49·4	53·6	57·7	61·8	65·9	70·0	74·
3/8	3½	44·4	48·8	53·2	57·7	62·1	66·5	71·	75·4	79·
3/8	3¾	47·5	52·5	57·0	61·8	66·5	71·3	76·	80·8	85·
3/8	4	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·
3/8	4¼	53·9	59·3	64·7	70·0	75·4	80·8	86·	91·6	97·
3/8	4½	57·0	62·7	68·4	74·2	79·9	85·6	91·3	97·0	102·7
3/8	4¾	60·2	66·	72·2	78·3	84·3	90·3	96·3	102·3	108·
3/8	5	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·
3/8	5¼	66·5	73·2	79·8	86·5	93·1	99·8	106·5	113·1	119·8
3/8	5½	69·7	76·7	83·7	90·6	97·6	104·5	111·5	118·5	125·5
3/8	5¾	72·	80·2	87·5	94·7	102·0	109·3	116·6	123·9	131·2
3/8	6	76·0	83·6	91·2	98·9	106·5	114·1	121·7	129·3	136·9
½	1	16·9	18·6	20·3	22·0	23·7	25·4	27·0	28·7	30·4
½	1¼	21·1	23·2	25·3	27·5	29·6	31·7	33·8	35·9	38·
½	1½	25·3	27·9	30·4	33·0	35·5	38·0	40·6	43·1	45·
½	1¾	29·6	32·5	35·5	38·5	41·4	44·4	47·3	50·3	53·
½	2	33·8	37·2	40·6	43·9	47·3	50·7	54·1	57·5	60·
½	2¼	38·0	41·8	45·6	49·4	53·2	57·0	60·8	64·6	68·
½	2½	42·2	46·5	50·7	54·9	59·1	63·4	65·6	71·8	76·
½	2¾	46·5	51·1	55·8	60·4	65·1	69·7	74·4	79·0	83·
½	3	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·
½	3¼	54·	60·4	65·9	71·4	76·9	82·4	87·9	93·3	98·
½	3½	59·2	65·1	71·0	76·9	82·8	88·7	94·6	100·6	106·
½	3¾	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·
½	4	67·6	74·4	84·1	87·9	94·6	101·4	108·2	114·9	121·

TABLE III.

FLAT IRON.

Thick.	Width.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
½	4¼	7·2	14·4	21·5	28·7	35·9	43·1	50·3	57·4	64·6
½	4½	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·8	68·4
½	4¾	8·0	16·1	24·1	32·1	40·1	48·2	56·2	64·2	72·2
½	5	8·4	16·9	25·3	33·8	42·2	50·7	59·1	67·6	76·0
½	5¼	8·9	17·7	26·6	35·5	44·4	53·2	62·1	71·0	79·9
½	5½	9·3	18·6	27·9	37·2	46·5	55·8	65·1	74·4	83·7
½	5¾	9·7	19·4	29·2	38·9	48·6	58·3	68·0	77·7	87·5
½	6	10·1	20·3	30·4	40·6	50·7	60·8	70·9	81·1	91·2
⅝	1	2·1	4·2	6·3	8·4	10·6	12·7	14·8	16·9	19·0
⅝	1¼	2·6	5·3	7·9	10·6	13·2	15·8	18·5	21·1	23·8
⅝	1½	3·2	6·3	9·5	12·7	15·8	19·0	22·2	25·4	28·5
⅝	1¾	3·7	7·4	11·1	14·8	18·5	22·2	25·9	29·6	33·3
⅝	2	4·2	8·4	12·7	16·9	21·1	25·3	29·9	33·8	38·0
⅝	2¼	4·8	9·5	14·3	19·0	23·8	28·5	33·3	38·0	42·8
⅝	2½	5·3	10·6	15·8	21·1	26·4	31·7	37·0	42·2	47·5
⅝	2¾	5·8	11·6	17·4	23·2	29·0	34·8	40·7	46·5	52·3
⅝	3	6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·6
⅝	3¼	6·9	13·7	20·6	27·5	34·3	41·2	48·1	54·9	61·8
⅝	3½	7·4	14·8	22·2	29·6	37·0	44·4	51·8	59·2	66·5
⅝	3¾	7·9	15·8	23·8	31·7	39·6	47·5	55·5	63·4	71·3
⅝	4	8·4	16·9	25·3	33·8	42·2	50·7	59·1	67·6	76·0
⅝	4¼	9·0	18·0	26·9	35·9	44·9	53·9	62·9	71·8	80·8
⅝	4½	9·5	19·0	28·5	38·0	47·5	57·0	66·5	76·1	85·6
⅝	4¾	10·0	20·1	30·1	40·1	50·2	60·2	70·2	80·3	90·3
⅝	5	10·6	21·1	31·7	42·3	52·8	63·4	73·9	84·5	95·1
⅝	5¼	11·1	22·2	33·3	44·4	55·5	66·5	77·6	88·7	99·8
⅝	5½	11·6	23·2	34·9	46·5	58·1	69·7	81·3	92·9	104·6
⅝	5¾	12·1	24·3	36·4	48·6	60·7	72·9	85·0	97·2	109·3

TABLE III.

FLAT IRON.

Thick.	Width.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
in.	ins.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
½	4¼	71·8	79·0	86·2	93·4	100·5	107·7	114·9	122·1	129·3
½	4½	76·0	83·6	91·2	98·9	106·5	114·1	121·7	129·3	136·9
½	4¾	80·3	88·3	96·3	104·3	112·4	120·4	128·4	136·4	144·5
½	5	84·5	92·9	101·4	109·8	118·3	126·7	135·2	143·0	152·1
½	5¼	88·7	97·6	106·5	115·4	124·2	133·1	142·0	150·8	159·7
½	5½	93·0	102·2	111·5	120·8	130·1	139·4	148·7	158·0	167·3
½	5¾	97·2	106·9	116·6	126·3	136·0	145·8	155·5	165·2	174·9
½	6	101·4	111·5	121·7	131·8	141·9	152·1	162·2	172·4	182·5
⅝	1	21·1	23·2	25·3	27·5	29·6	31·7	33·8	35·9	38·0
⅝	1¼	26·4	29·0	31·7	34·3	37·0	39·6	42·2	44·[illegible]	47·5
⅝	1½	31·7	34·8	38·0	41·2	44·4	47·5	50·7	53·9	57·0
⅝	1¾	37·0	40·7	44·4	48·1	51·8	55·5	59·2	62·8	66·5
⅝	2	42·2	46·5	50·7	54·9	60·1	63·4	67·6	71·8	76·0
⅝	2¼	47·5	52·3	57·0	61·8	66·5	71·3	76·0	80·8	85·5
⅝	2½	52·8	58·1	63·4	68·6	73·9	79·2	84·5	89·[illegible]	95·0
⅝	2¾	58·1	63·9	69·7	75·5	81·3	87·1	92·9	98·[illegible]	104·5
⅝	3	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·0
⅝	3¼	68·7	75·5	82·4	89·3	96·1	103·0	109·9	116·7	123·6
⅝	3½	73·9	81·3	88·7	96·1	103·5	110·9	118·3	125·7	133·1
⅝	3¾	79·2	87·1	95·1	103·0	110·9	118·8	126·8	134·7	142·6
⅝	4	84·5	92·9	101·4	109·8	118·3	126·7	135·2	143·6	152·1
⅝	4¼	89·8	98·8	107·8	116·7	125·7	134·7	143·7	152·6	161·6
⅝	4½	95·1	104·6	114·1	123·6	133·1	142·6	152·1	161·6	171·1
⅝	4¾	100·3	110·4	120·4	130·4	140·5	150·5	160·5	170·6	180·6
⅝	5	105·6	116·2	126·8	137·3	147·9	158·4	169·0	179·6	190·1
⅝	5¼	110·9	122·0	133·1	144·2	155·3	166·4	177·5	188·5	199·6
⅝	5½	116·2	127·8	139·4	151·0	162·6	174·3	185·9	197·5	209·1
⅝	5¾	121·5	133·6	145·7	157·9	170·0	182·2	194·3	206·5	218·6

TABLE III.

FLAT IRON.

Thick.	Width.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.
in.	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
$\frac{5}{8}$	6	12·7	25·3	38·0	50·7	63·4	76·0	88·7	101·4	114·1
$\frac{3}{4}$	1	2·5	5·1	7·6	10·1	12·7	15·2	17·7	20·3	22·8
$\frac{3}{4}$	$1\frac{1}{4}$	3·2	6·3	9·5	12·7	15·8	19·0	22·2	25·4	28·5
$\frac{3}{4}$	$1\frac{1}{2}$	3·8	7·6	11·4	15·2	19·0	22·8	26·6	30·4	24·2
$\frac{3}{4}$	$1\frac{3}{4}$	4·4	8·9	13·3	17·7	22·2	26·6	31·1	35·5	39·9
$\frac{3}{4}$	2	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
$\frac{3}{4}$	$2\frac{1}{4}$	5·7	11·4	17·1	22·8	28·5	34·2	39·9	45·6	51·3
$\frac{3}{4}$	$2\frac{1}{2}$	6·3	12·7	19·0	25·3	31·7	38·0	44·4	50·7	57·0
$\frac{3}{4}$	$2\frac{3}{4}$	7·0	13·9	20·9	27·9	34·9	41·8	48·8	55·8	62·7
$\frac{3}{4}$	3	7·6	15·2	22·8	30·4	38·0	45·6	53·2	60·9	68·4
$\frac{3}{4}$	$3\frac{1}{4}$	8·2	16·5	24·7	33·0	41·2	49·4	57·7	65·9	74·2
$\frac{3}{4}$	$3\frac{1}{2}$	8·9	17·7	26·6	35·5	44·4	53·2	62·1	71·0	79·9
$\frac{3}{4}$	$3\frac{3}{4}$	9·5	19·0	28·5	38·0	47·5	57·0	66·5	76·1	85·6
$\frac{3}{4}$	4	10·1	20·3	30·4	40·6	50·7	60·8	70·9	81·1	91·2
$\frac{3}{4}$	$4\frac{1}{4}$	10·8	21·5	32·3	43·1	53·9	64·6	75·4	86·2	97·0
$\frac{3}{4}$	$4\frac{1}{2}$	11·4	22·8	34·2	45·6	57·0	68·4	79·9	91·3	102·7
$\frac{3}{4}$	$4\frac{3}{4}$	12·0	24·1	36·1	48·2	60·2	72·2	84·3	96·3	108·4
$\frac{3}{4}$	5	12·7	25·3	38·0	50·7	63·4	76·0	88·7	101·4	114·0
$\frac{3}{4}$	$5\frac{1}{4}$	13·3	26·6	39·9	53·2	66·5	79·8	93·1	106·5	119·8
$\frac{3}{4}$	$5\frac{1}{2}$	13·9	27·9	41·8	55·8	69·7	83·7	97·6	111·5	125·5
$\frac{3}{4}$	$5\frac{3}{4}$	14·6	29·1	43·7	58·3	72·9	87·4	102·0	116·6	131·2
$\frac{3}{4}$	6	15·2	30·4	45·6	60·8	76·0	91·2	106·5	121·7	136·9
1	$1\frac{1}{2}$	5·1	10·1	15·2	20·3	25·3	30·4	35·5	40·6	45·6
1	2	6·8	13·5	20·3	27·0	33·8	40·6	47·8	54·1	60·8
1	3	10·1	20·3	30·4	40·6	50·7	60·8	70·9	81·1	91·2
1	4	13·5	27·0	40·6	54·1	67·6	81·1	94·6	108·1	121·7
1	5	16·9	33·8	50·7	67·6	84·5	101·4	118·3	135·2	152·1
1	6	20·3	40·6	60·8	81·1	101·4	121·7	141·9	162·2	182·5

TABLE III.

FLAT IRON.

Thick.	Width.	10 ft.	11 ft.	12 ft.	13 ft.	14 ft.	15 ft.	16 ft.	17 ft.	18 ft.
in	in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅝	6	126·7	139·4	152·1	164·8	177·4	190·1	202·8	215·4	228·1
¾	1	25·3	27·9	30·4	33·0	35·5	38·0	40·6	43·1	45·6
¾	1¼	31·7	34·9	38·0	41·2	44·4	47·5	50·7	53·9	57·0
¾	1½	38·0	41·8	45·6	59·4	53·2	57·0	60·8	64·6	68·4
¾	1¾	44·4	48·8	53·2	57·7	62·1	66·5	71·0	75·4	79·9
¾	2	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·2
¾	2¼	57·0	62·7	68·4	74·2	79·9	85·5	91·3	97·0	102·7
¾	2½	63·3	69·7	76·0	82·4	88·7	95·0	101·4	107·7	114·0
¾	2¾	69·7	76·7	83·7	90·6	97·6	104·5	111·5	118·5	125·5
¾	3	76·0	83·6	91·2	98·9	106·5	114·1	121·7	129·3	136·9
¾	3¼	82·4	90·6	98·9	107·1	115·3	123·6	131·8	140·0	148·3
¾	3½	88·7	97·6	106·5	115·4	124·2	133·1	142·0	150·8	159·7
¾	3¾	95·1	104·6	114·1	123·6	133·1	142·6	152·1	161·6	171·1
¾	4	101·4	111·5	121·7	131·8	141·9	152·1	162·2	172·4	182·5
¾	4¼	107·7	118·5	129·3	140·1	150·8	161·6	172·4	183·2	193·9
¾	4½	114·1	125·5	136·9	148·3	159·7	171·1	182·5	193·9	205·3
¾	4¾	120·4	132·4	144·5	156·5	168·6	180·6	192·6	204·7	216·7
¾	5	126·7	139·4	152·1	164·8	177·4	190·1	202·8	215·4	228·1
¾	5¼	133·1	146·4	159·7	173·0	186·3	199·6	212·9	226·2	239·5
¾	5½	139·4	153·3	167·3	181·2	195·2	209·2	223·1	237·0	250·9
¾	5¾	145·7	160·3	174·9	189·5	204·0	218·6	233·2	247·8	262·3
¾	6	152·1	167·3	182·5	197·7	212·9	228·1	243·3	258·5	273·7
1	1½	50·7	55·8	60·8	65·9	70·9	76·0	81·1	86·2	91·2
1	2	67·6	74·4	81·1	87·9	94·6	101·4	108·1	114·9	121·7
1	3	101·4	111·5	121·7	131·7	141·9	152·1	162·2	172·4	182·5
1	4	135·2	148·7	162·2	175·7	189·3	202·8	216·7	229·8	243·3
1	5	169·0	185·9	202·8	219·7	236·6	253·5	270·4	287·3	304·2
1	6	202·8	223·1	243·3	263·6	283·9	304·2	324·4	344·7	365·0

TABLE OF GRADIENTS

And Resistance per Ton for each.

Vertical Rise.		Gravity due to incline per ton.	Vertical Rise.		Gravity due to incline per ton.	Vertical Rise.		Gravity due to incline per ton.
Ratio.	Pr. Mile.		Ratio.	Pr. Mile.		Ratio.	Pr. Mile.	
one in	Feet.	lbs.	one in	Feet.	lbs.	one in	Feet.	lbs.
100	52·80	22·40	74	71·38	32·270	47	112·34	47·660
99	53·33	22·626	73	72·32	30·685	46	115·04	48·684
98	53·88	22·858	72	73·33	31·111	45	117·33	49·777
97	54·43	23·092	71	74·36	31·550	44	120·00	50·908
96	55·00	23·334	70	75·43	32·000	43	122·78	52·092
95	55·60	23·579	69	76·49	32·464	42	125·71	53·333
94	56·17	23·830	68	77·64	32·940	41	128·78	54·634
93	56·77	24·086	67	78·81	33·432	40	132·00	56·00
92	57·52	24·342	66	80·0	33·940	39	135·38	57·436
91	58·02	24·614	65	81·23	34·460	38	138·95	58·944
90	58·66	24·888	64	82·50	35·0	37	142·70	60·540
89	59·33	25·168	63	83·81	35·555	36	146·66	62·222
88	60·0	25·454	62	85·16	36·106	35	150·84	64·000
87	60·69	25·746	61	86·55	36·720	34	155·30	65·880
86	61·39	26·046	60	88·00	37·333	33	160·0	67·880
85·16	62·00	26·303	59	89·49	37·966	32	165·0	70·0
85	62·12	26·353	58	91·03	38·620	31	170·32	72·216
84	62·86	26·666	57	92·63	39·298	30	176·00	74·666
83	63·61	26·988	56	94·28	40·0	29	182·06	77·240
82	64·39	27·317	55	96·00	40·726	28	188·56	80·00
81	65·20	27·718	54	97·77	41·480	27	195·55	82·960
80	66·0	28·00	53	99·62	42·264	26	203·06	86·152
79	66·83	28·355	52	101·53	43·076	25	211·20	89·60
78	67·69	28·718	51	103·52	43·920	24	220·0	93·336
77	68·57	29·090	50	105·60	44·800	23	229·56	97·388
76	69·47	29·472	49	107·75	45·716	22	240	101·816
75	70·40	29·867	48	110·00	46·688	21	251·43	106·666

TO TAKE IMPRESSIONS FROM COINS, &C.—Make a thick solution of isinglass in water, and lay it hot on the metal; let it remain for twelve hours, then remove it, breathe on it, and apply gold or silver-leaf on the wrong side. Any color may be given to the isinglass instead of gold or silver, by simple mixture.

VARIATIONS IN TIDES.—The difference in time between high water averages about 49 minutes each day.

TABLE *of the Ultimate Breaking Weight, in tons, of cast-iron pillars, calculated from Professor Hodgkinson's Formula.*

The length includes every half-foot from 1 to 20, and the diameter every inch from 1 to 24.

LENGTH IN FEET.	DIAMETER OF CAST-IRON PILLARS IN INCHES.					
	1	2	3	4	5	6
	tons.	tons.	tons.	tons.	tons.	tons.
1	44·30	537	2312	6513	14544	28038
1½	22·23	269	1160	3269	7300	14073
2	13·63	165	711	2004	4476	8630
2½	9·33	113	487	1372	3064	5905
3	6·84	83	357	1006	2247	4331
3½	5·26	64	275	774	1729	3333
4	4·19	51	219	617	1378	2656
4½	3·43	41·6	179	505	1127	2174
5	2·87	34·8	150	422	943	1817
5½	2·44	29·6	127	359	802	1545
6	2·11	25·5	110	309	691	1333
6½	1·84	22·3	96	270	603	1163
7	1·62	19·6	84·6	238	532	1026
7½	1·44	17·5	75·2	212	473	912
8	1·29	15·6	67·4	190	424	817
8½	1·16	14·1	60·8	171	382	737
9	1·06	12·8	55·2	155	347	669
9½	·96	11·7	50·3	142	316	610
10	·88	10·7	46·1	130	290	559
10½	·81	9·86	42·4	119	267	515
11	·75	9·11	39·2	110	246	475
11½	·69	8·45	36·3	102	228	441
12	·65	7·86	33·8	95·3	212	410
12½	·60	7·33	31·5	88·9	198	383
13	·56	6·86	29·5	83·2	185	358
13½	·53	6·43	27·7	78·0	174	336
14	·50	6·05	26·0	73·3	163	315
14½	·47	5·70	24·5	69·1	154	297
15	·44	5·38	23·15	65·23	145·6	280·8
15½	·42	5·09	21·90	61·69	137·7	265·5
16	·40	4·82	20·75	58·45	130·5	251·6
16½	·377	4·57	19·69	55·47	123·8	238·8
17	·358	4·35	18·72	52·73	117·7	227·0
17½	·341	4·14	17·82	50·19	112·1	216·1
18	·325	3·94	16·98	47·85	106·8	205·9
18½	·310	3·77	16·21	45·67	101·9	196·6
19	·297	3·60	15·49	43·64	97·45	187·8
19½	·284	3·44	14·82	41·76	93·24	179·7
20	·272	3·30	14·20	40·00	89·32	172·2

TABLE *of the Ultimate Breaking Weight, in tons, of cast-iron pillars.* (Continued.)

LENGTH IN FEET.	DIAMETER OF CAST IRON PILLARS IN INCHES.					
	7	8	9	10	11	12
	tons.	tons.	tons.	tons.	tons.	tons.
1	48838	78982	120691	176361	248552	339982
1½	24513	39643	60579	88520	124756	170648
2	15031	24310	37147	54282	76501	104643
2½	10286	16635	25420	37145	52350	71607
3	7544	12202	18645	27246	38398	52523
3½	5805	9388	14347	20965	29546	40414
4	4626	7482	11433	16707	23546	32207
4½	3787	6124	9358	13675	19273	26363
5	3166	5120	7824	11433	16113	22039
5½	2692	4354	6653	9722	13703	18743
6	2322	3755	5738	8385	11818	16165
6½	2026	3277	5008	7319	10315	14109
7	1787	2889	4415	6452	9094	12439
7½	1589	2570	3927	5738	8087	11062
8	1424	2302	3519	5142	7247	9913
8½	1284	2077	3174	4638	6537	8942
9	1165	1885	2880	4209	5932	8114
9½	1063	1719	2627	3839	5411	7401
10	974	1575	2408	3519	4959	6783
10½	897	1450	2216	3238	4564	6243
11	828	1340	2048	2992	4217	5769
11½	768	1242	1898	2774	3910	5349
12	714	1156	1766	2581	3637	4975
12½	666	1078	1647	2408	3393	4642
13	623	1008	1541	2252	3174	4343
13½	585	946	1445	2112	2977	4073
14	550	889	1359	1986	2799	3828
14½	518	838	1280	1871	2637	3607
15	489·1	791·0	1208	1766	2489	3405
15½	462·6	748·1	1143	1671	2354	3220
16	438·3	708·8	1083	1583	2230	3051
16½	415·9	672·6	1028	1502	2117	2895
17	395·3	639·4	977·0	1428	2012	2752
17½	376·3	608·6	930·1	1359	1915	2620
18	358·7	580·2	886·5	1295	1826	2497
18½	342·4	553·8	846·2	1236	1743	2384
19	327·2	529·2	808·7	1182	1665	2278
19½	313·1	506·4	773·8	1131	1593	2179
20	299·9	485·0	741·2	1083	1526	2088

Note.—Example. Find the breaking weight of a cast-iron pillar whose external diameter is 17, and internal diameter 15 inches, and length 18 feet.

TABLE *of the Ultimate Breaking Weight, in tons, of cast-iron pillars.*
(Continued.)

LENGTH IN FEET.	DIAMETER OF CAST-IRON PILLARS IN INCHES.					
	13	14	15	16	17	18
	tons.	tons.	tons.	tons.	tons.	tons.
1	453524	592195	759158	957714	1191290	1463470
1½	227638	297241	381039	480707	597950	734563
2	139588	182269	233660	294769	366664	450443
2½	95522	124729	159895	201717	250912	308238
3	70064	91486	117281	147955	184040	226088
3½	53912	70396	90243	113846	141614	173966
4	42963	56100	71917	90726	112853	138638
4½	35137	45920	58867	74263	92375	113481
5	29400	38390	49213	62085	77228	94871
5½	25002	32647	41851	52798	65676	80680
6	21565	28158	36097	45538	56645	69586
6½	18821	24576	31505	39745	49439	60734
7	16593	21667	27776	35040	43587	53545
7½	14756	19269	24701	31163	38763	47619
8	13223	17267	22135	27924	34735	42671
8½	11928	15576	19967	25190	31333	38492
9	10824	14133	18118	22857	28432	34928
9½	9873	12892	16527	20850	25935	31861
10	9049	11815	15147	19109	23769	29200
10½	8329	10875	13941	17588	21877	26876
11	7695	10048	12882	16250	20214	24832
11½	7135	9317	11944	15067	18743	23025
12	6637	8667	11110	14016	17434	21418
12½	6192	8086	10365	13076	16265	19982
13	5793	7564	9697	12233	15216	18693
13½	5433	7094	9094	11472	14271	17531
14	5107	6669	8549	10785	13415	16481
14½	4811	6282	8054	10160	12638	15526
15	4542	5931	7603	9591	11930	14656
15½	4296	5609	7191	9071	11283	13862
16	4070	5314	6813	8595	10691	13103
16½	3863	5044	6466	8157	10146	12464
17	3671	4794	6146	7753	9424	11847
17½	3495	4563	5850	7380	9180	11277
18	3331	4350	5577	7035	8571	10750
18½	3180	4152	5323	6715	8353	10261
19	3039	3968	5087	6417	7983	9806
19½	2908	3797	4867	6140	7638	9383
20	2785	3637	4662	5881	7316	8987

Along the line marked 18 feet, and in the vertical lines numbered 17 and 15 inches, take the numbers 8751 and 5577; the difference of which, namely 3174, will be the breaking

TABLE *of the Ultimate Breaking Weight, in tons, of cast-iron Pillars.*
(Continued.)

LENGTH IN FEET.	DIAMETER OF CAST IRON PILLARS IN INCHES.					
	19	20	21	22	23	24
	tons.	tons.	tons.	tons.	tons.	tons.
1	1777940	2138516	2549140	3013880	3536910	4122530
1½	892404	1073880	1279490	1512760	1775280	2069230
2	547224	658204	784589	927630	1088610	1268880
2½	374471	450416	536902	634786	744947	868292
3	274670	330374	393810	465605	546409	636880
3½	211350	254212	303024	358269	420444	490059
4	168428	202586	241485	285511	335059	390543
4½	137865	165825	197666	233703	274260	319671
5	115257	138632	165251	195378	229286	267248
5½	98017	117894	140532	166157	194988	227273
6	84539	101684	121210	143307	168177	196023
6½	73784	88748	105789	125047	146781	171085
7	65051	78243	93266	110270	129406	150832
7½	57851	69584	82944	98067	115085	134140
8	51840	62353	74326	87876	103126	120200
8½	46763	56247	67047	79271	93028	108430
9	42433	51038	60840	71930	84414	98390
9½	38707	46557	55496	65614	77000	89750
10	35474	42669	50862	60134	70571	82255
10½	32651	39272	46814	55348	64954	75708
11	30168	36286	43254	51140	60014	69951
11½	27973	33645	40106	47417	55646	64860
12	26020	31297	37306	44108	51763	60333
12½	24275	29199	34805	41150	48292	56288
13	22710	27315	32560	38497	45178	52658
13½	21298	25618	30537	36104	42370	49385
14	20021	24082	28706	33940	39830	46424
14½	18862	22687	27043	31974	37523	43736
15	17806	21417	25529	30184	35421	41286
15½	16840	20255	24145	28547	33501	39049
16	15955	19191	22823	27047	31740	36997
16½	15142	18213	21711	25669	30128	35111
17	14393	17312	20636	24398	28632	33374
17½	13701	16480	19644	23225	27255	31768
18	13060	15709	18725	22139	25981	30283
18½	12466	14994	17873	21131	24799	28906
19	11913	14330	17081	20195	23700	27624
19½	11398	13710	16343	19322	22676	26430
20	10918	13133	15654	18508	21721	25317

weight in tons. For practical purposes the pillars should be calculated to bear one half more than the weight to which they are subjected.

TABLE OF STRENGTHS OF CAST-IRON SHAFTS.

The cube of the diameter of a journal or shaft of sufficient strength is directly as the *horse power*, and inversely as the number of revolutions of the shaft per minute. Mr. Robertson Buchanan deduced from several experiments that a journal suitable to a 50-horse engine, making 50 revolutions per minute, should be 7½ inches in diameter. It is from these data the following table has been computed.

Horse Power.	Number of Revolutions of Shaft per Minute.																					
	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	130
	in	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
10	7⅝	6⅝	5	5⅜	5⅛	4⅞	4⅝	4½	4⅜	4⅐	4	4	3⅞	3¾	3⅝	3⅝	3½	3½	3⅜	3¼	3¼	3⅛
20	9¼	8⅛	7⅜	6⅞	6⅝	6⅛	5⅞	5⅝	5⅝	5½	5⅛	5	4⅞	4¾	4⅝	4⅛	4½	4⅜	4¼	4⅛	4	3⅞
30	10⅝	9¼	8⅜	7⅞	7⅜	7	6¾	6½	6¼	6	5⅞	5⅝	5½	5⅜	5¼	5¼	5⅛	5	5	4¾	4⅝	4½
40	11⅝	10¼	9¼	8⅝	8⅛	7¾	7¼	7	6⅞	6⅝	6⅜	6¼	6⅛	6	5⅞	5¾	5⅝	5½	5⅜	5¼	5	4⅞
50	12⅝	11	10	9¼	8¾	8¼	8	7⅝	7⅜	7⅛	7	6¾	6½	6⅜	6¼	6⅛	6	6	5⅞	5⅝	5½	5⅜
60	13⅝	11¾	10⅝	9⅞	9¼	8⅞	8⅝	8⅛	7⅞	7⅝	7⅜	7⅛	7	6⅞	6⅝	6½	6⅜	6¼	6¼	6	5⅞	5⅝
70	14⅛	12¼	11¼	10⅜	9¾	9¼	8⅞	8½	8¼	8	7¾	7½	7¾	7⅛	7	6⅞	6¾	6⅝	6½	6⅜	6⅐	6
80	14¾	12⅞	11¾	10⅞	10¼	9¾	9¼	8⅞	8⅝	8⅜	8⅛	7⅞	7¾	7½	7⅜	7¼	7	7	6⅞	6⅝	6⅜	6¼
90	15¼	13⅜	12⅛	11¼	10⅝	10⅛	9⅝	9¼	9	8⅝	8½	8¼	8	7¾	7⅝	7½	7⅜	7½	7⅛	6⅞	6¾	6½
100	15	13⅞	12⅝	11¾	11	10½	10	9⅝	9¼	9	8¾	8½	8¼	8⅛	8	7¾	7⅝	7½	7⅜	7⅛	7	6¾
125	17⅛	15	13½	12⅝	11⅞	11¼	10¾	10⅜	10	9¾	9⅜	9⅛	9	8¾	8½	8⅜	8¼	8	7⅞	7⅝	7½	7¼
150	18⅛	15⅞	14⅜	13⅜	12⅝	12	11½	11	10⅝	10¼	10	9¾	9½	9¼	9⅛	9	8¾	8½	8⅜	8⅛	8	7¾
175	19⅛	16¾	15⅛	14⅛	13¼	12⅝	12	11½	11⅛	10⅞	10⅛	10¼	10	9¾	9½	9⅜	9¼	9	8⅞	8⅝	8⅜	8⅛

TABLE OF STRENGTHS OF CAST-IRON SHAFTS,
(Continued.)

Horse Power.	Number of Revolutions of Shaft per Minute.																					
	10	15	20	25	3.	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	130
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
200	20	17 1/2	15 7/8	14 3/4	13 7/8	13 1/8	12 5/8	12 1/8	11 3/4	11 3/8	11	10 3/4	10 1/2	10 1/4	10	9 7/8	9 5/8	9 3/8	9 1/4	9	8 3/4	8 1/2
225	20 7/8	18 1/8	16 1/2	15 3/8	14 3/8	13 3/4	13 1/8	12 5/8	12 1/8	11 3/4	11 3/8	11 1/8	10 7/8	10 5/8	10 3/8	10 1/4	10	9 7/8	9 5/8	9 3/8	9	8 7/8
250	21 1/2	18 7/8	17 1/8	15 7/8	15	14 1/4	13 5/8	13	12 5/8	12 1/4	11 7/8	11 1/2	11 1/4	11	10 3/4	10 1/2	10 3/8	10 1/8	10	9 5/8	9 3/8	9 1/8
275	22 1/4	19 3/8	17 5/8	16 3/8	15 3/8	14 5/8	14	13 1/2	13	12 5/8	12 1/4	11 7/8	11 5/8	11 3/8	11 1/8	10 7/8	10 3/4	10 1/2	10 3/8	10	9 3/4	9 1/2
300	22 7/8	20	18 1/8	16 7/8	15 7/8	15 1/8	14 3/8	13 7/8	13 3/8	13	12 5/8	12 1/4	12	11 3/4	11 1/2	11 1/4	11	10 7/8	10 5/8	10 1/4	10	9 3/4
350	24 1/8	21	19 1/8	17 3/4	16 3/4	15 7/8	15 1/8	14 5/8	14 1/8	13 5/8	13 1/4	12 7/8	12 5/8	12 1/4	12	11 7/8	11 5/8	11 3/8	11 1/8	10 7/8	10 1/2	10 1/4
400	25 1/4	22	20	18 5/8	17 1/2	16 5/8	15 7/8	15 1/4	14 3/4	14 1/4	13 7/8	13 1/2	13 1/8	12 7/8	12 5/8	12 3/8	12 1/8	12	11 3/4	11 3/8	11	10 3/4
450	26 1/4	22 7/8	20 3/4	19 3/8	18 1/4	17 1/4	16 1/2	15 7/8	15 3/8	14 7/8	14 3/8	14	13 3/4	13 3/8	13 1/8	12 7/8	12 5/8	12 3/8	12 1/8	11 3/4	11 3/8	11 1/8
500	27 1/8	23 3/4	21 1/2	20	18 3/4	17 7/8	17 1/8	16 3/8	15 7/8	15 3/8	15	14 1/2	14 1/8	13 7/8	13 1/2	13 1/4	13	12 3/4	12 5/8	12 1/4	11 7/8	11 1/2
550	28	24 1/2	22 1/2	20 5/8	19 3/8	18 1/2	17 5/8	17	16 3/8	15 7/8	15 3/8	15	14 5/8	14 3/8	14	13 3/4	13 1/2	13 1/4	13	12 5/8	12 1/4	11 3/4
600	28 7/8	25 1/4	22 7/8	21 1/4	20	19	18 1/8	17 1/2	16 7/8	16 3/8	15 7/8	15 1/2	15	14 3/4	14 [illegible]	14 1/8	13 7/8	13 5/8	13 3/8	13	12 5/8	12 1/4
650	29 5/8	25 7/8	23 1/2	21 7/8	20 1/2	19 1/2	18 5/8	18	17 3/8	16 3/4	16 1/4	15 7/8	15 1/2	15 1/8	14 3/4	14 1/2	14 1/4	14	13 3/4	13 3/8	13	12 5/8
700	30 3/8	26 1/2	24 1/8	22 3/8	21	20	19 1/8	18 3/8	17 3/4	17 1/4	16 3/4	16 1/4	15 7/8	15 1/2	15 1/8	14 7/8	14 5/8	14 3/8	14 1/8	13 5/8	13 1/4	12 7/8
750	31	27 1/8	24 5/8	22 7/8	21 1/2	20 1/2	19 1/2	18 7/8	18 1/8	17 5/8	17 1/8	16 5/8	16 1/4	15 7/8	15 1/2	15 1/4	15	14 5/8	14 3/8	14	13 5/8	13 1/4
800	31 3/4	27 3/4	25 1/2	23 3/8	22	21	20	19 1/4	18 1/2	18	17 1/2	17	16 5/8	16 1/4	15 7/8	15 1/2	15 1/4	15	14 3/4	14 1/4	13 7/8	13 1/2

TABLE OF STRENGTHS OF WROUGHT-IRON SHAFTS.

Horse Pow'r.	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	130
	NUMBER OF REVOLUTIONS OF SHAFT PER MINUTE.																					
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
10	6 3/8	5 9/16	5	4 11/16	4 3/8	4 3/16	4	3 13/16	3 3/4	3 9/16	3 1/2	3 1/4	3 3/16	3 1/4	3 5/32	3 3/32	3 1/32	3	2 15/16	2 13/16	2 3/4	2 11/16
20	8	7	6 3/8	5 7/8	5 9/16	5 1/4	5	4 13/16	4 11/16	4 1/2	4 3/8	4 5/16	4 3/16	4 3/32	4	3 7/8	3 13/16	3 25/32	3 3/4	3 9/16	3 1/2	3 1/4
30	9 3/16	8	7 5/16	6 3/4	6 3/8	5 15/16	5 3/4	5 9/16	5 3/8	5 3/16	5	4 15/16	4 3/4	4 11/16	4 9/16	4 7/16	4 3/8	4 5/16	4 1/4	4 1/16	4	3 15/16
40	10 1/8	8 13/16	8	7 1/2	7	6 5/8	6 3/8	6 1/8	6	5 3/4	5 1/2	5 7/16	5 1/4	5 1/8	5	4 15/16	4 13/16	4 3/4	4 9/16	4 1/2	4 7/16	4 5/16
50	10 7/8	9 1/2	8 5/8	8	7 1/2	7 3/16	7 3/16	6 9/16	6 3/8	6 1/8	6	5 13/16	5 11/16	5 1/2	5 7/16	5 5/16	5 1/4	5 1/8	5	4 7/8	4 3/4	4 5/8
60	11 9/16	10 1/8	9 1/4	8 1/2	8	7 5/8	7 5/16	6 7/8	6 3/4	6 9/16	6 7/16	6 3/16	6	5 15/16	5 13/16	5 11/16	5 1/2	5 7/16	5 3/8	5 3/16	5	4 7/8
70	12 3/16	10 5/8	9 5/8	9	8 1/2	8	7 11/16	7 3/8	7 1/8	7	6 3/4	6 1/2	6 3/8	6 1/4	6 1/16	5 15/16	5 7/8	5 3/4	5 5/8	5 1/2	5 5/16	5 3/16
80	12 3/4	11 1/8	10 1/8	9 7/16	8 3/16	8 7/16	8	7 3/4	7 1/2	7 1/4	7	6 15/16	6 5/8	6 1/2	6 3/8	6 1/4	6 1/8	6	5 7/8	5 3/4	5 1/2	5 7/16
90	13 1/4	11 1/2	10 1/2	9 3/4	9 1/4	8 13/16	8 3/8	8	7 3/4	7 1/2	7 5/16	7 1/8	6 15/16	6 3/4	6 5/8	6 1/2	6 3/8	6 1/4	6 1/8	5 15/16	5 11/16	5 5/8
100	13 3/4	12	10 7/8	10 1/8	9 9/16	9	8 5/8	8 5/16	8	7 3/4	7 9/16	7 3/8	7 3/16	7	6 7/8	6 3/4	6 9/16	6 7/16	6 3/8	6 3/16	6	5 13/16
125	14 13/16	12 15/16	11 3/4	10 7/8	10 1/4	9 3/4	9 5/16	9	8 5/8	8 3/8	8 1/8	7 15/16	7 3/4	7 1/2	7 7/16	7 1/4	7 1/8	7	6 7/8	6 5/8	6 7/16	6 5/16
150	15 3/4	13 3/4	12 1/2	11 9/16	11	10 3/16	10	9 5/8	9 1/8	9	8 5/8	8 7/16	8 5/16	8	7 7/8	7 11/16	7 9/16	7 7/16	7 5/16	7 1/16	6 7/8	6 11/16
175	16 1/2	14 1/2	13 1/8	12 3/8	11 1/2	10 7/8	10 7/16	10	9 5/8	9 3/8	9	8 7/8	8 5/8	8 3/8	8 1/4	8 1/16	7 15/16	7 13/16	7 5/8	7 7/16	7 3/16	7
200	17 3/8	15 1/8	13 3/4	12 3/4	12	11 5/8	10 7/8	10 1/2	10 1/8	9 7/8	9 [illegible]	9 1/4	9	8 13/16	8 5/8	8 1/8	8 5/16	8 3/16	8	7 3/4	7 7/16	7 3/8

TABLE OF STRENGTHS OF WROUGHT-IRON SHAFTS.

(Continued.)

Horse Pow'r.	Number of Revolutions of Shaft per Minute.																				
	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	130
		in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
225	18	15¾	14⅜	13¼	12½	11⅞	11⅜	11	10½	10 3/16	9 15/16	9⅝	9 7/16	9 3/16	9	8 13/16	8⅝	8½	8⅜	8	7⅞	7⅝
250	18⅝	16¼	14⅞	13¾	12 15/16	12¼	11¾	11 5/16	11	10 9/16	10¼	10	9 13/16	9½	9 5/16	9⅛	9	8 13/16	8⅝	8 7/16	8⅛	7 15/16
275	19¼	16⅞	15¼	14 3/16	13⅜	12¾	12⅛	11 11/16	11¼	10 15/16	10 9/16	10 5/16	10	9 15/16	9⅝	9 7/16	9¼	9	8 15/16	8½	8 7/16	8⅛
300	19 13/16	17⅜	15¾	14⅝	13¾	13	12½	12⅛	11 9/16	11¼	10 15/16	10⅝	10⅜	10⅛	9 15/16	9 11/16	9 9/16	9⅜	9 3/16	8 15/16	8⅝	8 7/16
350	20⅞	18¼	16½	15⅜	14½	13¾	13⅛	12 11/16	12 3/16	11⅞	11½	11 3/16	10 15/16	10⅝	10 7/16	10 3/16	10	9⅞	9 11/16	9⅜	9 1/16	8⅞
400	21 13/16	19	17⅜	16	15⅛	14⅜	13¾	13¼	12¾	12⅜	12	11 11/16	11 7/16	11 3/16	10 15/16	10⅝	10½	10 5/16	10⅛	9 13/16	9½	9¼
450	22⅝	19⅞	18	16¾	15¾	15	14¼	13¾	13	12⅞	12½	12⅛	11⅞	11 9/16	11 5/16	11	10 15/16	10 11/16	10½	10¼	9 15/16	9⅝
500	23½	20½	18⅝	17⅜	16¼	15½	14 13/16	14¼	13¾	13 5/16	12 15/16	12 9/16	12 5/16	12	11¾	11½	11 5/16	11	10⅞	10½	10¼	10
550	24¼	21¼	19¼	17⅞	16⅞	16	15¼	14⅝	14⅛	13¾	13⅜	13	12 11/16	12 7/16	12⅛	11⅞	11⅝	11 7/16	11¼	10 15/16	10 9/16	10 5/16
600	25	21 13/16	17⅞	18 7/16	17⅜	16 7/16	15¾	15⅛	14⅝	14⅛	13¾	13 7/16	13 3/16	12¾	12½	12¼	11 15/16	11 13/16	11 9/16	11¼	10 15/16	10⅝
650	25⅝	22 7/16	20⅜	19	17¾	17	16⅛	15 7/16	15	14½	14⅛	13¾	13 7/16	13 1/16	12⅞	12 9/16	12 5/16	12 1/16	11 15/16	11½	11 3/16	10 15/16
700	26 5/16	23	20⅞	19 7/16	18¼	17⅜	16½	16	15⅜	14 15/16	14½	14	13¾	13 7/16	13⅛	12⅞	12⅝	12 7/16	12 3/16	11 13/16	11½	11 3/16
750	26⅞	23½	21⅜	19⅞	18⅝	17¾	17	16⅜	15¾	15¼	14⅞	14 7/16	14	13¾	13 7/16	13 3/16	12 15/16	12 11/16	12½	12 1/16	11¾	11 7/16
800	27½	24	21 13/16	20¼	19	18⅛	17⅜	16 11/16	16 1/16	15 9/16	15⅛	14 9/16	14⅜	14	13¾	13 7/16	13¼	12 15/16	12¾	12 7/16	12	11 11/16

TABLE

Showing the Strength of the Teeth of Cast-Iron Wheels at a given Velocity.

Pitch of teeth in inches	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth in horse power at			
			3 feet per second.	4 feet per second.	6 feet per second.	8 feet per second.
3·99	1·9	7·6	20·57	27·43	41·14	54·85
3·78	1·8	7·2	17·49	23·32	34·98	46·64
3·57	1·7	6·8	14·73	19·65	29·46	39·28
3·36	1·6	6·4	12·28	16·38	24·56	32·74
3·15	1·5	6	10·12	13·50	20·24	26·98
2·94	1·4	5·6	8·22	10·97	16·44	21·92
2·73	1·3	5·2	6·58	8·78	13·16	17·54
2·52	1·2	4·8	5·18	6·91	10·36	13·81
2·31	1·1	4·4	3·99	5·32	7·98	10·64
2·1	1·0	4	3·00	4·00	6·00	8·00
1·89	·9	3·6	2·18	2·91	4·36	5·81
1·68	·8	3·2	1·53	2·04	3·06	3·08
1·47	·7	2·8	1·027	1·37	2·04	2·72
1·26	·6	2·4	·64	·86	1·38	1·84
1·05	·5	2	·375	·50	·75	1·00

Furniture Oil.—1. Linseed oil 1 pint, alkanet ½ oz. Digest in a warm place till colored, and strain.

2. The same, with ¼ pint of oil of turpentine.

3. Linseed oil 1 pint, alkanet root 1 oz., rose pink 1 oz. Let them stand in an earthen vessel all night.

4. A quart of linseed oil, 6 oz. of distilled vinegar, 8 oz. of spirit of turpentine, 1 oz. of muriatic acid, and 2 oz. of spirit of wine.

5. Linseed oil 8 oz., vinegar 4 oz., oil of turpentine, mucilage, rectified spirit, each ½ oz.; butter of antimony ¼ oz.; muriatic acid 1 oz. Mix.

6. Linseed oil 16 oz., black rosin 4 oz., vinegar 4 oz., rectified spirit 3 oz., butter of antimony 1 oz., spirit of salts 2 oz.; melt the rosin, add the oil, take it off the fire, and stir in the vinegar; let it boil for a few minutes, stirring it; when cool put it into a bottle, add the other ingredients, shaking all together. [The last two are especially used for reviving French polish.]

7. Linseed oil 1 pint, oil of turpentine ½ pint, rectified spirit 4 oz., powdered rosin 1½ oz., rose pink ½ oz. Mix.

8. Linseed oil 14 oz., vinegar 1½ oz., muriatic acid ½ oz. Mix.

TABLE

Showing how to ascertain the weights of Pipes, of various Metals, and any diameter required.

Thickness in parts of an inch.	Wrought iron.	Copper.		Lead.	
$\frac{1}{32}$	·326	11½ lbs. plate,	·38	2 lbs. lead,	·483
$\frac{1}{16}$	·653	23½ " "	·76	4 " "	·967
$\frac{3}{32}$	·976	35 " "	1·14	5½ " "	1·45
$\frac{1}{8}$	1·3	46½ " "	1·52	8 " "	1·933
$\frac{5}{32}$	1·627	58 " "	1·9	9½ " "	2·417
$\frac{3}{16}$	1·95	70 " "	2·28	11 " "	2·9
$\frac{7}{32}$	2·277	80½ " "	2·66	13 " "	3·383
$\frac{1}{4}$	2·6	93 " "	3·04	15 " "	3·867

Rule. To the interior diameter of the pipe, in inches, add the thickness of the metal; multiply the sum by the decimal numbers opposite the required thickness, and under the metal's name; also, by the length of the pipe in feet; and the product is the weight of the pipe in lbs.

1. Required the weight of a copper pipe whose interior diameter is 7½ inches, its length 6¼ feet, and the metal ⅛ of an inch in thickness.

$$7{\cdot}5 + {\cdot}125 = 7{\cdot}625 \times 1{\cdot}52 \times 6{\cdot}25 = 72{\cdot}4 \text{ lbs.}$$

2. What is the weight of a leaden pipe 18½ feet in length, 3 inches interior diameter, and the metal ¼ of an inch in thickness?

$$3 + {\cdot}25 = 3{\cdot}25 \times 3{\cdot}867 \times 18{\cdot}5 = 232{\cdot}5 \text{ lbs.}$$

Note.—Weight of a cubic inch of

Lead	equal	·4103	lb.
Copper, sheet	"	·3225	"
Brass, do.	"	·3037	"
Iron, do.	"	·279	"
Iron, cast	"	·263	"
Tin, do.	"	·2636	"
Zinc, do.	"	·26	"
Water	"	·03617	"

TO SOLDER TORTOISE-SHELL.—Bring the edges of the pieces of shell to fit each other, observing to give the same inclination of grain to each, then secure them in a piece of paper, and place them between hot irons or pincers; apply pressure, and let them cool. The heat must not be so great as to *burn* the shell, therefore try it first on a piece of white paper.

TABLE

Of the Weight of Cast-Iron Balls.

Diameter in inches.	Weight in lbs.	Diameter in inches.	Weight in lbs.	Diameter in inches.	Weight in lbs.
2	1·10	6	29·72	10	137·71
2¼	1·57	6¼	33·62	10¼	148·28
2½	2·15	6½	37·80	10½	159·40
2¾	2·86	6¾	42·35	10¾	171·05
3	3·72	7	47·21	11	183·29
3¼	4·71	7¼	52·47	11¼	196·10
3½	5·80	7½	58·06	11½	209·43
3¾	7·26	7¾	64·09	11¾	223·40
4	8·81	8	70·49	12	237·94
4¼	10·57	8¼	77·32	12¼	253·13
4½	12·55	8½	84·56	12½	268·97
4¾	14·76	8¾	92·24	12¾	285·37
5	17·12	9	100·39	13	302·41
5¼	19·93	9¼	108·98	13¼	320·80
5½	22·91	9½	118·06	13½	338·81
5¾	26·18	9¾	127·63	13¾	357·93

1. What will be the weight of a hollow ball or shell of cast-iron, the external diameter being 9½, and internal diameter 8¾ inches?

Opposite 9½ are 118·06, and
Opposite 8¾ are 92·24, subtract

25·82 lbs., weight required.

2. Requiring to remove a cast-iron ball 37·8 lbs. in weight, and in diameter 6½ inches, and replace it by one of lead of an equal weight, what must be the diameter of the leaden ball?

Weight of lead to that of cast-iron = 1·56,

$$\text{Then } \frac{6{\cdot}5^3}{1{\cdot}56} = \sqrt[3]{176} = 5{\cdot}6 \text{ inches, the diameter.}$$

To transfer Engravings to Plaster Casts.—Cover the plate with ink, and polish its surface in the usual way; then put a wall of paper round it, and when completed pour in some finely powdered plaster of Paris mixed in water; jerk the plate repeatedly to allow the air bubbles to fly upwards, and let it stand one hour: then take the cast off the plate, and a very perfect impression will be the result.

TABLE OF THE WEIGHT OF FLAT AND ROLLED IRON, *per foot in length.*

Thickness in inches and parts.	BREADTH IN INCHES AND PARTS OF AN INCH.															
	4	3¾	3½	3¼	3	2¾	2½	2¼	2	1¾	1½	1⅜	1¼	1	¾	½
⅛	1·68	1·57	1·47	1·36	1·26	1·15	1·05	0·94	0·84	0·73	0·63	0·57	0·52	0·42	0·31	0·21
3/16	2·52	2·36	2·20	2·04	1·89	1·73	1·57	1·41	1·26	1·10	0·94	0·86	0·78	0·63	0·47	0·31
¼	3·36	3·15	2·94	2·73	2·52	2·31	2·10	1·89	1·68	1·47	1·26	1·18	1·05	0·84	0·63	0·42
⅜	5·04	4·72	4·41	4·09	3·78	3·46	3·15	2·83	2·52	2·20	1·89	1·73	1·57	1·26	0·94	0·63
½	6·72	6·30	5·88	5·46	5·04	4·62	4·20	3·78	3·36	2·94	2·52	2·31	2·10	1·68	1·26	
⅝	8·40	7·87	7·35	6·82	6·30	5·77	5·25	4·72	4·20	3·67	3·15	2·88	2·62	2·10	1·57	
¾	10·08	9·45	8·82	8·19	7·56	6·93	6·30	5·66	5·04	4·41	3·78	3·46	3·15	2·52		
⅞	11·76	11·02	10·29	9·55	8·82	8·08	7·35	6·61	5·88	5·14	4·41	4·04	3·67	2·94		
1	13·44	12·60	11·76	10·92	10·08	9·24	8·40	7·56	6·72	5·87	5·04	4·62	4·20			
1⅛	15·12	14·16	13·20	12·28	11·34	10·39	9·55	8·50	7·56	6·60	5·67	5·19	4·72			
1¼	16·80	15·75	14·70	13·65	12·60	11·55	10·50	9·45	8·40	7·35	6·30	5·77				
1⅜	18·48	17·32	16·16	15·01	13·86	12·70	11·55	10·39	9·24	8·07						
1½	20·18	18·90	17·64	16·38	15·12	13·86	12·60	11·34	10·08	8·80						
1¾	23·54	22·05	20·58	19·11	17·64	16·17	14·70	13·22								
2	26·88	25·20	23·52	21·84	20·16	18·48	16·80	15·12								
2½	33·65	31·50	29·40	27·39	25·20	23·10										
3	40·32	37·80	35·28	32·76												
3½	47·04															

TABLE *of the Weight of Cast-Iron Pipes, in lengths.*

Bore.	Thick.	Long.	Weight.		
Inch.	In.	Ft.	C.	qr.	lb.
1	¼	3½			12
	⅜	3½			21
1½	¼	4½			21
	⅜	4½		1	4
2	¼	6		1	8
	⅜	6		2	0
2½	¼	6		1	16
	⅜	6		2	10
	½	6		3	10
3	¼	9		2	20
	⅜	9	1	0	6
	½	9	1	1	12
	⅝	9	1	3	6
	¾	9	2	1	0
3½	¼	9		3	0
	⅜	9	1	0	21
	½	9	1	2	14
	⅝	9	2	0	8
	¾	9	2	2	0
4	⅜	9	1	1	10
	½	9	1	3	12
	⅝	9	2	1	12
	¾	9	2	3	21
4½	⅜	9	1	2	2
	½	9	2	0	4
	⅝	9	2	2	14
	¾	9	3	0	21
5	⅜	9	1	2	22
	½	9	2	1	10
	⅝	9	2	3	17
	¾	9	3	1	24
5½	⅜	9	1	3	10
	½	9	2	2	0
	⅝	9	3	0	18
	¾	9	3	3	7
	1	9	5	0	12
6	⅜	9	2	0	0
	½	9	2	2	21
	⅝	9	3	1	17
	¾	9	4	0	16
	1	9	5	2	20

Bore.	Thick.	Long.	Weight.		
Inch.	In.	Ft.	C.	qr.	lb.
6½	⅜	9	2	0	16
	½	9	2	3	20
	⅝	9	3	2	21
	¾	9	4	1	21
	1	9	6	0	14
7	½	9	3	0	7
	⅝	9	3	3	20
	¾	9	4	3	5
	1	9	6	2	4
7½	½	9	3	1	6
	⅝	9	4	0	22
	¾	9	5	0	10
	1	9	7	0	0
8	½	9	3	2	4
	⅝	9	4	1	25
	¾	9	5	1	18
	1	9	7	1	16
8½	½	9	3	3	2
	⅝	9	4	2	26
	¾	9	5	2	22
	1	9	7	3	8
9	½	9	4	0	0
	⅝	9	5	0	4
	¾	9	6	0	2
	1	9	8	0	26
9½	½	9	4	0	18
	⅝	9	5	1	0
	¾	9	6	1	6
	1	9	8	2	20
10	½	9	4	1	10
	⅝	9	5	1	26
	¾	9	6	2	14
	1	9	9	0	8
10½	½	9	4	2	14
	⅝	9	5	3	7
	¾	9	7	0	0
	1	9	9	2	0
11	½	9	4	3	14
	⅝	9	6	0	11
	¾	9	7	1	7
	1	9	9	3	20

Bore.	Thick.	Long.	Weight.		
Inch.	In.	Ft.	C.	qr.	lb.
11½	½	9	5	0	7
	⅝	9	6	1	12
	¾	9	7	2	8
	1	9	10	1	2
12	½	9	5	0	24
	⅝	9	6	2	8
	¾	9	7	3	20
	1	9	10	3	0
12½	½	9	5	1	16
	⅝	9	6	3	9
	¾	9	8	1	0
	1	9	11	0	21
13	½	9	5	2	20
	⅝	9	7	0	14
	¾	9	8	2	7
	1	9	11	2	12
13½	½	9	5	3	7
	⅝	9	7	1	12
	¾	9	8	3	16
	1	9	11	3	24
14	½	9	6	0	4
	⅝	9	7	2	16
	¾	9	9	1	0
	1	9	12	1	14
14½	½	9	6	0	24
	⅝	9	7	3	14
	¾	9	9	2	2
	1	9	12	3	6
15	½	9	6	1	21
	¾	9	9	3	7
	1	9	13	0	26
	1¼	9	16	3	5
15½	½	9	6	2	14
	¾	9	10	0	10
	1	9	13	2	17
	1¼	9	17	1	6
16	½	9	7	0	22
	¾	9	10	1	20
	1	9	14	0	8
	1¼	9	17	3	14
	1½	9	21	3	4

TABLE

Of the weight of one foot length of Malleable Iron.

SQUARE IRON.		ROUND IRON.			
Scantling.	Weight.	Diameter.	Weight.	Circumfer.	Weight.
Inches.	Pounds.	Inches.	Pounds.	Inches.	Pounds.
1/4	0·21	1/4	0·16	1	0·26
3/8	0·47	3/8	0·37	1 1/4	0·41
1/2	0·84	1/2	0·66	1 1/2	0·59
5/8	1·34	5/8	1·03	1 3/4	0·82
3/4	1·89	3/4	1·48	2	1·05
7/8	2·57	7/8	2·02	2 1/4	1·34
1	3·36	1	2·63	2 1/2	1·65
1 1/8	4·25	1 1/8	3·33	2 3/4	2·01
1 1/4	5·25	1 1/4	4·12	3	2·37
1 3/8	6·35	1 3/8	4·98	3 1/4	2·79
1 1/2	7·56	1 1/2	5·93	3 1/2	3·24
1 5/8	8·87	1 5/8	6·96	3 3/4	3·69
1 3/4	10·29	1 3/4	8·08	4	4·23
1 7/8	11·81	1 7/8	9·27	4 1/2	5·35
2	13·44	2	10·55	5	6·61
2 1/4	17·01	2 1/4	13·35	5 1/2	7·99
2 1/2	21·00	2 1/2	16·48	6	9·51
2 3/4	25·41	2 3/4	19·95	6 1/2	11·18
3	30·24	3	23·73	7	12·96
3 1/2	41·16	3 1/4	27·85	7 1/2	14·78
4	53·76	3 1/2	32·32	8	16·92
4 1/2	68·04	3 3/4	37·09	8 1/2	19·21
5	84·00	4	42·21	9	21·53
6	120·96	4 1/2	53·41	10	26·43
7	164·64	5	65·93	12	31·99

FRESCO PAINTING.—Apply any colors that are not injured by lime (according to taste), on a fresh mortared or plastered wall.

TO TAKE FAC-SIMILES OF SIGNATURES.—Write your name on a piece of paper, and while the ink is wet sprinkle over it some finely powdered gum arabic, then make a rim round it, and pour on it some fusible alloy, in a liquid state. Impressions may be taken from the plates formed in this way, by means of printing ink and the copperplate press.

WATCHMAKER'S OIL, WHICH NEVER CORRODES OR THICKENS.—Take olive oil and put it into a bottle, then insert coils of thin sheet lead. Expose it to the sun for a few weeks, and pour off the clear oil.

TABLE
Of the Dimensions and Weight of Coppers, from 1 to 208 gallons.

The Dimensions taken from lag to brim.

Inches lag to brim.	Gallons.	Weight in lbs.	Inches lag to brim.	Gallons.	Weight in lbs.	Inches lag to brim.	Gallons.	Weight in lbs.
9¾	1	1½	24	15	22½	29½	29	43½
12½	2	3	24½	16	24	30	30	45
14	3	4½	25	17	25½	32	36	54
15½	4	6	25½	18	27	34	43	64½
16½	5	7½	26	19	28½	35	48	72
17½	6	9	26½	20	30	36	53	79½
18½	7	10½	26¾	21	31½	37	58	87
19½	8	12	27	22	33	38	63	94½
20½	9	13½	27¼	23	34½	39	67	100½
21	10	15	27½	24	36	40	71	106½
21½	11	16½	27¾	25	37½	45	104	156
22	12	18	28	26	39	50	146	219
22½	13	19½	28½	27	40½	55	208	312
23¾	14	21	29	28	42			

Weight of Cast-Iron Plates, per superficial foot.

From one-eighth of an inch to one inch thick.

⅛ inch.	¼ inch.	inch.	½ inch.	⅝ inch.	¾ inch	⅞ inch.	1 inch.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
4 13⅜	9 10⅝	14 8	19 5⅜	24 2¾	29 0	33 13⅜	38 10¾

THE MANNER OF SOLDERING FERRULES FOR TOOL HANDLES, &c.—Take your ferrule, lap round the jointing a small piece of brass wire, then just wet the ferrule, scatter on the joining ground borax, put it on the end of a wire, and hold it in the fire till the brass fuses. It will fill up the joining, and form a perfect solder. It may afterwards be turned in the lathe.

CAST ENGRAVINGS.—Take the engraved plate you intend to copy, and arrange a support of suitable materials round it, then pour on it the following alloy in a state of perfect fusion: tin 1 part; lead 64 parts; antimony 12 parts. These "cast plates" may be worked off on a common printing-press, and offer a ready mode of procuring cheap copies of the works of our celebrated artists.

TABLE
Of the Bore and Weight of Cocks.

Content of Copper.	Bore of Cock.	Weight of Cock.	Content of Copper	Bore of Cock.	Weight of Cock.
Gallons.	Inches.	Pounds.	Gallons.	Inches.	Pounds.
30	1½	7	200	2¾	30
50	1¾	8	260	3	34
80	2	12	340	3¼	44
120	2¼	19	420	3½	56
150	2½	26	430 and upwards.	3¾	70

Three-fourths of the diameter of the bore, taken at the hinder part, will give the diameter of the cock at the mouth.

TABLE
Of the Weight of Lead, per superficial foot.
From one-sixteenth of an inch to one inch thick.

Thickness.	Weight.	Thickns.	Weight.	Thickns.	Weight.	Thickness.	Weight.
inch.	lbs.	inch.	lbs.	inch.	lbs.	inch.	lbs.
1-16th	3¾	1-8th	7½	1-4th	14¾	3-4ths.	44¼
1-12th	5	1-6th	10	1-3rd	19¼	1 inch	59
1-10th	6	1-5th	12	1-half	29½		

Weight of Lead Pipe of the usual thicknesses.
Per foot in length.

½-inch bore . . 1 lb. 1 oz.
¾ " . . 1 lb. 8 oz. — 1 lb. 12 oz. — 2 lbs.
1 " . . 2 lbs. — 2 lbs. 11 oz. — 2 lbs. 14 oz.
1¼ " . . 3 lbs. — 3 lbs. 11 oz. — 4 lbs. 7 oz.
1½ " . . 4 lbs. — 4 lbs. 11 oz. — 5 lbs. 9 oz.
2 " . . 5 lbs. 9 oz. — 7 lbs. — 8 lbs. 5 oz.
2½ " . . 7 lbs. — 8 lbs. 9 oz. —10 lbs.

Weight of Copper Tubing.
Of the usual thickness.

When the inside diameter is ¼ of an inch, 3 ounces; ⅜ of an inch, 5 ounces; ½ of an inch, 6 ounces; ⅝ of an inch, 8 ounces; and ¾ of an inch, 10 ounces per foot.

STRENGTH OF MATERIALS.

Materials of construction are liable to four different kinds of strain, viz., stretching, crushing, transverse action, and torsion or twisting: the first of which depends upon the body's tenacity alone; the second, on its resistance to compression; the third on its tenacity and compression combined; and the fourth, on that property by which it opposes any acting force tending to change from a straight line, to that of a spiral direction, the fibres of which the body is composed.

In bodies, the power of tenacity and resistance to compression, in the direction of their length is as the cross-section of their area multiplied by the results of experiments on similar bodies, as exhibited in the following table:

TABLE

Showing the Tenacities, Resistances to Compression, and other Properties of the common Materials of Construction.

Names of Bodies.	Absolute.		Compared with Cast Iron.		
	Tenacity in lbs. per sq. inch.	Resistance to compression in lbs per sq. in.	Its strength is	Its extensibility is	Its stiffness is
Ash	14130	—	0 23	2·6	0.089
Beech	12225	8548	0·15	2·1	0·073
Brass	17968	10304	0·435	0·9	0·49
Brick	275	562	—	—	—
Cast iron	13434	86397	1·000	1·0	1·000
Copper (wrought)	33000	—	—	—	—
Elm	9720	1033	0·21	2·9	0·073
Fir, or Pine, white	12346	2028	0·23	2·4	0·1
" " red	11800	5375	0·3	2·4	0·1
" " yellow	11835	5445	0·25	2·9	0·087
Granite	—	10910	—	—	—
Gun-metal (copper 8, and tin 1	35838	—	0·65	1·25	0·535
Malleable iron	56000	—	1.12	0·86	1·3
Larch	12240	5568	0·136	2·3	0·058
Lead	1824	—	0·096	2·5	0·0385
Mahogany, Honduras	11475	8000	0·24	2·9	0·487
Marble	551	6060	—	—	—
Oak	11880	9504	0·25	2·8	0·093
Rope (1 in. in circum.)	200	—	—	—	—
Steel	128000	—	—	—	—
Tin (cast)	4736	—	0·182	0·75	0·25
Zinc (sheet)	9120	—	0·365	0 5	0·76

TABLE

Of the Comparative Strength and Weight of Ropes and Chains.

Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons and cwt.	Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons and cwt.
$3\frac{1}{2}$	$2\frac{3}{4}$	$\frac{5}{16}$	$5\frac{1}{2}$	1 $5\frac{1}{2}$	10	23	$\frac{7}{8}$	43	10 0
$4\frac{1}{4}$	$4\frac{3}{4}$	$\frac{3}{8}$	8	1 $16\frac{3}{4}$	$10\frac{3}{4}$	28	$\frac{15}{16}$	49	11 11
5	$5\frac{3}{4}$	$\frac{7}{16}$	$10\frac{1}{2}$	2 10	$11\frac{1}{2}$	$30\frac{1}{2}$	1 in.	56	13 8
$5\frac{3}{4}$	7	$\frac{1}{2}$	14	3 $5\frac{1}{2}$	$12\frac{1}{4}$	36	$1\frac{1}{16}$	63	14 18
$6\frac{1}{2}$	$9\frac{3}{4}$	$\frac{9}{16}$	18	4 $3\frac{1}{2}$	13	39	$1\frac{1}{8}$	71	16 14
7	$11\frac{1}{4}$	$\frac{5}{8}$	22	5 2	$13\frac{3}{4}$	45	$1\frac{3}{16}$	79	18 11
8	15	$\frac{11}{16}$	27	6 $4\frac{1}{2}$	$14\frac{1}{2}$	$48\frac{1}{2}$	$1\frac{1}{4}$	87	20 8
$8\frac{3}{4}$	19	$\frac{3}{4}$	32	7 7	$15\frac{1}{4}$	56	$1\frac{5}{16}$	96	22 13
$9\frac{1}{2}$	21	$\frac{13}{16}$	37	8 $18\frac{1}{2}$	16	60	$1\frac{3}{8}$	106	24 18

Note.—It must be understood, and also borne in mind, that in estimating the amount of tensile strain to which a body is subjected, the weight of the body itself must also be taken into account; for according to its position so may it approximate to its whole weight, in tending to produce extension within itself; as in the almost constant application of ropes and chains to great depths, considerable heights, &c.

Resistance to Lateral Pressure, or Transverse Action.

TABLE

Of Data, containing the Results of Experiments on the Elasticity and Strength of various Species of Timber.

Species of Timber.	Value of E.	Value of S.	Species of Timber.	Value of E.	Value of S.
Teak, . . .	174·7	2462	Elm, ...	50·64	1013
Poona, . . .	122·26	2221	Pitch pine, .	88·68	1632
English Oak, .	105	1672	Red pine, . .	133	1341
Canadian do., .	155·5	1766	New Eng. fir,	158·5	1102
Dantzic do., .	86·2	1457	Riga do., . .	90	1100
Adriatic do., .	70·5	1383	Mar Forest do.	63	1200
Ash, ...	119	2026	Larch, . . .	76	900
Beech, . . .	98	1556	Norway spruce,	105·47	1474

The strength of a square or rectangular beam to resist lateral pressure, acting in a perpendicular direction to its length, is as the breadth and square of the depth, and inversely as the length. Thus, a beam twice the breadth of another, all other circumstances being alike, equals twice the strength of the other; or twice the depth, equal four times the strength, and twice the length, equal only half the strength, &c., according to the rule.

To find the dimensions of a beam capable of maintaining a given weight, with a given degree of deflection, when supported at both ends.

RULE. Multiply the weight to be supported in lbs. by the cube of the length in feet; divide the product by 32 times the tabular value of E, multiplied into the given deflection in inches; and the quotient is the breadth multiplied by the cube of the depth in inches.

Note 1.—When the beam is intended to be square, then the fourth root of the quotient is the breadth and depth required.

Note 2.—If the beam is to be cylindrical, multiply the quotient by 1·7, and the fourth root of the product is the diameter.

EXAMPLE. The distance between the supports of a beam of Riga fir is 16 feet, and the weight it must be capable of sustaining in the middle of its length is 8000 lbs., with a deflection of not more than ¾ of an inch; what must be the depth of the beam, supposing the breadth 8 inches?

$$\frac{16 \times 8000}{90 \times 32 \times \cdot 75} = 15175 \div 8 = \sqrt[3]{1897} = 12{\cdot}35 \text{ in., the depth.}$$

To determine the absolute strength of a rectangular beam of timber, when supported at both ends, and loaded in the middle of its length, as beams in general ought to be calculated to, so that they may be rendered capable of withstanding all accidental cases of emergency.

RULE. Multiply the tubular value of S by four times the depth of the beam in inches, and by the area of the cross section in inches; divide the product by the distance between the supports in inches, and the quotient will be the absolute strength of the beam in lbs.

Note 1.—If the beam be not laid horizontally, the distance between the supports, for calculation, must be the horizontal distance.

Note 2.—One fourth of the weight obtained by the rule is the greatest weight that ought to be applied in practice as permanent load.

Note 3.—If the load is to be applied at any other point than the middle, then the strength will be as the product of the two distances is to the square of half the length of the beam between the supports; or, twice the distance from one end, multiplied by twice from the other, and divided by the whole length, equal the effective length of the beam

EXAMPLE. In a building 18 feet in width, an engine boiler of 5½ tons is to be fixed, the centre of which is to be 7 feet from the wall and having two pieces of red pine, 10 inches by 6, which I can lay across the two walls for the purpose of slinging it at each end, may I with sufficient confidence apply them, so as to effect this object?

$$\frac{2240 \times 5{\cdot}5}{4} = 6160 \text{ lbs. to carry at each end.}$$

And 18 feet — 7 = 11, double each, or 14 and 22, then

$$\frac{14 \times 22}{13} = 17 \text{ feet, or 204 inches, effective length of beam.}$$

Tabular value of S, red pine, $= \frac{1341 \times 4 \times 10 \times 60}{204} = 15776$ lbs.

The absolute strength of each piece of timber at that point.

To determine the dimensions of a rectangular beam capable of supporting a required weight, with a given degree of deflection, when fixed at one end.

RULE. Divide the weight to be supported. in lbs., by the tabular value of E, multiplied by the breadth and deflection, both in inches; and the cube root of the quotient, multiplied by the length in feet, equal the depth required in inches.

EXAMPLE. A beam of ash is intended to bear a load of 700 lbs. at its extremity, its length being 5 feet, its breadth 4 inches, and the deflection not to exceed $\frac{1}{2}$ of an inch.

Tabular value of E $= 119 \times 4 \times {\cdot}5 = 238$ the divisor;

then $700 \div 238 = \sqrt[3]{2{\cdot}94} \times 5 = 7{\cdot}25$ inches, depth of the beam.

To find the absolute strength of a rectangular beam, when fixed at one end and loaded at the other.

RULE. Multiply the value of S by the depth of the beam, and by the area of its section, both in inches: divide the product by the leverage in inches, and the quotient equal the absolute strength of the beam in lbs.

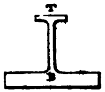
EXAMPLE. A beam of Riga fir, 12 inches by $4\frac{1}{2}$, and projecting $5\frac{1}{2}$ feet from the wall; what is the greatest weight it will support at the extremity of its length?

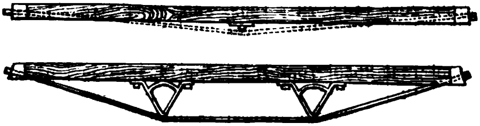
Tabular value of S $= 1100$
$12 \times 4{\cdot}5 = 54$ sectional area,

Then, $\frac{1100 \times 12 \times 54}{78} = 9138{\cdot}4$ lbs.

When fracture of a beam is produced by vertical pressure, the fibres of the lower section of fracture are separated by extension, whilst at the same time those of the upper portion are destroyed by compression; hence exists a point in section where neither the one nor the other takes place, and which is distinguished as the point

of neutral axis. Therefore, by the law of fracture thus established, and proper data of tenacity and compression given, as in the table (p. 135), we are enabled to form metal beams of strongest section with the least possible material. Thus, in cast iron, the resistance to compression is nearly as 6½ to 1 of tenacity; consequently a beam of cast iron, to be of strongest section, must be of the following form, and a parabola in the direction of its length, the quantity of material in the bottom flange being about 6½ times that of the upper. But such is not the case with beams of timber; for although the tenacity of timber be on an average twice that of its resistance to compression, its flexibility is so great that any considerable length of beam, where columns cannot be situated to its support, requires to be strengthened or trussed by iron rods, as in the following manner:





and these applications of principle not only tend to diminish deflection, but the required purpose is also more effectively attained, and that by lighter pieces of timber.

To ascertain the absolute strength of a cast-iron beam of the preceding form, or that of strongest section.

Rule. Multiply the sectional area of the bottom flange in inches by the depth of the beam in inches, and divide the product by the distance between the supports, also in inches; and 514 times the quotient equal the absolute strength of the beam in cwts.

The strongest form in which any given quantity of matter can be disposed is that of a hollow cylinder; and it has been demonstrated that the maximum of strength is obtained in cast iron when the thickness of the annulus or ring amounts to ⅕th of the cylinder's external diameter; the relative strength of a solid to that of a hollow cylinder being as the diameters of their sections.

Tortoise-shell ground for Metal.—Cover the plates intended to represent the transparent parts of the tortoise-shell with a thin coat of vermilion in seed-lac varnish. Then brush over the whole with a varnish composed of linseed oil boiled with umber until it is almost black. The varnish may be thinned with oil of turpentine before it is used. When the work is done it may be set in an oven, with the same precautions as the black varnish

Force in Pile-Driving.—In a sandy soil the greatest force of a pile-driver will not drive a pile over fifteen feet.

TABLE

Showing the Weight or Pressure a Beam of Cast Iron, 1 inch in breadth, will sustain, without destroying its elastic force, when it is supported at each end, and loaded in the middle of its length, and also the deflection in the middle which that weight will produce.

Length.	6 feet.		7 feet.		8 feet.		9 feet.		10 feet.	
Depth in inches.	Weight in lbs.	Deflection in inches.	Weight in lbs.	Deflection in inches.	Weight in lbs.	Deflection in inches.	Weight in lbs.	Deflection in inches.	Weight in lbs.	Deflection in inches.
3	1278	·24	1089	·33	954	·426	855	·54	765	·66
3½	1739	·205	1482	·28	1298	·365	1164	·46	1041	·57
4	2272	·18	1936	·245	1700	·32	1520	·405	1360	·5
4½	2875	·16	2450	·217	2146	·284	1924	·36	1721	·443
5	3560	·144	3050	·196	2650	·256	2375	·32	2125	·4
6	5112	·12	4356	·168	3816	·213	3420	·27	3060	·33
7	6958	·103	5929	·14	5194	·183	4655	·23	4165	·29
8	9088	·09	7744	·123	6784	·16	6080	·203	5440	·25
9			9801	·109	8586	·142	7695	·18	6885	·22
10			12100	·098	10600	·128	9500	·162	8500	·2
11					12826	·117	11495	·15	10285	·182
12					15264	·107	13680	·135	12240	·17
13							16100	·125	14400	·154
14							18600	·115	16700	·143
	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.	
6	2548	·48	2184	·65	1912	·85	1699	1·08	1580	1·34
7	3471	·41	2975	·58	2603	·73	2314	·93	2082	1·14
8	4532	·36	3884	·49	3396	·64	3020	·81	2720	1·00
9	5733	·32	4914	·44	4302	·57	3825	·72	3438	·89
10	7083	·28	6071	·39	5312	·51	4722	·64	4250	·8
11	8570	·26	7346	·36	6428	·47	5714	·59	5142	·73
12	10192	·24	8736	·33	7648	·43	6796	·54	6120	·67
13	11971	·22	10260	·31	8978	·39	7980	·49	7182	·61
14	13883	·21	11900	·28	10412	·36	9255	·46	8330	·57
15	15937	·19	13660	·26	11952	·34	10624	·43	9562	·53
16	18128	·18	15536	·24	13584	·32	12080	·40	10880	·5
17	20500	·17	17500	·23	15353	·3	13647	·38	12282	·47
18	22932	·16	19656	·21	17208	·28	15700	·36	13752	·44

Note.—This table shows the greatest weight that ever ought to be laid upon a beam for permanent load; and, if there be any liability to jerks, &c., ample allowance must be made; also, the weight of the beam itself must be included.

To find the weight of a cast-iron beam of given dimensions.

Rule. Multiply the sectional area in inches by the length in feet, and by 3·2, the product equal the weight in lbs.

Ex. Required the weight of a uniform rectangular beam of cast iron, 16 feet in length, 11 inches in breadth, and 1½ inch in thickness.

$$11 \times 1{\cdot}5 \times 16 \times 3{\cdot}2 = 844{\cdot}8 \text{ lbs.}$$

Resistance of Bodies to Flexure by Vertical Pressure.

When a piece of timber is employed as a column or support, its tendency to yielding by compression is different according to the proportion between its length and area of its cross section; and supposing the form that of a cylinder whose length is less than seven or eight times its diameter, it is impossible to bend it by any force applied longitudinally, as it will be destroyed by splitting before that bending can take place; but when the length exceeds this, the column will bend under a certain load, and be ultimately destroyed by a similar kind of action to that which has place in the transverse strain.

Columns of cast iron, and of other bodies, are also similarly circumstanced, this law having recently been fully developed by the experiments of Mr. Hodgkinson on columns of different diameters, and of different lengths.

When the length of a cast-iron column with flat ends equals about thirty times its diameter, fracture will be produced wholly by bending of the material. When of less length, fracture takes place partly by crushing and partly by bending. But, when the column is enlarged in the middle of its length from one and a half to twice its diameter at the ends, by being cast hollow, the strength is greater by $\frac{1}{7}$th than in a solid column containing the same quantity of material.

To determine the dimensions of a support or column to bear, without sensible curvature, a given pressure in the direction of its axis.

Rule.—Multiply the pressure to be supported in lbs. by the square of the column's length in feet, and divide the product by twenty times the tabular value of E; and the quotient will be equal to the breadth multiplied by the cube of the least thickness, both being expressed in inches.

Note 1.—When the pillar or support is a square, its side will be the fourth root of the quotient.

2 If the pillar or column be a cylinder, multiply the tabular value of E by 12, and the fourth root of the quotient equal the diameter.

Ex. 1. What should be the least dimensions of an oak support, to bear a weight of 2240 lbs., without sensible flexure, its breadth being 3 inches, and its length 5 feet?

Tabular value of E = 105,

$$\text{and } \frac{2240 \times 5^2}{20 \times 105 \times 3} = \sqrt[3]{8{\cdot}888} = 2{\cdot}05 \text{ inches.}$$

Ex. 2. Required the side of a square piece of Riga fir, 9 feet in length, to bear a permanent weight of 6000 lbs.

Tabular value of E = 96,

$$\text{and } \frac{6000 \times 9^2}{20 \times 96} \times \sqrt[4]{253} = 4 \text{ inches nearly.}$$

TABLE

Of the Dimensions of Cylindrical Columns of Cast Iron to sustain a given load or pressure with safety.

Diameter in inches.	LENGTH OR HEIGHT IN FEET.										
	4	6	8	10	12	14	16	18	20	22	24
	WEIGHT OR LOAD IN CWTS.										
2	72	60	49	40	32	26	22	18	15	13	11
2½	119	105	91	77	65	55	47	40	34	29	25
3	178	163	145	128	111	97	84	73	64	56	49
3½	247	232	214	191	172	156	135	119	106	94	83
4	326	310	288	266	242	220	198	178	160	144	130
4½	418	400	379	354	327	301	275	251	229	208	189
5	522	501	479	452	427	394	365	337	310	285	262
6	607	592	573	550	525	497	469	440	413	386	360
7	1032	1013	989	959	924	887	848	808	765	725	686
8	1333	1315	1289	1259	1224	1185	1142	1097	1052	1005	959
9	1716	1697	1672	1640	1603	1561	1515	1467	1416	1364	1311
10	2119	2100	2077	2045	2007	1964	1916	1865	1811	1755	1697
11	2570	2550	2520	2490	2450	2410	2358	2305	2248	2189	2127
12	3050	3040	3020	2970	2930	2900	2880	2780	2730	2670	2600

Practical Utility of the preceding Table.

Ex. Wanting to support the front of a building with cast-iron columns 18 feet in length, 8 inches in diameter, and the metal 1 inch in thickness; what weight may I confidently expect each column capable of supporting without tendency to deflection?

Opposite 8 inches diameter and under 18 feet = 1097
Also opposite 6 in. diameter and under 18 feet = 440

= 657 cwt.

Note.—The strength of cast iron as a column being			1·0000
"	steel	"	= 2·518
"	wrought iron	"	= 1·745
"	(oak) Dantzic	"	= ·1068
"	red deal	"	= ·0785

Elasticity of Torsion, or Resistance of Bodies to Twisting.

The angle of flexure by torsion is as the length and extensibility of the body directly, and inversely as the diameter; hence the length of a bar or shaft being given, the power, and the leverage the power acts with, being known, and also the number of degrees of torsion that will not affect the action of the machine, to determine the diameter in cast iron, with a given angle of flexure.

Rule. Multiply the power in lbs. by the length of the shaft in feet, and by the leverage in feet; divide the product by fifty-five times the number of degrees in the angle of torsion; and the fourth root of the quotient equal the shaft's diameter in inches.

Ex. Required the diameters for a series of shafts 35 feet in length, and to transmit a power equal to 1245 lbs., acting at the circumference of a wheel $2\frac{1}{2}$ feet radius, so that the twist of the shafts on the application of the power may not exceed one degree.

$$\frac{1245 \times 35 \times 2·5}{55 \times 1} = \sqrt[4]{1981} = 6·67 \text{ inches in diameter.}$$

Relative Strength of Metals to resist Torsion.

Cast iron	= 1·	Swedish bar iron .	= 1·05
Copper	= ·48	English do. . .	= 1·12
Yellow Brass . .	= ·511	Shear steel . . .	= 1·96
Gun-metal . . .	= ·55	Cast do.	= 2·1

Map Colors.

YELLOW.

1. Dissolve gamboge in water.
2. Make a decoction of French berries, strain, and add a little gum arabic.

RED.

1. Make a decoction of Brazil dust in vinegar, and add a little gum and alum.
2. Make an infusion of cochineal, and add a little gum.

BLUE.

A weak mixture of sulphate of indigo and water, to which add a little gum.

GREEN.

1. Dissolve crystals of verdigris in water, and add a little gum.
2. Dissolve sap green in water, and add gum.

TABLE

Of the Weight of a Superficial Foot of Plate or Sheet Iron, Copper, and Brass, in pounds.

Thickness in parts of an inch.	Iron.	Thickness by the wire gauge. No.	Iron.	Copper.	Brass.	Thickness by the wire gauge. No	Iron.	Copr.	Brass.
$\frac{1}{32}$	1·25	1	12·5	14·5	13·75	16	2·5	2·9	2·75
$\frac{1}{16}$	2·5	2	12	13·9	13·2	17	2·18	2·52	2·4
$\frac{1}{8}$	5	3	11	12·75	12·1	18	1·86	2·15	2·04
$\frac{3}{16}$	7·5	4	10	11·6	11	19	1·7	1·97	1·87
$\frac{1}{4}$	10	5	8·74	10·1	9·61	20	1·54	1·78	1·69
$\frac{5}{16}$	12·5	6	8·12	9·4	8·93	21	1·4	1·62	1·54
$\frac{3}{8}$	15	7	7·5	8·7	8·25	22	1·25	1·45	1·37
$\frac{7}{16}$	17·5	8	6·86	7·9	7·54	23	1·12	1·3	1·23
$\frac{1}{2}$	20	9	6·24	7·2	6·86	24	1	1·16	1·1
$\frac{9}{16}$	22·5	10	5·62	6·5	6·18	25	·9	1·04	·99
$\frac{5}{8}$	25	11	5	5·8	5·5	26	·8	·92	·88
$\frac{11}{16}$	27·5	12	4·38	5·08	4·81	27	·72	·83	·79
$\frac{3}{4}$	30	13	3·75	4·34	4·12	28	·64	·74	·7
$\frac{7}{8}$	35	14	3·12	3·6	3·43	29	·56	·64	·61
1	40	15	2·82	3·27	3·1	30	·5	·58	·55

Note.—No. 1 wire gauge equal $\frac{5}{16}$ths of an inch.
" 4 " $\frac{1}{4}$ "
" 7 " $\frac{3}{16}$ "
" 11 " $\frac{1}{8}$ "
" 16 " $\frac{1}{16}$ "
" 22 " $\frac{1}{32}$ "

The great variety of thicknesses into which copper is manufactured, cause in trade the weight to be named whereby to determine the thickness required, the unit being that of a common sheet, so designated, viz. 4 feet by 2 feet, in lbs., thus:

A 70 lb. plate is $\frac{3}{16}$ths of an inch in thickness.
" $46\frac{1}{2}$ " $\frac{1}{8}$ " "
" 23 " $\frac{1}{16}$ " "
" $11\frac{1}{2}$ " $\frac{1}{32}$ " "
" 6 " $\frac{1}{64}$ " "

The thickness of lead is also in common determined or understood by the weight, the unit being that of a square or superficial foot; thus:

4	lbs. lead is	$\frac{1}{16}$th	of an inch	in thickness.
6	"	$\frac{1}{10}$	"	"
$7\frac{1}{2}$	"	$\frac{1}{8}$	"	"
11	"	$\frac{3}{16}$	"	"
15	"	$\frac{1}{4}$	"	"

Comparative Weights of Different Bodies.

Bar iron being 1,		Cast iron being 1,	
Cast iron	= ·95	Bar iron	= 1·0
Steel	= 1·02	Steel	= 1·08
Copper	= 1·16	Brass	= 1·16
Brass	= 1·09	Copper	= 1·21
Lead	= 1·48	Lead	= 1·56

1. Suppose I have an article of plate iron, the weight of which is 728 lbs., but want the same of copper, and of similar dimensions, what will be its weight?

$$728 \times 1{\cdot}16 = 844{\cdot}48 \text{ lbs.}$$

2. A model of dry pine, weighing $32\frac{1}{2}$ lbs., and in which the iron for its construction forms no material portion of the weight, what may I anticipate its weight to be in cast iron?

$$32{\cdot}5 \times 16 = 520 \text{ lbs.}$$

Note.—It frequently occurs, in the formation or construction of models, that neither the quality nor condition of the timber can be properly estimated; and, in such cases, it may be a near enough approximation to reckon 15 lbs. of cast iron to each lb. of model.

SILVERING POWDER, &c., for silvering copper, covering the worn parts of plated goods, &c.—1. Nitrate of silver 30 gr., common salt 30 gr., cream of tartar $3\frac{1}{2}$ dr. Mix. Moistened with water, and rubbed on dial plates or other copper articles, it coats them with silver.

2. Silver precipitated from its nitric solution by copper 20 gr., alum 30 gr., cream of tartar 2 dr., salt 2 dr.

3. Precipitated silver $\frac{1}{2}$ oz., common salt 2 oz., muriate of ammonia 2 oz., corrosive sublimate 1 dr. Make it into a paste with water. Copper utensils are previously boiled with tartar and alum, and rubbed with this paste, then made red hot, and afterwards polished.

4. Dissolve muriate of silver in a solution of hyposulphite of soda, and mix this with prepared hartshorn, or other suitable powder.

PLATINA FOR SPRINGS.—Platinum 1 part; gold 12 parts. Add the platinum to the gold in a state of fusion.

Tables by which to facilitate the Mensuration of Timber.

1. *Flat or Board Measure.*

Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.
¼	·0208	4	·3334	8	·6667
½	·0417	4¼	·3542	8¼	·6875
¾	·0625	4½	·375	8½	·7084
1	·0834	4¾	·3958	8¾	·7292
1¼	·1042	5	·4167	9	·75
1½	·125	5¼	·4375	9¼	·7708
1¾	·1459	5½	·4583	9½	·7917
2	·1667	5¾	·4792	9¾	·8125
2¼	·1875	6	·5	10	·8334
2½	·2084	6¼	·5208	10¼	·8542
2¾	·2292	6½	·5416	10½	·875
3	·25	6¾	·5625	10¾	·8959
3¼	·2708	7	·5833	11	·9167
3½	·2916	7¼	·6042	11¼	·9375
3¾	·3125	7½	·625	11½	·9583
		7¾	·6458	11¾	·9792

Application and Use of the Table.

1. Required the number of square feet in a board or plank 16½ feet in length, and 9¾ inches in breadth.

Opposite 9¾ is ·8125 × 16·5 = 13·4 square feet.

2. A board 1 foot 2¾ inches in breadth, and 21 feet in length; what is its superficial content in square feet?

Opposite 2¾ is ·2292, to which add the 1 foot.

Then 1·2292 × 21 = 25·8 square feet.

3. In a board 15½ inches at one end, 9 inches at the other, and 14½ feet in length, how many square feet?

$$\frac{15{\cdot}5+9}{2} = 12\tfrac{1}{4}, \text{ or } 1{\cdot}0208; \text{ and } 1{\cdot}0208 \times 14{\cdot}5 = 14{\cdot}8 \text{ square feet.}$$

TO GIVE IRON A TEMPER TO CUT PORPHYRY.—Make your iron red hot, and plunge it into distilled water from nettles, acanthus, and pilosella, or in the very juice pounded out from these plants.

PASTE FOR CLEANING METALS.—Take oxalic acid 1 part; rottenstone 6 parts. Mix with equal parts of train oil and spirits of turpentine to a paste.

2. *Cubic or Solid Measure.*

Mean ¼ girth in inches.	Cubic feet in each lineal foot.	Mean ¼ girth in inches.	Cubic feet in each lineal foot.	Mean ¼ girth in inches.	Cubic feet in each lineal foot
6	·25	14	1·361	22	3·362
6¼	·272	14¼	1·41	22¼	3·438
6½	·294	14½	1·46	22½	3·516
6¾	·317	14¾	1·511	22¾	3·598
7	·340	15	1·562	23	3·673
7¼	·364	15¼	1·615	23¼	3·754
7½	·39	15½	1·668	23½	3·835
7¾	·417	15¾	1·722	23¾	3·917
8	·444	16	1·777	24	4·
8¼	·472	16¼	1·833	24¼	4·084
8½	·501	16½	1·89	24½	4·168
8¾	·531	16¾	1·948	24¾	4·254
9	·562	17	2·006	25	4·34
9¼	·594	17¼	2·066	25¼	4·428
9½	·626	17½	2·126	25½	4·516
9¾	·659	17¾	2·187	25¾	4·605
10	·694	18	2·25	26	4·694
10¼	·73	18¼	2·318	26¼	4·785
10½	·766	18½	2·376	26½	4·876
10¾	·803	18¾	2·442	26¾	4·969
11	·84	19	2·506	27	5·062
11¼	·878	19¼	2·574	27¼	5·158
11½	·918	19½	2·64	27½	5·252
11¾	·959	19¾	2·709	27¾	5·348
12	1·	20	2·777	28	5·444
12¼	1·042	20¼	2·898	28¼	5·542
12½	1·085	20½	2·917	28½	5·64
13¾	1·129	20¾	2·99	28¾	5·74
13	1·174	21	3·062	29	5·84
13¼	1·219	21¼	3·136	29¼	5·941
13½	1·265	21½	3·209	29½	6·044
13¾	1·313	21¾	3·285	29¾	6·146

In the cubic estimation of timber, custom has established the rule of ¼ the mean girt being the side of the square considered as the cross sectional dimensions; hence, multiply the number of cubic feet per lineal foot, as in the Table of Cubic Measure, opposite the ¼ girth, and the product is the solidity of the given dimensions in cubic feet.

Suppose the mean ¼ girth of a tree 21¼ inches, and its length 16 feet, what are its contents in cubic feet?

$$3·136 \times 16 = 50·176 \text{ cubic feet.}$$

CAST METAL CYLINDERS.

The Cylinders are solid, each 1 foot in length.

Diameter.	Iron.	Copper.	Brass.	Lead.
inches.	lbs.	lbs.	lbs.	lbs.
1	2·5	3·0	2·9	3·9
2	9·8	12·0	11·4	15·5
3	22·1	27·0	25·8	34·8
4	39·3	47·9	45·8	61·9
5	61·4	74·9	71·6	96·7
6	88·4	107·8	103·0	139·3
7	120·3	146·8	140·2	189·6
8	157·1	191·7	183·2	247·7
9	198·8	242·7	231·8	313·4
10	245·4	299·5	286·2	387·0

CAST-IRON PIPES.

Table showing the Weight of Pipes 1 foot long, of bores from 1 inch to 12 inches in diameter, advancing by ¼ of an inch; and of thicknesses from ¼ of an inch to 1¼ inches, advancing by ⅛ of an inch.

bore.	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼
in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1	3·1	5·1	7·4	10·0	12·9	16·1	19·6	23·5	27·6
1¼	3·7	6·0	8·6	11·5	14·7	18·3	22·1	26·2	30·7
1½	4·3	6·9	9·8	13·0	16·6	20·4	24·5	29·0	33·7
1¾	4·9	7·8	11·1	14·6	18·4	22·6	27·0	31·8	36·8
2	5·5	8·8	12·3	16·1	20·3	24·7	29·5	34·5	39·9
2¼	6·1	9·7	13·5	17·6	22·1	26·8	31·9	37·3	43·0
2½	6·7	10·6	14·7	19·2	23·9	28·9	34·4	40·0	46·0
2¾	7·4	11·5	16·0	20·7	25·7	31·1	36·8	42·8	49·1
3	8·0	12·4	17·2	22·2	27·6	33·3	39·3	45·6	52·2
3¼	8·6	13·3	18·4	23·8	29·5	35·4	41·7	48·3	55·2
3½	9·2	14·2	19·6	25·3	31·3	37·6	44·2	51·1	58·3
3¾	9·8	15·2	20·9	26·9	33·1	39·7	46·6	53·8	61·4
4	10·4	16·1	22·1	28·4	35·0	41·9	49·1	56·6	64·4
4¼	11·1	17·1	23·4	30·0	36·9	44·1	51·6	59·4	67·6
4½	11·7	18·0	24·5	31·4	38·7	46·2	54·0	62·1	70·6
4¾	12·3	18·9	25·8	33·0	40·5	48·3	56·5	64·9	73·6
5	12·9	19·8	27·0	34·5	42·3	50·5	58·9	67·6	76·7
5¼	13·5	20·7	28·2	36·1	44·2	52·6	61·4	70·4	79·8
5½	14·1	21·6	29·5	37·6	46·0	54·8	63·8	73·2	82·8

CAST-IRON PIPES.

(Continued.)

bore.	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼
in.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
5¾	14·7	22·6	30·7	39·1	47·9	56·9	66·3	76·0	85·9
6	15·3	23·5	31·9	40·7	49·7	59·1	68·7	78·7	88·8
6¼	16·0	24·4	33·1	42·2	51·5	61·2	71·2	81·2	92·0
6½	16·6	25·3	34·4	43·7	53·4	63·4	73·4	84·2	95·1
6¾	17·2	26·2	35·6	45·3	55·2	65·3	76·1	87·0	98·2
7	17·8	27·2	36·8	46·8	56·8	67·7	78·5	89·7	101·2
7¼	18·4	28·1	38·1	48·1	58·9	69·8	81·0	92·5	104·2
7½	19·0	29·0	39·1	49·9	60·7	72·0	83·5	95·3	107·4
7¾	19·6	29·7	40·5	51·4	62·6	74·1	85·9	98·0	110·5
8	20·0	30·8	41·7	52·9	64·4	76·2	88·4	100·8	113·5
8¼	20·9	31·7	43·0	54·5	66·3	78·4	90·8	103·5	116·6
8½	21·7	32·9	44·4	56·2	68·3	80·8	93·5	106·5	119·9
8¾	22·1	33·6	45·4	57·5	70·0	82·7	95·7	109·1	122·7
9	22·7	34·5	46·6	59·1	71·8	84·8	98·2	111·8	125·8
9¼	23·3	35·4	47·9	60·6	73·6	87·0	100·6	114·6	128·9
9½	23·9	36·4	49·1	62·1	75·5	89·1	103·1	117·4	131·9
9¾	24·6	37·3	50·3	63·7	77·3	91·3	105·5	120·1	135·0
10	25·2	38·2	51·5	65·2	79·2	93·4	108·0	122·8	138·1
10¼	25·8	39·1	52·8	66·7	81·0	95·6	110·4	125·6	141·1
10½	26·4	40·0	54·0	68·3	82·8	97·7	112·9	128·4	144·2
10¾	27·0	41·0	55·2	69·8	84·7	99·9	115·4	131·2	147·3
11	27·6	41·9	56·5	71·3	86·5	102·0	117·8	133·9	150·3
11¼	28·2	42·8	57·7	72·9	88·4	104·2	120·3	136·7	153·4
11½	28·8	43·7	58·9	74·4	90·2	106·3	122·7	139·4	156·4
11¾	29·5	44·6	60·1	75·9	92·0	108·5	125·2	142·2	159·5
12	30·1	45·6	61·4	77·5	93·6	110·6	127·6	145·0	162·6

Strength of Journals of Shafts.

Mr. Buchanan's rule is: The cube root of the weight in cwts. is nearly equal to the diameter of the journal; it being prudent to make the journal a little more than less, and to make a due allowance for wearing.

Ex. What is the diameter of a journal of a water-wheel shaft, 13 feet long, the weight of the wheel being 15 tons?

By Mr. B.'s rule,

$$\sqrt[3]{15 \times 20} = 6{\cdot}7, \text{ or 7 inches diameter.}$$

By Mr. Tredgold's rule,

Weight in the middle, $\frac{3360}{500} \times 13 = 873\ \sqrt[3]{873} = 9\frac{1}{2}$ inches diam.

Weight equally distributed, $33600 \times 13 = 436800\ \sqrt[3]{\frac{436800}{10}} =$ 7·65 inches.

To resist Torsion or Twisting.

It is obvious that the strength of revolving shafts* is directly as the cubes of their diameters and revolutions; and inversely as the resistance they have to overcome.

Mr. Robertson Buchanan, in his Essay on the Strength of Shafts, gives the following data, deduced from several experiments, viz.: That the fly-wheel shaft of a 50-horse-power engine, at 50 revolutions per minute, requires to be $7\frac{1}{2}$ inches diameter; and therefore the cube of this diameter, which is = 421·875, serves as a multiplier to all other shafts in the same proportion: and, taking this as a standard, he gives the following multipliers, viz.:

For the shaft of a steam-engine, water-wheel, or any shaft connected with a first power,	400
For shafts in inside of mills, to drive smaller machinery, or connected with the shafts above,	200
For the small shafts of a mill or machinery,	100

From the foregoing, the following rule is derived, viz.: The number of horse power a shaft is equal to is directly as the cube of the diameter and number of revolutions; and inversely as the above multipliers.

Ex. 1. When the fly-wheel shaft of a 45-horse-power steam engine makes 90 revolutions per minute, what is the diameter of the journal?

$$\frac{45 \times 400}{90} = 200\ \sqrt[3]{200} = 5\tfrac{8}{10} \text{ inches diameter.}$$

Ex. 2. The velocity of a shaft is 80 revolutions per minute, and its diameter is 3 inches; what is its power?

$$\frac{3^3 \times 80}{400} = 5{\cdot}4 \text{ horse power.}$$

Ex. 3. What will be the diameter of the shaft in the first example, when used as a shaft of the second mover.†

$$\frac{5{\cdot}8}{1{\cdot}25} = 4{\cdot}64, \text{ or } \frac{\sqrt[3]{45 \times 200}}{90} = 4\tfrac{6}{10} \text{ inches diameter.}$$

* Shafts here are understood as the journals of shafts, the bodies of shafts being generally made square.

† The diameters of the second movers will be found by dividing the numbers in the table by 1·25, and the diameters of the third movers, by dividing the numbers by 1·56.

TABLE *of the diameters of shafts, being the first movers, or having 400 for their multipliers.*

Horse Power.	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105
	REVOLUTIONS.																			
	INCHES DIAMETER.																			
4	5·5	4·8	4·5	4·	3·7	3·8	3·5	3·3	3·2	3·1	3·	2·9	2·9	2·8	2·7	2·7	2·6	2·6	2·6	2·5
5	5·9	5·1	4·7	4·4	4·1	3·9	3·7	3·6	3·5	3·3	3·3	3·2	3·1	3·	3·	2·9	2·9	2·8	2·8	2·7
6	6·3	5·5	5·	4·6	4·4	4·1	4·	3·8	3·7	3·6	3·5	3·5	3·4	3·3	3·2	3·2	3·	3·	2·9	2·9
7	6·6	5·8	5·2	4·9	4·6	4·4	4·2	4	3·9	3·7	3·6	3·6	3·5	3·4	3·4	3·3	3·3	3·2	3·1	3·1
8	6·9	6·	5·5	5·1	4·8	4·6	4·4	4·2	4·1	4·	3·9	3·8	3·7	3·6	3·5	3·5	3·4	3·4	3·3	3·2
9	7·2	6·3	5·7	5·5	5·	4·8	4·5	4·4	4·2	4·1	4·	3·8	3·7	3·7	3·6	3·6	3·5	3·5	3·4	3·3
10	7·4	6·6	5·9	5·6	5·2	4·9	4·7	4·6	4·4	4·2	4·1	4·	3·9	3·8	3·7	3·7	3·6	3·6	3·5	3·4
12	7·9	6·9	6·3	5·8	5·6	5·4	5·2	5·	4·8	4·6	4·4	4·3	4·2	4·1	4·	3·9	3·8	3·8	3·7	3·6
14	8·3	7·2	6·7	6·2	5·9	5·6	5·4	5·2	5·	4·7	4·5	4·4	4·4	4·3	4·2	4·1	4·	4·	3·9	3·8
16	8·7	7·6	7·1	6·6	6·1	5·8	5·6	5·4	5·2	5·	4·8	4·7	4·6	4·5	4·4	4·4	4·3	4·2	4·1	4·
18	9·	7·9	7·5	7·	6·6	6·2	5·8	5·6	5·4	5·2	5·	4·9	4·8	4·7	4·6	4·5	4·4	4·3	4·2	4·2
20	9·3	8·1	7·7	7·2	6·8	6·4	5·9	5·7	5·6	5·4	5·2	5·1	5·	4·8	4·6	4·6	4·5	4·5	4·4	4·4
25	10·	8·5	8·	7·4	7·1	6·8	6·3	6·	5·9	5·6	5·5	5·4	5·3	5·2	5·1	4·9	4·8	4·7	4·6	4·6
30	10·7	9·3	8·4	7·9	7·4	7·1	6·9	6·7	6·5	6·3	5·9	5·8	5·7	5·6	5·5	5·3	5·2	5·1	5·	4·9
35	11·4	9·8	8·9	8·4	7·9	7·4	7·1	6·9	6·6	6·5	6·3	6·1	5·9	5·7	5·6	5·5	5·4	5·3	5·2	5·2
40	11·7	10·5	9·3	8·8	8·3	7·8	7·4	7·2	6·9	6·7	6·6	6·4	6·2	6·	5·9	5·8	5·7	5·6	5·6	5·5
45	12·	10·6	9·7	9·2	8·7	8·1	7·6	7·4	7·	6·8	6·7	6·5	6·4	6·2	6·1	6·	5·9	5·8	5·7	5·6
50	12·6	11·	10·	9·8	9·	8·5	8·	7·8	7·4	7·3	7·2	6·9	6·8	6·6	6·5	6·4	6·2	6·	5·9	5·8
55	13·4	11·4	10·4	9·8	9·1	8·8	8·4	8·	7·5	7·4	7·3	7·2	7·	6·7	6·6	6·5	6·3	6·2	6·1	6·
60	13·6	12·	10·8	10·	9·3	9·	8·6	8·2	7·7	7·6	7·4	7·3	7·2	6·9	6·8	6·8	6·7	6·6	6·4	6·2

It is a well known fact, that a cast-iron rod will sustain more torsional pressure than a malleable iron rod of the same dimensions; that is, a malleable iron rod will be twisted by a less weight than what is required to wrench a cast-iron rod of the same dimensions.

When the strength of malleable is less than that of cast iron to resist torsion, it is stronger than cast iron to resist lateral pressure, and that is in proportion as 9 is to 14.

From the foregoing, it is easy for the millwright to make his shafts of the iron best suited to overcome the resistance to which they will be subject, and the proportion of the diameters of their journals, according to the iron of which they are made.

Ex. What will be the diameter of a malleable iron journal to sustain an equal weight with a cast-iron journal of 7 inches diameter?

$7^3 = 343$.

As $14 : 343 :: 9 : 220\frac{1}{2}$; now $\sqrt[3]{220\cdot5} = 6\cdot04$ inches diameter.

Strength of Wheels.

The arms of wheels are as levers fixed at one end, and loaded at the other; and, consequently, the greatest strain is upon the end of the arm next the axle. For that reason, all arms of wheels should be strongest at that part, and tapering toward the rim.

The rule for the breadth and thickness of arms, according to their length and number in the wheel, is as follows: Multiply the power or weight acting at the end of the arm by the cube of its length; the product of which, divided by 2656 times the number of arms multiplied by the deflection, will give the breadth and cube of the depth.

Ex. Suppose the force acting at the circumference of a spur-wheel to be 1600 lbs., the radius of wheel 6 feet, and number of arms 8, and let the deflection not exceed $\frac{1}{10}$th of an inch.

$$\frac{1600 \times 6^3}{2656 \times 8 \times 1} = 163 = \text{breadth and cube of the depth.}$$

Let the breadth be 2·5 inches; therefore $\frac{163}{2\cdot5} = 65\cdot2$; which is equal to the cube of the depth. Now the cube root of 65·2 is nearly 4·03 inches; this, consequently, is the depth or dimension of each arm in the direction of the force.

Note.—When the depth at the rim is intended to be half that of the axes, use 1640 as a divisor instead of 2656.

The teeth are as beams, or cantilevers, fixed at one end, and loaded at the other. The rule applying directly to them where the length of the beam is the length of the teeth and the depth the thickness of the teeth. For the better explanation of the rule the following example is given.

Ex. The greatest power acting at the pitch line of the wheel is

6000 lbs., and the thickness of the teeth $1\frac{1}{2}$ inch, the length of the teeth being 0·25 feet; it is required to determine the breadth of the teeth.

$$\frac{6000 \times 0{\cdot}25}{212 \times 1{\cdot}5^2} = \frac{1500}{477} = 3{\cdot}2 \text{ inches, the breadth required.}$$

In order that the teeth may be capable of offering a sufficient resistance after being worn by friction, the breadth thus found should be doubled; therefore, in the above example, the breadth should be 6·4, or say $6\frac{1}{2}$ inches.

The following data are gleaned from experiments, which are therefore valuable, and of much use to the practical mechanic:

Rule. Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length; the quotient will be the proportional strength in horse power, with a velocity of 2·27 feet per second.

Ex. What is the power of a wheel, the teeth of which are 6 inches broad, 1·5 inch thick, and 1·8 inch long, and revolving at the velocity of 3 feet per second?

$$\frac{5^2 \times 6}{1{\cdot}8} \times \frac{13{\cdot}5}{1{\cdot}8} = 7{\cdot}5, \text{ strength at } 2{\cdot}27 \text{ feet per second, then}$$

$$2{\cdot}27 : 7{\cdot}5 :: 3 = \frac{7{\cdot}5 \times 3}{2{\cdot}27} = 9{\cdot}91 \text{ horse power}$$

Rule. The pitch is found by multiplying the thickness by 2·1, and the length is found by multiplying the thickness by 1·2.

Ex. The thickness being 2 inches, what is the pitch and length?

$$2 \times 2{\cdot}1 = 4{\cdot}2, \text{ pitch.}$$
$$2 \times 1{\cdot}2 = 2{\cdot}4, \text{ length.}$$

For table of the proportions of wheels, see next page.

Note.—The breadth of the teeth, as commonly executed by the best mechanics seems to be from about twice to thrice the pitch.

BEAN SHOT COPPER.—Take copper, melt it, and pour it in a small stream into boiling water.

FEATHER SHOT COPPER.—Take copper, melt it, and pour it in a small stream into cold water.

TO PRESERVE WALLS FROM DAMPNESS.—When the walls are about two feet high, use for one row of stones or bricks a mixture of tar, pitch, and fine sand, in the same way as mortar. The composition must be previously melted to a proper consistence.

TO PREVENT IRON FROM RUSTING.—Warm your iron till you cannot bear your hand on it without burning yourself. Then rub it with new and clean white wax. Put it again to the fire till it has soaked in the wax. When done rub it over with a piece of serge. This prevents the iron from rusting afterwards.

TABLE *of the Proportions of Wheels.*

Pitch in in.	Thickness in inches.	Breadth in in.	Length in in.	Horse power, at 2·27 feet per second.	Horse power, at 3 feet per second.	Horse power, at 6 feet per second.
4·2	2·	8·	2·40	13·33	17·61	35·22
3·99	1·9	7·6	2·28	12·03	15·90	31·80
3·78	1·8	7·2	2·16	10·80	14·27	28·54
3·57	1·7	6·8	2·04	9·63	12·72	25·54
3·36	1·6	6·4	1·92	8·53	11·27	22·54
3·15	1·5	6·	1·80	7·50	9·91	19·82
2·94	1·4	5·6	1·68	6·53	8·63	17·26
2·73	1·3	5·2	1·56	5·63	7·44	14·88
2·52	1·2	4·8	1·44	4·80	6·34	12·68
2·31	1·1	4·4	1·32	4·03	5·32	10·64
2·10	1·	4·	1·20	3·33	4·40	8·81
1·89	·9	3·6	1·08	2·70	3·57	7·14
1·68	·8	3·2	·96	2·13	2·81	5·62
1·47	·7	2·8	·84	1·63	2·15	4·30
1·26	·6	2·4	·72	1·20	1·59	3·18
1·05	·5	2·	·60	·83	1·10	2·20

ALLOYS, OR MISCELLANEOUS METALS.

Chaudet's Medal Metal.

Copper 100 parts; tin 4·17. Cast in moulds formed of cupel bone ash.

Lead in Grains.

Lead, melt it, and pour it in a small stream from a height of three or four feet into cold water.

Bell Metals.

1. Copper 25 parts; tin 5. Mix.
2. Copper 72 parts; tin 26. Mix.
3. Copper 78 parts; tin 22. Mix.

Common Bell Metal.

Copper 100 parts; tin 50 Mix.

Parisian Bell Metal.

Copper 72 parts; tin 26½; iron 1½. This alloy is used for the bells of small ornamental clocks.

Bath Metal.

Brass 32 parts; spelter 9. Mix.

Another.

Brass 35 parts; zinc 9. Mix.

Brass.

Copper 3 parts. Melt, then add zinc 1 part.

Button Makers' Fine Brass.

Brass 8 parts; zinc 5. Mix.

Button Makers' Common Brass.

Button brass 6 parts; tin 1; lead 1. Mix.

Bright Brass Color.

Brass reduced to fine powder.

Red Brass Color.

Copper filings 8 parts; bole 2. Mix.

Fine Brass.

Copper 2 parts; zinc 1. Mix.

Brass for Wire.

Copper 34 parts; calamine 56. Mix.

To give Plates of Copper a Brass Color.

Expose the plates, after being sufficiently heated, to the fumes of zinc.

To Brass Copper Vessels.

Argol 1 part; amalgam of zinc 1; muriatic acid 2; water to fill the vessel. Mix.

Brass or Hard Solder.

Brass 2 parts; zinc 1. A little tin is occasionally added.

Jewellers' Metal.

Copper 30 parts; brass 10; tin 7. Mix.

Fusible Alloys.

1. Bismuth 8 parts; lead 5; tin 3. This is fusible at boiling water heat.

2. Zinc, lead, and bismuth equal parts. This may be fused in a bit of writing paper, and will melt even in hot water.

3. Lead 3 parts; tin 2; bismuth 5. Mix. This alloy melts at 197° Fah. In using this composition to make casts of seals, gems, &c., it should be employed at the lowest possible temperature at which it will keep fluid; for this purpose it is as well to let it become pasty, and then forcibly impress the substances together.

4. Bismuth 2 parts; tin 3 parts; lead 5. Melt. This alloy fuses in boiling water.

German Silver.

1. Nickel 1 part; zinc 1; copper 2.

When intended for rolling into plates, use the following:

2. Nickel 25 parts; zinc 20; copper 60; to which may be added 3 of lead.

3. Pure copper 55 parts; nickle 23; zinc 17; iron 3; tin 2.

Fine White German Silver.

Iron 1 part; nickel 10; zinc 10; copper 20. Mix.

German Silver for Castings, &c.

Lead 3 parts; nickel 20: zinc 20; copper 60. Mix.

Genuine German Silver.

Copper 40½ parts; nickel 31½: zinc 25½; iron 2½. Mix.

Gilding Metal.

Copper 4 parts: brass 1: tin 1. Fuse together.

Another.

Copper 14 parts; zinc 6; tin 4.

To Separate Gold from Gilt Copper or Silver.

Take a solution of borax in water, apply to the gilt surface, and sprinkle over it some finely powdered sulphur; make the article red hot, and quench it in water; then scrape off the gold, and recover it by means of lead.

Gold in Grains.

Gold 3 parts; silver 1. Granulate by pouring it in a small stream, from a moderate height, into cold water; then dissolve the silver with nitric acid, and wash well in pure water; next heat the grains, to give them a proper lustre.

Common Gold.

Spanish copper 16 parts: silver 1: gold 2. Melt together.

Onion's Fusible Metal.

Tin 2 parts; lead 3: bismuth 5. Melt. This alloy melts at 197° Fah. The addition of a little mercury renders it still more fusible.

Alloy for Flute Key Valves.

Lead 4 parts; antimony 2. Fuse.

Pewter.

1. Tin 100 parts; antimony 17. Mix.

2. Zinc 1 part; copper 3: lead 8: tin 60. Melt the copper, then add the rest.

3. *Fine.* Tin 50 parts: antimony 4: bismuth 1; copper 1. Mix, as before.

4. *French.* Lead 9 parts: tin 41. Mix.

Keller's Medal Alloy.

Tin 9 parts; copper 89; zinc 2.

Gun Metal.

Brass 100 parts; spelter 13; tin 6. Mix.

Another.

Copper 9 parts; tin 1

Pinchbeck.

1. Brass 2 parts; copper 3. Melt under charcoal dust.

2. Copper 5 parts: zinc 1 Melt the copper, then add the zinc

Tin Filings.

Take grain tin, melt it in an iron vessel, and stir it, while cooling, until it becomes a powder: then sift it.

Tin in Grains.

Take Cornish grain tin, melt it, and pour it into a wooden box, well rubbed on the inside with whiting or chalk; close the cover, and continue shaking it violently until the tin is reduced to powder; then wash it in clean water, and dry it immediately.

Mosaic Gold, or Molu.

Take copper and zinc, equal parts. Melt at the lowest temperature that will fuse the former; then mix by stirring, and add more zinc, until the fused alloy becomes perfectly white; lastly, pour it into moulds. The proportion of zinc to the copper is from 50 to 55 per cent., exclusive of what is lost by the heat employed.

Hard White Metal.

Tin 1 part; spelter 3; brass 20. Mix.

Turners' Brass.

Brass 98 parts; lead 2. Mix.

Titania, or Britannia Metal.

1. Plate brass 2 parts; tin 2; bismuth 2; antimony 2; copper 1; arsenic 1. Mix, and add this alloy, at discretion, to melted tin.

2. *Spanish.* Of Spanish Titania metal there are two kinds. The first is made thus: Antimony 4 parts; tin 2; arsenic 1. The second is made in the following manner: Scrap iron 1 part; antimony 2; nitre a little. Melt, and harden one pound of tin with 2 oz. of this composition. A little arsenic improves the color of this alloy.

Tutenag.

Tin 2 parts; bismuth 1. Fuse.

Type Metal.

Lead 11 parts; antimony 2. Fuse.

Ring Gold.

Spanish copper 6 parts; silver 3; gold 5; Mix.

Prince Rupert's Metal.

Copper 2 parts; melt, and add zinc 1 part.

White Metal.

Brass 1 part; tin 2; antimony 4.

Another.

Lead 20 parts; bismuth 12; antimony 1. Fuse.

Yellow Dipping Metal.

Copper 19 parts; spelter 6. Mix.

A Metal that resembles Silver.

Tin ½ oz.; copper 1 lb. This alloy will make a pale bell metal that will roll and ring very near to sterling silver.

Silver Dust.

Take silver, dissolve it in nitric acid, and precipitate it with slips of bright copper; wash the powder in spirits, and dry it.

Imitation Platina.

Pale brass 8 parts; spelter 5. Mix.

Dessaussy's Steel.

Copper 100 parts; tin 14. This alloy may be hardened and sharpened in a similar way to steel.

Stereotype Metal.

Lead 18 parts; antimony 4 parts; bismuth 2 parts. Melt.

Another.

Lead 16 parts; antimony 3 parts; tin 5 parts; copper 2 parts.

Another.

Lead 20 parts; tin 8; antimony 1.

Speculum Metal.

Copper 43 parts; tin 20. Mix.

Another.

Copper 7 parts; melt, and add zinc 3 parts, tin 4.

Prince's Metal.

Copper 3 parts; zinc 1.

Another.

Brass 8 parts; zinc 1.

Another.

Zinc and copper, equal parts. Mix.

To make Iron resemble Gold.

Take of linseed oil 3 oz.; tartar 2 oz.; yolk of eggs, boiled hard and beaten, 2 oz.; aloes ½ oz.; saffron 5 grains; turmeric 2 grains. Boil together in an earthen vessel, and with it wash the iron, and it will look like gold. Should there not be linseed oil enough more may be added.

Queen's Metal.

Lead 1 part; bismuth 1; antimony 1; tin 9. Mix.

Another.

Tin 9 parts; bismuth 1; lead 2; antimony 1. Mix by melting.

Another.

Tin 1000 parts; regulus of antimony 80; bismuth 10; copper 40. Melt the copper, then expertly add the rest, and mix well together.

Purified Quicksilver.

Quicksilver 1 part; iron filings 1. Distil in an iron retort, into a vessel containing water.

Mock Gold.

Platina 7 parts; copper 16; zinc 1. Fuse together.

Bronze Metals.

For medals, and small castings—copper 95 parts; tin 4.

Another.

Copper 89 parts; tin 8; zinc 3.

Another.

Ancient. Copper 100 parts; tin 7; lead 7.

Another.

Kelly's. Copper 91 parts; zinc 6; tin 2; lead 1.

Blanched Copper.

Copper 8 parts; arsenic ½ part.

Manheim Gold.

Copper 3 parts; zinc 1. Melt separately, then suddenly mix them, and stir well.

Red Tombac.

Copper 11 parts; zinc 2. Mix.

FURNITURE PASTE.

1. Melt 1 pound of beeswax with ½ pint of linseed oil, and add ½ oz. alkanet root; keep it at a moderate heat till sufficiently colored, then remove from the fire, add ½ pint of oil of turpentine, strain through muslin; and put it into small gallipots to cool.

2. Scrape 4 oz. of wax, and put it into a pipkin with as much oil of turpentine as will cover it, and ½ oz. of powdered resin; melt with a gentle heat, and stir in sufficient Indian red to color it.

3. Equal weights of beeswax, spirit of turpentine, and linseed oil.

BRONZE POWDER.

The best methods of preparing these powders are probably kept secret. The following are some of the published recipes:

1. Gold leaf, or alloys of gold, reduced to powder by grinding them with sulphate of potash, or with honey, and washing away the extraneous matter with hot water, and drying the metallic powder.

2. Dutch metal, and other similar alloys, treated in the same way.

3. Verdigris 4 oz.; tutty 2 oz.; sublimate 1 dr.; borax 1 dr.; nitre 1 dr. Mix them into a paste with oil, and fuse the mixture in a crucible. This has failed in some hands, perhaps from the tutty being factitious.

4. Mix together 100 parts of sulphate of copper, and 50 of crystallized carbonate of soda; apply heat till they unite. Powder the mass, when cold, and add 15 parts of copper filings; mix well, and keep it at a white heat for twenty minutes. Wash and dry the product.

BALLS FOR SCOURING—BREECHES BALLS, CLOTHES BALLS.

1. Bathbrick 4 parts; pipeclay 8; pumice 1; softsoap 1; ochre, umber, or other color, to bring it to the desired shade, q. s.; ox-gall to form a paste. Make it into balls, and dry them.

2. Pipeclay 4 oz.; fuller's-earth ½ oz.; whiting ½ oz.; white pepper ½ oz; ox-gall sufficient to form it into a paste.

3. Pipeclay 3 oz.; white pepper 1 dr.; starch 1 dr, orris powder 1½ dr. It may be kept in powder, or formed into balls, as above.

MENSURATION OF CIRCLES.

TABLE *of the Diameters, Circumferences, and Areas of Circles.*

Diam. in inches.	Circum. in inches.	Area in square inches.	Diam. in inches.	Circum. in inches.	Area in square inches.	Diam. in inches.	Circum. in inches.	Area in square inches.
			4	12·566	12·566	9	28·274	63·617
$\frac{1}{16}$	·1963	·00306	$\frac{1}{8}$	12·959	13·364	$\frac{1}{8}$	28·667	65·396
$\frac{1}{8}$	·3927	·01227	$\frac{1}{4}$	13·351	14·186	$\frac{1}{4}$	29·059	67·200
$\frac{3}{16}$	·5890	·02761	$\frac{3}{8}$	13·744	15·033	$\frac{3}{8}$	29·452	69·029
$\frac{1}{4}$	·7854	·04909	$\frac{1}{2}$	14·137	15·904	$\frac{1}{2}$	29·845	70·882
$\frac{5}{16}$	·9817	·07670	$\frac{5}{8}$	14·529	16·800	$\frac{5}{8}$	30·237	72·759
$\frac{3}{8}$	1·1781	·11044	$\frac{3}{4}$	14·922	17·720	$\frac{3}{4}$	30·630	74·652
$\frac{7}{16}$	1·3744	·15033	$\frac{7}{8}$	15·315	18·665	$\frac{7}{8}$	31·023	76·588
$\frac{1}{2}$	1·5708	·19635	5	15·708	19·635	10	31·416	78·540
$\frac{9}{16}$	1·7671	·24850	$\frac{1}{8}$	16·100	20·629	$\frac{1}{8}$	31·808	80·515
$\frac{5}{8}$	1·9635	·30680	$\frac{1}{4}$	16·493	21·647	$\frac{1}{4}$	32·201	82·516
$\frac{11}{16}$	2·1598	·37122	$\frac{3}{8}$	16·886	22·690	$\frac{3}{8}$	32·594	84·540
$\frac{3}{4}$	2·3562	·44172	$\frac{1}{2}$	17·278	23·758	$\frac{1}{2}$	32·986	86·590
$\frac{13}{16}$	2·5525	·51849	$\frac{5}{8}$	17·671	24·850	$\frac{5}{8}$	33·379	88·664
$\frac{7}{8}$	2·7489	·60132	$\frac{3}{4}$	18·064	25·967	$\frac{3}{4}$	33·772	90·762
$\frac{15}{16}$	2·9452	·69030	$\frac{7}{8}$	18·457	27·108	$\frac{7}{8}$	34·164	92·885
1	3·141	·785	6	18·849	28·274	11	34·557	95·033
$\frac{1}{8}$	3·534	·994	$\frac{1}{8}$	19·242	29·464	$\frac{1}{8}$	34·950	97·205
$\frac{1}{4}$	3·927	1·227	$\frac{1}{4}$	19·635	30·679	$\frac{1}{4}$	35·343	99·402
$\frac{3}{8}$	4·319	1·484	$\frac{3}{8}$	20·027	31·919	$\frac{3}{8}$	35·735	101·623
$\frac{1}{2}$	4·712	1·767	$\frac{1}{2}$	20·420	33·183	$\frac{1}{2}$	36·128	103·869
$\frac{5}{8}$	5·105	2·073	$\frac{5}{8}$	20·813	34·471	$\frac{5}{8}$	36·521	106·139
$\frac{3}{4}$	5·497	2·405	$\frac{3}{4}$	21·205	35·784	$\frac{3}{4}$	36·913	108·434
$\frac{7}{8}$	5·890	2·761	$\frac{7}{8}$	21·598	37·122	$\frac{7}{8}$	37·306	110·753
2	6·283	3·141	7	21·991	38·484	12	37·699	113·097
$\frac{1}{8}$	6·675	3·546	$\frac{1}{8}$	22·383	39·871	$\frac{1}{8}$	38·091	115·466
$\frac{1}{4}$	7·068	3·976	$\frac{1}{4}$	22·776	41·282	$\frac{1}{4}$	38·484	117·859
$\frac{3}{8}$	7·461	4·430	$\frac{3}{8}$	23·169	42·718	$\frac{3}{8}$	38·877	120·276
$\frac{1}{2}$	7·854	4·908	$\frac{1}{2}$	23·562	44·178	$\frac{1}{2}$	39·270	122·718
$\frac{5}{8}$	8·246	5·411	$\frac{5}{8}$	23·954	45·663	$\frac{5}{8}$	39·662	125·184
$\frac{3}{4}$	8·639	5·939	$\frac{3}{4}$	24·347	47·173	$\frac{3}{4}$	40·055	127·676
$\frac{7}{8}$	9·032	6·491	$\frac{7}{8}$	24·740	48·707	$\frac{7}{8}$	40·448	130·192
3	9·424	7·068	8	25·132	50·265	13	40·840	132·732
$\frac{1}{8}$	9·817	7·669	$\frac{1}{8}$	25·525	51·848	$\frac{1}{8}$	41·233	135·297
$\frac{1}{4}$	10·210	8·295	$\frac{1}{4}$	25·918	53·456	$\frac{1}{4}$	41·626	137·886
$\frac{3}{8}$	10·602	8·946	$\frac{3}{8}$	26·310	55·088	$\frac{3}{8}$	42·018	140·500
$\frac{1}{2}$	10·995	9·621	$\frac{1}{2}$	26·703	56·745	$\frac{1}{2}$	42·411	143·139
$\frac{5}{8}$	11·388	10·320	$\frac{5}{8}$	27·096	58·426	$\frac{5}{8}$	42·804	145·802
$\frac{3}{4}$	11·781	11·044	$\frac{3}{4}$	27·489	60·132	$\frac{3}{4}$	43·197	148·489
$\frac{7}{8}$	12·173	11·793	$\frac{7}{8}$	27·881	61·862	$\frac{7}{8}$	43·589	151·201

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
14	43·98	153·93	19	59·69	283·52	24	75·39	452·39
1/8	44·37	156·69	1/8	60·08	287·27	1/8	75·79	457·11
1/4	44·76	159·48	1/4	60·47	291·03	1/4	76·18	461·86
3/8	45·16	162·29	3/8	60·86	294·83	3/8	76·57	466·63
1/2	45·55	165·13	1/2	61·26	298·64	1/2	76·96	471·43
5/8	45·94	167·98	5/8	61·65	302·48	5/8	77·36	476·25
3/4	46·33	170·87	3/4	62·04	306·35	3/4	77·75	481·10
7/8	46·73	173·78	7/8	62·43	310·24	7/8	78·14	485·97
15	47·12	176·71	20	62·83	314·16	25	78·53	490·87
1/8	47·51	179·67	1/8	63·22	318·09	1/8	78·93	495·79
1/4	47·90	182·65	1/4	63·61	322·06	1/4	79·32	500·74
3/8	48·30	185·66	3/8	64·01	326·05	3/8	79·71	505·71
1/2	48·69	188·69	1/2	64·40	330·06	1/2	80·10	510·70
5/8	49·08	191·74	5/8	64·79	334·10	5/8	80·50	515·72
3/4	49·48	194·82	3/4	65·18	338·16	3/4	80·89	520·76
7/8	49·87	197·93	7/8	65·58	342·25	7/8	81·28	525·83
16	50·26	201·06	21	65·97	346·36	26	81·68	530·93
1/8	50·65	204·21	1/8	66·36	350·49	1/8	82·07	536·04
1/4	51·05	207·39	1/4	66·75	354·65	1/4	82·46	541·18
3/8	51·44	210·59	3/8	67·15	358·84	3/8	82·85	546·35
1/2	51·83	213·82	1/2	67·54	363·05	1/2	83·25	551·54
5/8	52·22	217·07	5/8	67·93	367·28	5/8	83·64	556·76
3/4	52·62	220·35	3/4	68·32	371·54	3/4	84·03	562·00
7/8	53·01	223·65	7/8	68·72	375·82	7/8	84·43	567·26
17	53·40	226·98	22	69·11	380·13	27	84·82	572·55
1/8	53·79	230·33	1/8	69·50	384·46	1/8	85·21	577·87
1/4	54·19	233·70	1/4	69·90	388·82	1/4	85·60	583·20
3/8	54·58	237·10	3/8	70·29	393·20	3/8	86·00	588·57
1/2	54·97	240·52	1/2	70·68	397·60	1/2	86.39	593·95
5/8	55·37	243·97	5/8	71·07	402·03	5/8	86·78	599·37
3/4	55·76	247·45	3/4	71·47	406·49	3/4	87·17	604·80
7/8	56·15	250·94	7/8	71·86	410·97	7/8	87·57	610·26
18	56·54	254·46	23	72·25	415·47	28	87·96	615·75
1/8	56·94	258·01	1/8	72·64	420·00	1/8	88·35	621·26
1/4	57·33	261·58	1/4	73·04	424·55	1/4	88·75	626·79
3/8	57·72	265·18	3/8	73·43	429·13	3/8	89·14	632·35
1/2	58·11	268·80	1/2	73·82	433·73	1/2	89·53	637·94
5/8	58·51	272·44	5/8	74·21	438·36	5/8	89·92	643·54
3/4	58·90	276·11	3/4	74·61	443·01	3/4	90·32	649·18
7/8	59·29	279·81	7/8	75·00	447·69	7/8	90·71	654·83

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
29	91·10	660·52	34	106·8	907·92	39	122·5	1194·59
$\frac{1}{8}$	91·49	666·22	$\frac{1}{8}$	107·2	914·61	$\frac{1}{8}$	122·9	1202·26
$\frac{1}{4}$	91·89	671·95	$\frac{1}{4}$	107·5	921·32	$\frac{1}{4}$	123·3	1209·95
$\frac{3}{8}$	92·28	677·71	$\frac{3}{8}$	107·9	928·06	$\frac{3}{8}$	123·7	1217·67
$\frac{1}{2}$	92·67	683·49	$\frac{1}{2}$	108·3	934·82	$\frac{1}{2}$	124·0	1225·42
$\frac{5}{8}$	93·06	689·29	$\frac{5}{8}$	108·7	941·60	$\frac{5}{8}$	124·4	1233·18
$\frac{3}{4}$	93·46	695·12	$\frac{3}{4}$	109·1	948·41	$\frac{3}{4}$	124·8	1240·98
$\frac{7}{8}$	93·85	700·98	$\frac{7}{8}$	109·5	955·25	$\frac{7}{8}$	125·2	1248·79
30	94·24	706·86	35	109·9	962·11	40	125·6	1256·64
$\frac{1}{8}$	94·64	712·76	$\frac{1}{8}$	110·3	968·99	$\frac{1}{8}$	126·0	1264·50
$\frac{1}{4}$	95·03	718·69	$\frac{1}{4}$	110·7	975·90	$\frac{1}{4}$	126·4	1272·39
$\frac{3}{8}$	95·42	724·64	$\frac{3}{8}$	111·1	992·84	$\frac{3}{8}$	126·8	1280·31
$\frac{1}{2}$	95·81	730·61	$\frac{1}{2}$	111·5	989·80	$\frac{1}{2}$	127·2	1288·25
$\frac{5}{8}$	96·21	736·61	$\frac{5}{8}$	111·9	996·78	$\frac{5}{8}$	127·6	1296·21
$\frac{3}{4}$	96·60	742·64	$\frac{3}{4}$	112·3	1003·71	$\frac{3}{4}$	128·0	1304·20
$\frac{7}{8}$	96·99	748·69	$\frac{7}{8}$	112·7	1010·81	$\frac{7}{8}$	128·4	1312·21
31	97·38	754·76	36	113·0	1017·87	41	128·8	1320·25
$\frac{1}{8}$	97·78	760·86	$\frac{1}{8}$	113·4	1024·95	$\frac{1}{8}$	129·1	1328·32
$\frac{1}{4}$	98·17	766·99	$\frac{1}{4}$	113·8	1032·06	$\frac{1}{4}$	129·5	1336·40
$\frac{3}{8}$	98·56	773·14	$\frac{3}{8}$	114·2	1039·19	$\frac{3}{8}$	129·9	1344·51
$\frac{1}{2}$	98·96	779·31	$\frac{1}{2}$	114·6	1046·39	$\frac{1}{2}$	130·3	1352·65
$\frac{5}{8}$	99·35	785·51	$\frac{5}{8}$	115·0	1053·52	$\frac{5}{8}$	130·7	1360·81
$\frac{3}{4}$	99·74	791·73	$\frac{3}{4}$	115·4	1060·73	$\frac{3}{4}$	131·1	1369·00
$\frac{7}{8}$	100·13	797·97	$\frac{7}{8}$	115·8	1067·95	$\frac{7}{8}$	131·5	1377·21
32	100·5	804·24	37	116·2	1075·21	42	131·9	1385·44
$\frac{1}{8}$	100·9	810·54	$\frac{1}{8}$	116·6	1082·48	$\frac{1}{8}$	132·3	1393·70
$\frac{1}{4}$	101·3	816·86	$\frac{1}{4}$	117·0	1089·79	$\frac{1}{4}$	132·7	1401·98
$\frac{3}{8}$	101·7	823·21	$\frac{3}{8}$	117·4	1097·11	$\frac{3}{8}$	133·1	1410·29
$\frac{1}{2}$	102·1	829·57	$\frac{1}{2}$	117·8	1104·46	$\frac{1}{2}$	133·5	1418·62
$\frac{5}{8}$	102·4	835·97	$\frac{5}{8}$	118·2	1111·84	$\frac{5}{8}$	133·9	1426·98
$\frac{3}{4}$	102·8	842·39	$\frac{3}{4}$	118·5	1119·24	$\frac{3}{4}$	134·3	1435·36
$\frac{7}{8}$	103·2	848·83	$\frac{7}{8}$	118·9	1126·66	$\frac{7}{8}$	134·6	1443·77
33	103·6	855·30	38	119·3	1134·11	43	135·0	1452·20
$\frac{1}{8}$	104·0	861·79	$\frac{1}{8}$	119·7	1141·59	$\frac{1}{8}$	135·4	1460·65
$\frac{1}{4}$	104·4	868·30	$\frac{1}{4}$	120·1	1149·08	$\frac{1}{4}$	135·8	1469·13
$\frac{3}{8}$	104·8	874·84	$\frac{3}{8}$	120·5	1156·61	$\frac{3}{8}$	136·2	1477·63
$\frac{1}{2}$	105·2	881·41	$\frac{1}{2}$	120·9	1164·15	$\frac{1}{2}$	136·6	1486·17
$\frac{5}{8}$	105·6	888·00	$\frac{5}{8}$	121·3	1171·73	$\frac{5}{8}$	137·0	1494·72
$\frac{3}{4}$	106·0	894·61	$\frac{3}{4}$	121·7	1179·32	$\frac{3}{4}$	137·4	1503·30
$\frac{7}{8}$	106·4	901·25	$\frac{7}{8}$	122·1	1186·94	$\frac{7}{8}$	137·8	1511·90

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
44	138·2	1520·53	46	144·5	1661·90	48	150·7	1809·56
1/8	138·6	1529·18	1/8	144 9	1670 95	1/8	151·1	1818·99
1/4	139·0	1537·86	1/4	145·2	1680·01	1/4	151·5	1828·46
3/8	139·4	1546·55	3/8	145·6	1689·10	3/8	151·9	1837·93
1/2	139·8	1555·28	1/2	146·0	1698·23	1/2	152·3	1847·45
5/8	140·1	1564·03	5/8	146·4	1707·37	5/8	152·7	1856·99
3/4	140·5	1572·81	3/4	146·8	1716·54	3/4	153·1	1866·55
7/8	140·9	1581·61	7/8	147·2	1725 73	7/8	153·5	1876·13
45	141·3	1590·43	47	147·6	1734·94	49	153·9	1885·74
1/8	141 7	1599·28	1/8	148·0	1744·18	1/8	154·3	1895·37
1/4	142·1	1608·15	1/4	148·4	1753·45	1/4	154·7	1905·03
3/8	142·5	1617·04	3/8	148·8	1762·73	3/8	155·1	1914·70
1/2	142·9	1625·97	1/2	149·2	1772·05	1/2	155·5	1924·42
5/8	143·3	1634·92	5/8	149·6	1781·39	5/8	155·9	1934·15
3/4	143·7	1643·89	3/4	150·0	1790·76	3/4	156·2	1943·91
7/8	144·1	1652·88	7/8	150·4	1800·14	7/8	156·6	1953·69

Diam. in.	Circum. inches.	Area in square in.	Area in square feet.	Diam. in.	Circum. inches.	Area in square in.	Area in square feet.
50	157·0	1963·5	13·63	55	172·7	2375·8	16·49
1/4	157·8	1983·1	13·77	1/4	173·5	2397·4	16·64
1/2	158·6	2002·9	13·90	1/2	174·3	2419·2	16·80
3/4	159·4	2022·8	14·04	3/4	175·1	2441·0	16·95
51	160·2	2042·8	14·18	56	175·9	2463·0	17·10
1/4	161·0	2062·9	14·32	1/4	176 7	2485·0	17·25
1/2	161·7	2083·0	14·46	1/2	177·5	2507 1	17·41
3/4	162·5	2103·3	14 60	3/4	178·2	2529·4	17·56
52	163·3	2123·7	14·74	57	179·0	2551·7	17·72
1/4	164·1	2144·1	14 89	1/4	179·8	2574·1	17·87
1/2	164 9	2164·7	15·03	1/2	180·6	2596·7	18·03
3/4	165·7	2185·4	15·17	3/4	181·4	2619·3	18·19
53	166·5	2206·1	15·32	58	182·2	2642·0	18·34
1/4	167·2	2227·0	15·46	1/4	182·9	2664·9	18·50
1/2	168·0	2248·0	15·61	1/2	183·7	2687·8	18·68
3/4	168·8	2269 0	15·75	3/4	184·5	2710·8	18 82
54	169·6	2290·2	15·90	59	185·3	2733·9	18·98
1/4	170·4	2311·4	16·05	1/4	186·1	2757·1	19·14
1/2	171·2	2332·8	16·20	1/2	186·9	2780·5	19·30
3/4	172·0	2354·2	16·34	3/4	187·7	2803·9	19 47

Diam. in.	Circum. inches.	Area in square in.	Area in square feet.	Diam. in.	Circum. inches.	Area in square in.	Area in square feet.
60	188·4	2827·4	19·63	69	216·7	3739·2	25·96
¼	189·2	2851·0	19·79	¼	217·5	3766·4	26·15
½	190·0	2874·7	19·96	½	218·3	3793·6	26·34
¾	190 8	2898·5	20·12	¾	219·1	3821·0	26·53
61	191·6	2922·4	20·29	70	219·9	3848·4	26·72
¼	192·4	2946·4	20·46	¼	220·6	3875·9	26·91
½	193·2	2970·5	20·62	½	221·4	3903 6	27·10
¾	193·9	2994·7	20·79	¾	222·2	3931·3	27·30
62	194·7	3019·0	20·96	71	223·0	3959 2	27·49
¼	195·5	3043·4	21·13	¼	223·8	3987·1	27·68
½	196·3	3067·9	21·20	½	224·6	4015·1	27·87
¾	197·1	3092·5	21·47	¾	225·4	4043·2	28·07
63	197·9	3117·2	21·64	72	226·1	4071·5	28·27
¼	198·7	3142·0	21·81	¼	226·9	4099·8	28 47
½	199·4	3166 9	21·98	½	227·7	4128 2	28·66
¾	200·2	3191·9	22·16	¾	228·5	4156·7	28·86
64	201·0	3216·9	22·34	73	229·3	4185·3	29·06
¼	201 8	3242·1	22·51	¼	230·1	4214·1	29·26
½	202·6	3267·4	22·68	½	230·9	4242·9	26·46
¾	203·4	3292·8	22·86	¾	231·6	4271·8	29·66
65	204·2	3318·3	23·04	74	232·4	4300·8	29·86
¼	204·9	3343·8	23·22	¼	233·2	4329·9	30·06
½	205·7	3369·5	23·39	½	234·0	4359·1	30·26
¾	206·5	3395·3	23·57	¾	234·8	4388·4	30·47
66	207·3	3421·2	23·75	75	235·6	4417·8	30·67
¼	208·1	3447·1	23·93	¼	236·4	4447·3	30·88
½	208·9	3473·2	24·11	½	237·1	4476·9	31·09
¾	209·7	3499·3	24·30	¾	237·9	4506·6	31·30
67	210·4	3525·6	24·48	76	238·7	4536·4	31·50
¼	211·2	3552·0	24·66	¼	239·5	4566·3	31·71
½	212 0	3578·4	24·84	½	240·3	4596·3	31·91
¾	212·8	3605·0	25·03	¾	241·1	4626·4	32·12
68	213·6	3631·6	25 22	77	241·9	4656·6	32·33
¼	214·4	3658·4	25·40	¼	242·6	4686 9	32·54
½	215·1	3685·2	25·59	½	243·4	4717·2	32·75
¾	215 9	3712·2	25·77	¾	244·2	4747·7	32·96

Diam. in.	Circum. inches.	Area in square in.	Area in square feet.	Diam. in.	Circum. inches.	Area in square in.	Area in square feet.
78	245·0	4778·3	33·18	87	273·3	5944·6	41·28
¼	245·8	4809·0	33·39	¼	274·1	5978·9	41·52
½	246·6	4839·8	33·60	½	274·8	6013·2	41·75
¾	247·4	4870·7	33·81	¾	275·6	6047·6	41·99
79	248·1	4901·6	34·03	88	276·4	6082·1	42·23
¼	248·9	4932·7	34·24	¼	277·2	6116·7	42·47
½	249·7	4963·9	34·46	½	278·0	6151·4	42·71
¾	250·5	4995·1	34·68	¾	278·8	6186·2	42·95
80	251·3	5026·5	34·90	89	279·6	6221·1	43·20
¼	252·1	5058·0	35·12	¼	280·3	6256·1	43·44
½	252·8	5089·5	35·34	½	281·1	6291·2	43·68
¾	253·6	5121·2	35·56	¾	281·9	6326·4	43·92
81	254·4	5153·0	35·78	90	282·7	6361·7	44·17
¼	255·2	5184·8	36·00	¼	283·5	6397·1	44·42
½	256·0	5216·8	36·22	½	284·3	6432·6	44·66
¾	256·8	5248·8	36·44	¾	285·1	6468·2	44·81
82	257·6	5281·0	36·67	91	285·8	6503·8	45·16
¼	258·3	5313·2	36·90	¼	286·6	6539·6	45·41
½	259·1	5345·6	37·12	½	287·4	6575·5	45·66
¾	259·9	5378·0	37·34	¾	288·2	6611·5	45·91
83	260·7	5410·6	37·57	92	289·0	6647·6	46·16
¼	261·5	5443·2	37·79	¼	289·8	6683·8	46·41
½	262·3	5476·0	38·02	½	290·5	6720·0	46·66
¾	263·1	5508·8	38·25	¾	291·3	6756·4	46·91
84	263·8	5541·7	38·48	93	292·1	6792·9	47·17
¼	264·6	5574·8	38·71	¼	292·9	6829·4	47·43
½	265·4	5607·9	38·94	½	293·7	6866·1	47·68
¾	266·2	5641·1	39·07	¾	294·5	6902·9	47·93
85	267·0	5674·5	39·40	94	295·3	6939·7	48·19
¼	267·8	5707·9	39·63	¼	296·0	6976·7	48·45
½	268·6	5741·4	39·87	½	296·8	7013·8	48·70
¾	269·3	5775·0	40·10	¾	297·6	7050·9	48·96
86	270·1	5808·8	40·33	95	298·4	7088·2	49·22
¼	270·9	5842·6	40·57	¼	299·2	7125·5	49·48
½	271·7	5876·5	40·80	½	300·0	7163·0	49·64
¾	272·5	5910·5	41·04	¾	300·8	7200·5	50·00

Diam. in.	Circum. inches.	Area in square in.	Area in square feet.
96	301·5	7238·2	50·26
¼	302·3	7275·9	50·52
½	303·1	7313·8	50·78
¾	303·9	7351·7	51·05
97	304·7	7389·8	51·35
¼	305·5	7427·9	51·57
½	306·3	7466·2	51·84
¾	307·0	7504·5	52·11
98	307·8	7542·9	52·38
¼	308·6	7581·5	52·65
½	309·4	7620·1	52·91
¾	310·2	7658·8	53·18
99	311·0	7697·7	53·45
¼	311·8	7736·6	53·72
½	312·5	7775·6	53·99
¾	313·3	7814·7	54·26
100	314·1	7854·0	54·54
101	317·3	8011·7	55·63
102	320·4	8091·2	56·74
103	323·5	8332·3	57·86
104	326·7	8494·9	58·99
105	329·8	8659·0	60·13
106	333·0	8824·7	61·28
107	336·1	8992·0	62·44
108	339·2	9160·9	63·61
109	342·4	9331·1	64·80
110	345·5	9503·3	65·99
111	348·7	9676·9	67·20
112	351·8	9852·0	68·41
113	355·0	10028·7	69·64
114	358·1	10207·0	70·88
115	361·2	10386·9	72·13
116	364·4	10568·3	73·39
117	367·5	10751·3	74·66
118	370·7	10935·9	75·94
119	373·8	11122·0	77·23
120	376·6	11309·7	78·54

Diam. in.	Circum. inches.	Area in square in.	Area in square feet.
121	380·1	11499·0	79·85
122	383·2	11689·9	81·18
123	386·4	11882·3	82·51
124	389·5	12076·3	83·86
125	392·7	12271·8	85·22
126	395·8	12469·0	86·59
127	398·9	12667·7	87·97
128	402·1	12867·9	89·36
129	405·2	13069·8	90·76
130	408·4	13273·2	92·17
131	411·5	13478·2	92·59
132	414·6	13684·8	95·03
133	417·8	13892·9	96·47
134	420·9	14102·6	97·93
135	424·1	14313·9	99·40
136	427·2	14526·7	100·88
137	430·3	14741·1	102·36
138	433·5	14957·1	103·87
139	436·6	15174·7	105·37
140	439·8	15393·8	106·90
141	442·9	15614·5	108·43
142	446·1	15836·8	109·97
143	449·2	16060·6	111·53
144	452·3	16286·0	113·09
145	455·5	16513·0	114·67
146	458·6	16741·5	116·26
147	461·8	16971·7	117·86
148	464·9	17203·4	119·46
149	468·0	17436·6	121·08
150	471·2	17671·5	122·71
151	474·3	17907·9	124·36
152	477·5	18145·9	126·01
153	480·6	18385·4	127·67
154	483·8	18626·5	129·35
155	486·9	18869·2	131·03

TABLE

Of the Circumferences and Areas of Circles, from 1 *to* 50 *feet, advancing by an inch.*

Diam. ft. & in.	Circumference in feet and in.		Area in feet.	Diam. ft. & in.	Circumference in feet and in.		Area in feet.
1 *ft.*	3	1 5/8	·7854	3	13	4 1/8	14·1862
1	3	4 5/8	·9217	4	13	7 1/4	14·7479
2	3	8	1·0690	5	13	10 1/2	15·3206
3	3	11	1·2271	6	14	1 5/8	15·9043
4	4	2 1/8	1·3962	7	14	4 5/8	16·4986
5	4	5 3/8	1·5761	8	14	7 7/8	17·1041
6	4	8 1/2	1·7671	9	14	11	17·7205
7	4	11 5/8	1 9689	10	15	2 1/8	18·3476
8	5	2 3/4	2·1816	11	15	5 1/4	18·9858
9	5	5 7/8	2·4052	5 *ft.*	15	8 1/2	19·6350
10	5	9	2·6398	1	15	11 5/8	20·2947
11	6	2 1/4	2 8852	2	16	2 3/4	20·9656
2 *ft.*	6	3 3/8	3·1416	3	16	5 3/4	21·6475
1	6	6 1/2	3·4087	4	16	9	22·3400
2	6	9 5/8	3·6869	5	17	0 1/8	23 0437
3	7	0 3/4	3·9760	6	17	3 1/4	23·7583
4	7	3 7/8	4·2760	7	17	6 3/8	24·4835
5	7	7	4·5869	8	17	9 5/8	25·2199
6	7	10 1/4	4·9087	9	18	0 3/4	25 9672
7	8	1 3/8	5·2413	10	18	3 7/8	26·7251
8	8	4 1/2	5·5850	11	18	7 1/8	27·4943
9	8	7 5/8	5·9395	6 *ft.*	18	10 1/8	28·2744
10	8	10 3/4	6 3049	1	19	1 1/4	29·0649
11	9	1 7/8	6·6813	2	19	4 3/8	29·8668
3 *ft.*	9	5	7·0686	3	19	7 1/2	30·6796
1	9	8 1/4	7·4666	4	19	10 5/8	31·5029
2	9	11 3/8	7·8757	5	20	1 7/8	32·3376
3	10	2 1/2	8 2957	6	20	4 7/8	33·1831
4	10	5 5/8	8·7265	7	20	8 1/8	34·0391
5	10	8 3/4	9·1683	8	20	11 1/2	34·9065
6	10	11 7/8	9·6211	9	21	2 3/8	35·7847
7	11	3	10·0846	10	21	5 1/2	36·6735
8	11	6 1/8	10·5591	11	21	8 3/4	37·5736
9	11	9 3/8	11·0446	7 *ft.*	21	11 7/8	38·4846
10	12	5 1/2	11·5409	1	22	3	39 4060
11	12	3 5/8	12·0481	2	22	6 1/8	40·3388
4 *ft.*	12	6 3/4	12·5664	3	22	9 1/4	41·2825
1	12	9 7/8	13·0952	4	23	0 3/8	42·2367
2	13	1	13·6353	5	23	2 1/8	43·2022

Diam. ft. & in.	Circumference in feet and in.	Area in feet.	Diam. ft. & in.	Circumference in feet and in.	Area in feet.
6	23 6¾	44·1787	3	45 4⅛	99·4021
7	23 11	45·1656	4	35 7¼	100·8797
8	24 1⅛	46·1638	5	35 10⅝	102·3689
9	24 4⅛	47·1730	6	36 1½	103·8691
10	24 7¼	48·1926	7	36 4½	105·3794
11	24 10⅜	49·2236	8	36 7¾	106·9013
8 *ft.*	25 1½	50·2656	9	36 10⅞	108·4342
1	25 4⅝	51·3178	10	37 2¾	109·9772
2	25 7⅞	52·3816	11	37 5¼	111·5319
3	25 11	53·4562	12 *ft.*	37 8¾	113·0976
4	26 2⅛	54·5412	1	37 11½	114·6732
5	26 5¼	55·6377	2	38 2⅝	116·2607
6	26 8⅜	56·7451	3	38 5¾	117·8590
7	26 11½	57·8628	4	38 8⅞	119·4674
8	27 2¾	58 9920	5	39 0	121·0876
9	27 5¾	60·1321	6	39 3¼	122 7187
10	27 9	61·2826	7	39 6⅜	124·3598
11	28 0⅛	62·4445	8	39 9½	126 0127
9 *ft.*	28 3¼	63·6174	9	40 0⅝	127·6765
1	28 6⅜	64·8006	10	40 3¾	129·3504
2	28 9½	65·9951	11	40 6⅞	131·0360
3	29 0⅝	67·2007	13 *ft.*	40 10	132·7326
4	29 3¾	68·4166	1	41 1⅛	134·4391
5	29 7	69·6440	2	41 4⅜	136·1574
6	29 10⅛	70·8823	3	41 7½	137·8867
7	30 1¼	72·1309	4	41 10⅝	139·6260
8	30 4⅜	73·3910	5	42 1⅝	141·3771
9	30 7½	74·6620	6	42 4⅞	143·1391
10	30 11⅝	75·9433	7	42 8	144·9111
11	31 1¾	77·2362	8	42 11⅛	146·6949
10 *ft.*	31 5	78·5400	9	43 2¼	148·4896
1	31 8⅛	79·8540	10	43 5½	150·2943
2	31 11¼	81·1795	11	43 8⅝	152·1109
3	32 2⅜	82·5160	14 *ft.*	43 11¾	153·9384
4	32 5½	83·8627	1	44 2⅞	155·7758
5	32 8⅝	85·2211	2	44 6	157·6250
6	32 11¾	86·5903	3	44 9⅛	159·4852
7	33 2⅞	87·9697	4	45 0¼	161·3553
8	33 6⅛	89·3608	5	45 3½	163·2373
9	33 9¼	90·7627	6	45 6⅝	165·1303
10	34 0⅜	92·1749	7	45 9¾	167·0331
11	34 3½	93 5986	8	46 0⅞	168·9479
11 *ft.*	34 6⅝	95·0334	9	46 4	170·8735
1	34 9¾	96·4783	10	46 7⅛	172·8091
2	35 0⅞	97·9347	11	46 11¼	174·7565

Diam. ft. & in.	Circumference in feet and in.	Area in feet.	Diam. ft. & in.	Circumference in feet and in.	Area in feet.
15 *ft.*	47 1½	176·7150	9	58 10¾	276·1171
1	47 4⅝	178·6832	10	59 2	278·5761
2	47 7¾	180·6634	11	59 5⅛	281·0472
3	47 10⅞	182·6545	19 *ft.*	59 8¼	283·5294
4	48 2⅛	184·6555	1	59 11½	286·0210
5	48 5⅛	186·6684	2	60 2½	288·5249
6	48 8¼	188·6923	3	60 5⅝	291·0397
7	48 11⅜	190·7260	4	60 8¾	293·5641
8	49 2⅝	192·7716	5	60 11⅞	296·1107
9	49 5¾	194·8282	6	61 3⅛	298·6483
10	49 8⅞	196·8946	7	61 6¼	301·2054
11	50 0	198·9730	8	61 9½	303·7747
16 *ft.*	50 3⅛	201·0624	9	61 0½	306·3550
1	50 6¼	203·1615	10	62 3⅝	308·9448
2	50 9⅝	205·2726	11	62 6¾	311·5469
3	51 0⅝	207·3946	20 *ft.*	62 9⅞	314·1600
4	51 3¾	209·5264	1	63 1⅛	316·7824
5	51 6⅞	211·6703	2	63 4¼	319·4173
6	51 10	213·8251	3	63 7⅜	322·0630
7	52 1⅛	215·9896	4	63 11½	324·7182
8	52 4¼	218·1662	5	64 1⅝	327·3858
9	52 7⅜	220·3537	6	64 4¾	330·0643
10	52 10½	222·5510	7	64 7⅞	332·7522
11	53 1⅝	224·7603	8	64 11	335·4525
17 *ft.*	53 4⅞	226·9806	9	65 2¼	338·1637
1	53 8	229·2105	10	65 5⅜	340·8844
2	53 11⅛	231·4525	11	65 8¼	343·6174
3	54 2⅛	233·7055	21 *ft.*	65 11⅝	346·3614
4	54 5⅜	235·9682	1	66 2¾	349·1147
5	54 8½	238·2430	2	66 5⅞	351·8804
6	54 11⅝	240·5287	3	66 9	354·6571
7	55 2⅞	242·8241	4	66 0⅛	357·4432
8	55 6	245·1316	5	67 3⅜	360·2417
9	55 9⅛	247·4500	6	67 6½	363·0511
10	56 0¼	249·7781	7	67 9⅝	365·8698
11	56 3½	252·1184	8	68 0¾	368·7011
18 *ft.*	56 6½	254·4696	9	68 3⅞	371·5432
1	56 9⅝	256·8303	10	68 7	374·3947
2	57 0⅞	259·2033	11	68 10¼	377·2587
3	57 4	261·5872	22 *ft.*	69 1⅜	380·1336
4	57 7⅛	263·9807	1	69 4½	383·0177
5	57 10¼	266·3864	2	69 7⅝	385·9144
6	58 1⅜	268·8031	3	69 10¾	388·8220
7	58 4½	271·2293	4	70 1⅞	391·7389
8	58 7⅝	273·6678	5	70 5	394·6683

Diam. ft. & in.	Circumference in feet and in.		Area in feet.
6	70	8¼	397·6087
7	70	11⅜	400·5583
8	71	2½	403·5204
9	71	5⅝	406·4935
10	71	8¾	409·4759
11	71	11⅞	412·4707
23 *ft.*	72	3	415·4766
1	72	6⅛	418·4915
2	72	9⅜	421·5192
3	73	0½	424·5577
4	73	3⅝	427·6055
5	73	6¾	430·6658
6	73	9⅞	433·7371
7	74	1	436·8175
8	74	4⅛	439·9106
9	74	7¼	443·0146
10	74	10⅝	446·1278
11	75	1⅝	449·2536
24 *ft.*	75	4¾	452 3904
1	75	7⅞	455·5362
2	75	11	458·6948
3	76	2⅛	461·8642
4	76	5¼	465·0428
5	76	8½	468·2341
6	76	11⅝	471·4363
7	77	2¾	474·6476
8	77	5⅞	477·8716
9	77	9	481·1065
10	78	0⅛	484·3506
11	78	3¼	487·6073
25 *ft.*	78	6¾	490·8750
1	78	9½	494·1516
2	79	0¾	497·4411
3	79	3⅞	500·7415
4	79	7⅛	504·0510
5	79	11⅛	507·3732
6	80	1¼	510 7063
7	80	4⅜	514·0484
8	80	7⅝	517·4034
9	80	10¾	520·7692
10	81	1⅞	524·1441
11	81	5	527·5318
26 *ft.*	81	8⅛	530·9304
1	81	11½	534·3379
2	82	2⅝	537·7583
3	82	5¼	541·1896
4	82	8⅝	544·6299
5	82	11⅞	548 0830
6	83	3	551·5471
7	83	6⅛	555·0201
8	83	9¼	558·5059
9	84	0⅜	562·0027
10	84	3½	565·5084
11	84	6⅝	569·0270
27 *ft.*	84	9⅞	572·5566
1	85	1	576·0949
2	85	4¼	579·6463
3	85	8⅛	583·2085
4	85	11⅜	586·7796
5	86	1½	590·3637
6	86	4⅝	593·9587
7	86	7⅞	597 5625
8	86	11	601·1793
9	87	2⅛	604 8070
10	87	5¼	608·4436
11	87	8¾	612·0931
28 *ft.*	87	11½	615 7536
1	88	2⅝	619 4228
2	88	5¾	623·1050
3	88	9	626·7982
4	89	0⅛	630 5002
5	89	3¼	634·2152
6	89	6⅜	637·9411
7	89	9½	641·6758
8	90	0⅝	645·4235
9	90	3¾	649·1821
10	90	6⅞	652·9495
11	90	11⅛	656·7300
29 *ft.*	91	1¼	660·5214
1	91	4⅜	664 8214
2	91	7½	668·1346
3	91	10⅝	671·9587
4	92	1¾	675·7915
5	92	4⅞	679·6375
6	92	8⅛	683·4943
7	92	11⅛	687·3598
8	93	2⅜	691·2385
9	93	5½	695·1280
10	93	8⅝	699·0263
11	93	11⅞	702·9377

Diam. ft. & in.	Circumference in feet and in.		Area in feet.	Diam. ft. & in.	Circumference in feet and in.		Area in feet.
30 *ft.*	94	2⅞	706·8600	9	106	0¼	894·6196
1	94	6	710·7909	10	106	3⅝	899·0413
2	94	8¼	714·7350	11	106	6⅝	903·4763
3	95	9⅜	718·6900	34 *ft.*	106	9¾	907·9224
4	95	3½	722·6537	1	107	0⅞	912·3767
5	95	6⅝	726·6305	2	107	4	916·8445
6	95	9¾	730·6183	3	107	7⅛	921·3232
7	96	0⅞	734·6147	4	107	10¼	925·8103
8	96	4	738·6242	5	108	1⅝	930·3108
9	96	7¼	742·6447	6	108	4⅝	934·8223
10	96	10⅜	746·6738	7	108	7¾	939·3421
11	97	1½	750·7161	8	108	10⅞	943·8753
31 *ft*	97	4⅝	754·7694	9	109	2	948·4195
1	97	7¾	758·8311	10	109	5⅛	952·9720
2	97	10⅞	762·9062	11	109	8¼	957·5380
3	98	2	766·9921	35 *ft.*	109	11⅝	962·1150
4	98	5⅛	771·0866	1	110	2⅝	966·7001
5	98	8⅜	775·1944	2	110	5¾	971·2989
6	98	11½	779·3131	3	110	8⅞	975·9085
7	99	2⅝	783·4403	4	111	0	980·5264
8	99	5¾	787·5808	5	111	3⅛	985·1579
9	99	8⅞	791·7322	6	111	6¼	989·8003
10	100	0	795·8922	7	111	9⅝	994·4509
11	100	3⅛	800·0654	8	112	0½	999·1151
32 *ft.*	100	6⅜	804·2496	9	112	3¾	1003·7902
1	100	9½	808·4422	10	112	6⅞	1008·4736
2	101	0⅝	812·6481	11	112	10	1013·1705
3	101	3¾	816·8650	36 *ft*	113	1⅛	1017·8784
4	101	6⅞	821·0904	1	113	4¼	1022·5944
5	101	10	825·3291	2	113	7⅜	1027·3240
6	102	1⅛	829·5787	3	113	10⅝	1032·0646
7	102	4⅜	833·8368	4	114	1¾	1036·8134
8	102	7½	838·1082	5	114	4⅞	1041·5758
9	102	10⅝	842·3905	6	114	8	1046·3491
10	103	1¾	846·6813	7	114	11⅛	1051·1306
11	103	4⅞	850·9855	8	115	2¼	1055·9257
33 *ft.*	103	8	855·3006	9	115	5⅜	1060·7317
1	103	11⅛	859·6240	10	115	9½	1065·5459
2	104	2¼	863·9609	11	115	11⅝	1070·3738
3	104	5⅜	868·3087	37 *ft.*	116	2⅞	1075·2126
4	104	8⅝	872·6649	1	116	6	1080·0594
5	104	11¾	877·0346	2	116	9⅛	1084·9201
6	105	2⅞	881·4151	3	117	0¼	1089·7915
7	105	6	885·8040	4	117	3½	1094·6711
8	105	9⅛	890·2064	5	117	6½	1099·5644

Diam. ft. & in.	Circumference in feet and in.	Area in feet.	Diam. ft. & in.	Circumference in feet and in.	Area in feet.
6	117 9 5/8	1104·4687	3	129 7	1336·4071
7	118 0 3/4	1109·3810	4	129 10 1/8	1341·8101
8	118 4	1114·3071	5	130 1 3/8	1347·2271
9	118 7 1/8	1119·2440	6	130 4 1/2	1352·6551
10	118 10 1/4	1124·1891	7	130 7 5/8	1358·0908
11	119 1 3/8	1129·1478	8	130 10 3/4	1363·5406
38 *ft.*	119 4 1/2	1134·1176	9	131 1 7/8	1369·0012
1	119 7 5/8	1139·0953	10	131 5	1374·4697
2	119 10 3/4	1144·0868	11	131 8 1/8	1379·9521
3	120 2	1149·0892	42 *ft.*	131 11 3/8	1385·4456
4	120 5	1154·0997	1	132 2 1/2	1390·2467
5	120 8 1/8	1159·1239	2	132 5 5/8	1396·4619
6	120 11 3/8	1164·1591	3	132 8 3/4	1401·9880
7	121 2 1/2	1169·2023	4	132 11 7/8	1407·5219
8	121 5 5/8	1174·2592	5	133 3	1413·0698
9	121 8 3/4	1179·3271	6	133 6 1/8	1418·6287
10	121 11 7/8	1184·4030	7	133 9 1/4	1424·1952
11	122 3 1/8	1189·4927	8	134 0 1/2	1429·7759
39 *ft.*	122 6 1/4	1194·5934	9	134 3 5/8	1435·3675
1	122 9 1/2	1199·7195	10	134 6 3/4	1440·9668
2	123 0 1/2	1204·8244	11	134 9 7/8	1446·5802
3	123 3 5/8	1209·9577	43 *ft.*	135 1	1452·2046
4	123 6 3/4	1215·0990	1	135 4 1/8	1457·8365
5	123 9 7/8	1220·2542	2	135 7 1/4	1463·4827
6	124 1 1/8	1225·4203	3	135 10 1/2	1469·1397
7	124 4 1/4	1230·5943	4	136 1 5/8	1474·8044
8	124 7 3/8	1235·7822	5	136 4 3/4	1480·4833
9	124 10 1/2	1240·9810	6	136 7 7/8	1486·1731
10	125 1 5/8	1246·1878	7	136 11	1491·8705
11	125 4 3/4	1251·4084	8	137 2 1/8	1497·5821
40 *ft.*	125 7 7/8	1256·6400	9	137 5 1/4	1503·3046
1	125 11	1261·8794	10	137 8 3/8	1509·0348
2	126 2 1/4	1267·1327	11	137 11 5/8	1514·7791
3	126 5 3/8	1272·3970	44 *ft.*	138 2 3/4	1520·5344
4	126 8 1/2	1277·6692	1	138 5 7/8	1526·2971
5	126 11 5/8	1282·9553	2	138 9	1532·0742
6	127 2 3/4	1288·2523	3	139 0 1/8	1537·8622
7	127 5 7/8	1293·5572	4	139 3 1/4	1543·6578
8	127 9	1298·8760	5	139 6 3/8	1549·4776
9	128 0 1/4	1304·2057	6	139 9 5/8	1555·2883
10	128 3 3/8	1309·5433	7	140 0 3/4	1561·1165
11	128 6 1/2	1314·8949	8	140 3 7/8	1566·9591
41 *ft.*	128 9 5/8	1320·2574	9	140 7 1/8	1572·8125
1	129 0 3/4	1325·6276	10	140 10 1/8	1578·6735
2	129 3 7/8	1331·0119	11	141 1 1/4	1584·5488

Diam. ft. & in.	Circumference in feet and in.	Area in feet.	Diam. ft. & in.	Circumference in feet and in.	Area in feet.
45 *ft.*	141 4⅝	1590·4350	7	149 5⅞	1778·2795
1	141 7⅝	1596·3286	8	149 8⅞	1784·5148
2	141 10¾	1602·2366	9	150 0⅛	1790·7610
3	142 1⅞	1608·1555	10	150 3¼	1797·0145
4	142 5	1614·0819	11	150 6⅜	1803·2826
5	142 8⅛	1620·0226	48 *ft.*	150 9½	1809·5616
6	142 11¼	1625·9743	1	151 0⅝	1815·8477
7	143 2⅝	1631·9334	2	151 3¾	1822·1485
8	143 5½	1637·9068	3	151 6⅞	1828·4602
9	143 8¾	1643·8912	4	151 10⅛	1834·7791
10	143 11⅞	1649·8831	5	152 1¼	1841·1127
11	144 3	1655·8892	6	152 4⅜	1847·4571
46 *ft.*	144 6⅛	1661·9064	7	152 7½	1853·8087
1	144 9¼	1667·9308	8	152 10⅝	1860·1750
2	145 0⅜	1673·9698	9	153 1¾	1866·5521
3	145 3½	1680·0196	10	153 4⅞	1872·9365
4	145 6⅝	1686·0769	11	153 8⅛	1879·3355
5	145 9⅞	1692·1485	49 *ft.*	153 11¼	1885·7454
6	146 1⅛	1698·2311	1	154 2¾	1892·1724
7	146 4⅛	1704·3210	2	154 5½	1898·5041
8	146 7¼	1710·4254	3	154 8⅝	1905·0367
9	146 10⅜	1716·5407	4	154 11⅞	1911·4965
10	147 1½	1722·6634	5	155 2⅞	1917·9609
11	147 4⅝	1728·8005	6	155 6	1924·4263
47 *ft.*	147 7¾	1734·9486	7	155 9¼	1930·9188
1	147 11	1741·1039	8	156 0⅛	1937·3159
2	148 2⅛	1747·2738	9	156 3½	1943·9140
3	148 5¼	1753·4545	10	156 6⅝	1950·4392
4	148 8⅜	1759·6426	11	156 9¾	1956·9691
5	148 11½	1765·8452	50 *ft.*	157 0⅞	1963·5000
6	149 2⅝	1772·0587			

To Preserve Steel Goods.—Caoutchouc 1 part; turpentine 16 parts. Dissolve with a gentle heat, then add boiled oil 8 parts. Mix by bringing them to the heat of boiling water; apply it to the steel with a brush, in the way of varnish. It may be removed with turpentine. The oil may be wholly omitted.

Size.—*Oil size* is made by grinding yellow ochre or burnt red ochre with boiled linseed oil, and thinning it with oil of turpentine. *Water size* (for burnished gilding) is parchment size ground with yellow ochre.

Silica and Carbon.—Silica is the base of the mineral world. Carbon the base of the organized.

IVORY.

How to Soften it.—Take 3 oz. spirits of nitre, and 15 of spring water; mix together; drop in the Ivory, and let it soak. In three or four days it will be so soft as to obey your fingers.

How to Dye Ivory when Softened.—If you desire to dye Ivory when thus softened, dissolve, in spirits of wine, such colors as you wish to use. When the spirit of wine is sufficiently tinged with the color you have put in plunge in your Ivory, and leave it there till it is dyed to suit you. Then take out the Ivory and give it what form you please.

How to Harden Ivory.—To harden the Ivory afterwards, wrap it up in a sheet of white paper, cover it with dry, decrepitated salt, and lay it by for twenty-four hours, when it will be restored to its original hardness.

To re-Whiten Ivory which has Turned a Brown Yellow.—There are two ways of doing this, namely: 1. Slack some lime in water, into which drop the ivory; decant it gently, and boil till it looks quite white. 2. To polish it afterwards, set it in the turner's wheel, and after having worked it, take some rushes and pumice stone, mix a subtile powder with water, and rub till it becomes perfectly smooth: then heat it by turning it over a piece of linen or sheepskin, and when hot rub it with a little whitening diluted with olive oil; then rub it with a little dry whitening alone, and finally with a piece of soft white rag, and the Ivory will look remarkably white.

How to Dye Ivory Black.—Immerse the Ivory in a boiling solution of logwood, then take it out, and wash it in a solution of copperas.

Blue.—There are two ways of reaching this color. The first is to soak the Ivory in a solution of verdigris in nitric acid, which will make it green; then dip it into a solution of boiling hot pearlash, and it will turn blue. The second way is as follows: Immerse the Ivory in a solution of sulphate of indigo and water, partly neutralized with potash.

Green.—Steep blued Ivory in a solution of nitro-muriate of tin, and then in a decoction of fustic. Another and a more instantaneous plan is to immerse it in a solution of acetate of copper.

Yellow.—Steep the Ivory in a bath of neutral chromate of potash, and afterwards in a boiling solution of acetate of lead.

Red.—Soak the Ivory for a short time in a solution of tin, and then in a decoction of cochineal.

Violet.—Moisten the Ivory with a solution of tin, as before; then immerse it in a decoction of logwood.

Purple.—Soak the Ivory in a solution of sal ammoniac into four times its weight of nitrous acid.

Fluid for Marking Ivory.—Take nitrate of silver, 2 parts; nitric acid, 1 part; water, 7 parts. Mix.

Etching Fluid for Ivory.—Take of diluted sulphuric acid and diluted muriatic acid, equal parts. Mix.

Etching Varnish for Ivory.—White wax, 2 parts; tears of mastic, 2 parts. Mix

To Gild Ivory.—Immerse it in a solution of nitro-muriate of gold, and then, while yet damp, expose it to hydrogen gas. Wash it afterwards in clean water. Another plan of gilding Ivory is by immersing it in a fresh solution of proto-sulphate of iron, and afterwards in a solution of chloride of gold.

To Polish Ivory.—Use a rubber and putty and water.

The hardest, toughest, whitest, and most translucent ivory has the preference in the market; and the tusks of the sea horse are considered to afford the best. Ivory has the same constituents as the teeth of animals: three-fourths being phosphate, with a little carbonate of lime; one-fourth cartilage. With regard to dyeing Ivory, it may in general be observed, that the colors penetrate better before the surface is polished than afterwards. Should any dark spots appear, they may be cleared up by rubbing them with chalk; after which the Ivory should be dyed once more to produce a perfect uniformity of shade. On taking it out of the boiling hot dye bath, it should be plunged immediately into cold water, to prevent the chance of fissures being caused by the heat.

CENTRE,

In a general sense, denotes a point equally remote from the extremes of a line, surface, or solid.

Centre of Attraction

Of a body, is that point into which if all its matter is collected, its action upon any remote particle would still be the same.

Centre of Equilibrium

Is the same, in respect to bodies immersed in a fluid, as the centre of gravity is to bodies in free space.

Centre of Friction

Is that point in the base of a body on which it revolves, into which if the whole surface of the base and the mass of the body were collected, and made to revolve about the centre of the base of the given body, the angular velocity destroyed by its friction would be equal to the angular velocity destroyed in the given body by its friction in the same time.

Centre of Gravity

Of any body, or system of bodies, is that point upon which the body, or system of bodies, acted upon only by the force of gravity, will balance itself in all positions; hence it follows, that if a line or plane, passing through the centre of gravity, be supported, the body or system will be also supported.

Centre of Gyration

Is that point into which, if the whole mass were collected, a given force, applied at a given distance, would produce the same angular velocity in the same time as if the bodies were disposed at their respective distances.

This point differs from the *Centre of Oscillation* only in this, that, in the latter case, the motion is produced by the gravity of the body; but, in the former, the body is put in motion by some other force, acting at one place only.

COHESION

Is that species of attraction which, uniting particle to particle, retains together the component parts of the same mass; being thus distinguished from *Adhesion*, or that species of attraction which takes place between the surfaces of similar or dissimilar bodies. The absolute cohesion of solids is measured by the force necessary to pull them asunder. Thus, if a rod of iron be suspended in a vertical position, having weight attached to its lower extremity till the rod breaks, the whole weight attached to the rod, at the time of fracture, will be the measure of its cohesive force, or absolute cohesion.

The particles of solid bodies, in their natural state, are arranged in such a manner, that they are in equilibrium in respect to the forces which operate on them; therefore, when any new force is applied, it is evident that the equilibrium will be destroyed, and that the particles will move among themselves till it be restored. When the new force is applied to pull the body asunder, the body becomes longer in the direction of the force, which is called the *extension;* and its area, at right angles to the direction of the force, contracts. When the force is applied to compress the body, it becomes shorter in the direction of the force, which is called the *compression;* and the area of its section, at right angles to the force, expands. In either case, a part of the heat, or any fluid that occupies the pores or interstices of the body, before the new force was made to act upon it, will be expelled.

PLATINA-MOHR.—Zinc two parts: platinum one part. Melt and reduce the alloy to powder, which must be treated with dilute sulphuric acid until all the zinc is washed out; then wash it with water, digest it in a ley of potash, and again wash it with water. This powder possesses the property of converting alcohol into vinegar.

THE VELOCITY OF SOUND.—It has been ascertained, by careful investigation, that sound passes in water at a speed of 4,708 feet per second.

MECHANICAL LAWS OF ELASTIC FLUIDS.

Boyle's or Mariotte's Law.

The elastic force of a gas or air at a given temperature is inversely proportional to the space which it occupies.

Let p = elastic force of a gas when it occupies the space s.
P = do. do. S.

$$\therefore P = \frac{p\,s}{S}$$

The elastic force of any gas at a given temperature is proportional to its density.

The density of any body is the weight of a cubic unit of it, usually one cubic foot.

Let p = the elastic force when the density is d.
And k = do. do. unit.

$$\therefore p = k\,d.$$

Dalton's and Gay-Lussac's Law.

All gases, under the same pressure, undergo equal expansions for equal increments of temperature.

It was ascertained by these eminent philosophers, that 100 measures of air expand to 137·5 measures on being heated from 32° to 212° of Fahrenheit's thermometer, hence

37·5 = increments of 100 measures for 180 degrees of heat.

$\frac{37\cdot5}{100}$ = do. 1 180 do.

$\frac{\cdot375}{180}$ = do. 1 1 do.

$$= \frac{1}{480} = a.$$

Let V = volume of any gas at the temperature t.
V' = do. do. t'.

$$\text{Then, } V' = \frac{1 + a\,(t' - 32)}{1 + a\,(t - 32)} \cdot V \text{ accurately.}$$

$$= \left\{ 1 + a\,(t' - t) \right\} V \text{ very nearly.}$$

Amonton's Law.

This law is the relation between the elastic force, the density, and the temperature, of any gas. If, then, the volume of a gas be constant, its elastic force will increase; and, if the elastic force is constant, its volume will increase for every increase of temperature. It is important to connect these quantities by an equation.

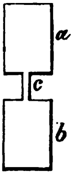
Put $p =$ elastic force of a gas at the temperature $\theta°$ and density d.

$$\text{Then, } p = k\,d\,(1 + a\,\theta)$$

where k is a constant quantity depending on the nature of the gas, and $a = \frac{1}{480}$.

When a light and heavy gas are once mixed, they do not exhibit any tendency to separate; in this respect they differ from mixed liquids.

Dalton's Experiment.



The vessel a contains a light gas, as hydrogen, and the vessel b contains a heavy gas, as carbonic acid; the two gases are allowed to communicate by a narrow tube c, an interchange speedily takes place of a part of their contents, which their relative position might be supposed would prevent. Contrary to gravity, the heavy gas ascends, and the light one descends, till, in a few hours, the gases become perfectly mixed, and the proportion of the two gases is the same in both vessels.

Gases diffuse into the atmosphere and into each other with different degrees of rapidity. The velocity with which air will rush into a vacuum is 1348 feet per second.

To determine the velocity with which the air of the atmosphere will rush into a space containing rarer air:

Let $v =$ velocity of air, of density (d), rushing into a void.

$V =$ velocity of air rushing into air of density D.

$$\therefore V = v\left(1 - \frac{D}{d}\right)$$

There will always be a current so long as (D) and (d) are unequal.

Illuminating Gases.

Pure hydrogen burns with too feeble a flame to be employed for the purpose of illumination. Carburetted hydrogen has the property of precipitating its carbon; in the act of burning, its solid particles become incandescent, and diffuse a strong light. The more carbon the gas contains the more brightly does it burn.

Two measures of hydrogen gas, with one measure of the vapor of carbon, form the carburetted hydrogen found in coal mines, and is also evolved in ditches, from decomposing vegetable matter. Another kind of carburetted hydrogen, called olefiant gas, is formed by two measures of hydrogen and two measures of gaseous carbon. This gas burns with a brighter flame than the common carburetted hydrogen.

The best substances for furnishing a gas rich in luminiferous materials are pit coal, resin, oil, fats of all kinds, tar, wax, &c.

The volume of gas discharged from the end of a pipe is directly proportional to the square of its diameter, and inversely as the square root of its length.

Let n = number of cubic feet of gas discharged per hour through a length of pipes l feet and diameter D.

$$\therefore n = \frac{3162\,D^2}{\sqrt{l}}$$

This formula is applicable only when the gas is transmitted through the pipes, without being let off in its way by burners. If the main send off branches for burners, then, for the same length, the diameter may be reduced; or, for a like diameter, the length may be increased.

STAINS, TO REMOVE.—Stains of *iodine* are removed by rectified spirit. *Ink* stains by oxalic acid or superoxalate of potash. *Iron moulds* by the same; but if obstinate, it has been recommended to moisten them with *ink*, then remove them in the usual way.

Red spots on black cloth, from acids, are removed by spirits of hartshorn, or other solutions of ammonia.

Stains of Marking Ink, or Nitrate of Silver, to remove 1. Wet the stain with fresh solution of chloride of lime, and after ten or fifteen minutes, if the marks have become white, dip the part in solution of ammonia or of hyposulphite of soda. In a few minutes wash with clean water.

2. Stretch the stained linen over a basin of hot water, and wet the mark with tincture of iodine.

BROWNING, OR BRONZING LIQUIDS, FOR GUN BARRELS.—1. Aquafortis ½ oz., sweet spirit of nitre ½ oz., spirit of wine 1 oz., sulphate of copper 2 oz., water 30 oz., tincture of muriate of iron 1 oz. Mix.

2. Sulphate of copper 1 oz., sweet spirit of nitre 1 oz., water 1 pint. Mix. In a few days it will be fit for use.

3. Sweet spirit of nitre 3 oz., gum benzoin 1½ oz., tincture of muriate of iron ½ oz., sulphate of copper 2 dr., spirit of wine ½ oz. Mix, and add 2 lbs. of soft water.

4. Tincture of muriate of iron ½ oz., spirit of nitric ether ½ oz., sulphate of copper 2 scruples, rain water ½ pint. The above are applied with a sponge, after cleaning the barrel with lime and water. When dry, they are polished with a stiff brush, or iron scratch brush.

BRONZING LIQUIDS FOR TIN CASTINGS.—Wash them over, after being well cleaned and wiped, with a solution of 1 part sulphate of iron, and 1 of sulphate of copper, in 20 parts of water; afterwards with a solution of 4 parts verdigris in 11 of distilled vinegar; leave for an hour to dry, and then polish with a soft brush and colcothar.

SOLVENTS FOR GUTTA PERCHA.—Benzole readily dissolves it: so do chloroform and bisulphuret of carbon.

TABLE

Of Squares, Cubes, Square and Cube Roots of Numbers.

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
1	1	1	1·0	1·0	1
2	4	8	1·414213	1·25992	2
3	9	27	1·732050	1·44225	3
4	16	64	2·0	1·58740	4
5	25	125	2·236068	1 70997	5
6	36	216	2·449489	1·81712	6
7	49	343	2·645751	1·91293	7
8	64	512	2·828427	2·0	8
9	81	729	3·0	2·08008	9
10	100	1000	3·162277	2·15443	10
11	121	1331	3·316624	3·22398	11
12	144	1728	3·464101	2·28942	12
13	169	2197	3·605551	2 35133	13
14	196	2744	3·741657	2·41014	14
15	225	3375	3·872983	2·46621	15
16	256	4096	4·0	2·51984	16
17	289	4913	4·123105	2·57128	17
18	324	5832	4·242640	2·62074	18
19	361	6859	4·358898	2·66840	19
20	400	8000	4·472136	2·71441	20
21	441	9261	4·582575	2·75892	21
22	484	10648	4·690415	2·80203	22
23	529	12167	4·795831	2·84386	23
24	576	13824	4·898979	2·88449	24
25	625	15625	5·0	2·92401	25
26	676	17576	5·099019	2·96249	26
27	729	19683	5·196152	3·0	27
28	784	21952	5·291502	3·03658	28
29	841	24389	5·385164	3·07231	29
30	900	27000	5·477225	3·10723	30
31	961	29791	5·567764	3·14138	31
32	1024	32768	5·656854	3·17480	32
33	1089	35937	5·744562	3·20753	33
34	1156	39304	5·830951	3·23961	34
35	1225	42875	5·916079	3·27106	35
36	1296	46656	6·0	3·30192	36
37	1369	50653	6 082762	3·33222	37
38	1444	54872	6·164414	3·36197	38
39	1521	59319	6·244998	3·39121	39
40	1600	64000	6·324555	3·41995	40
41	1681	68921	6·403124	3·44821	41

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
42	1764	74088	6·480740	3·47602	42
43	1849	79507	6·557438	3·50339	43
44	1936	85184	6·633249	3·53034	44
45	2025	91125	6·708203	3·55689	45
46	2116	97336	6·782330	3·58304	46
47	2209	103823	6·855654	3·60882	47
48	2304	110592	6·928203	3·63424	48
49	2401	117649	7·0	3·65930	49
50	2500	125000	7·071067	3·68403	50
51	2601	132651	7·141428	3·70842	51
52	2704	140608	7·211102	3·73251	52
53	2809	148877	7·280109	3·75628	53
54	2916	157464	7·348469	3·77976	54
55	3025	166375	7·416198	3·80295	55
56	3136	175616	7·483314	3·82586	56
57	3249	185193	7.549834	3·84850	57
58	3364	195112	7·615773	3·87087	58
59	3481	205379	7·681145	3·89299	59
60	3600	216000	7·745966	3·91486	60
61	3721	226981	7·810249	3·93649	61
62	3844	238328	7·874007	3·95789	62
63	3969	250047	7·937253	3·97905	63
64	4096	262144	8·0	4·0	64
65	4225	274625	8·062257	4·02072	65
66	4356	287496	8·124038	4·04124	66
67	4489	300763	8·185352	4·06154	67
68	4624	314432	8·246211	4·08165	68
69	4761	328509	8·306623	4·10156	69
70	4900	343000	8·366600	4·12128	70
71	5041	357911	8·426149	4·14081	71
72	5184	373248	8·485281	4·16016	72
73	5329	389017	8·544003	4·17933	73
74	5476	405224	8·602325	4·19833	74
75	5625	421875	8·660254	4·21716	75
76	5776	438976	8·717797	4·23582	76
77	5929	456533	8·774964	4·25432	77
78	6084	474552	8·831760	4·27265	78
79	6241	493039	8·888194	4·29084	79
80	6400	512000	8·944271	4·30886	80
81	6561	531441	9·0	4·32674	81
82	6724	551368	9·055385	4·34448	82
83	6889	571787	9·110433	4·36207	83
84	7056	592704	9·165151	4·37951	84
85	7225	614125	9·219544	4·39682	85
86	7396	636056	9·273618	4·41400	86
87	7569	658503	9·327379	4·43104	87

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
88	7744	681472	9·380831	4·44796	88
89	7921	704969	9·433981	4·46474	89
90	8100	729000	9·486833	4·48140	90
91	8281	753571	9·539392	4·49794	91
92	8464	778688	9·591663	4·51435	92
93	8649	804357	9·643650	4·53065	93
94	8836	830584	9·695359	4·54683	94
95	9025	857375	9·746794	4·56290	95
96	9216	884736	9·797959	4·57885	96
97	9409	912673	9·848857	4·59470	97
98	9604	941192	9·899494	4·61043	98
99	9801	970299	9·949874	4·62606	99
100	10000	1000000	10·0	4·64158	100
101	10201	1030301	10·049875	4·65700	101
102	10404	1061208	10·099504	4·67232	102
103	10609	1092727	10·148891	4·68754	103
104	10816	1124864	10·198039	4·70266	104
105	11025	1157625	10·246950	4·71769	105
106	11236	1191016	10·295630	4·73262	106
107	11449	1225043	10·344080	4·74745	107
108	11664	1259712	10·392304	4·76220	108
109	11881	1295029	10·440306	4·77685	109
110	12100	1331000	10·488088	4·79141	110
111	12321	1367631	10·535653	4·80589	111
112	12544	1404928	10·583005	4·82028	112
113	12769	1442897	10·630145	4·83458	113
114	12996	1481544	10·677078	4·84880	114
115	13225	1520875	10·723805	4·86294	115
116	13456	1560896	10·770329	4·87699	116
117	13689	1601613	10·816653	4·89097	117
118	13924	1643032	10·862780	4·90486	118
119	14161	1685159	10·908712	4·91868	119
120	14400	1728000	10·954451	4·93242	120
121	14641	1771561	11·0	4·94608	121
122	14884	1815848	11·045361	4·95967	122
123	15129	1860867	11·090536	4·97318	123
124	15376	1906624	11·135528	4·98663	124
125	15625	1953125	11·180339	5·0	125
126	15876	2000376	11·224972	5·01329	126
127	16129	2048383	11·269427	5·02652	127
128	16384	2097152	11·313708	5·03968	128
129	16641	2146689	11·357816	5·05277	129
130	16900	2197000	11·401754	5·06579	130
131	17161	2248091	11·445523	5·07875	131
132	17424	2299968	11·489125	5·09164	132
133	17689	2352637	11·532562	5·10446	133
134	17956	2406104	11·575836	5·11722	134

Number.	Square.	Cube.	Square Root.	Cube Root.	Numb
135	18225	2460375	11·618950	5·12992	135
136	18496	2515456	11·661903	5·14256	136
137	18769	2571353	11·704699	5·15513	137
138	19044	2628072	11·747340	5·16764	138
139	19321	2685619	11·789826	5·18010	139
140	19600	2744000	11·832159	5·19249	140
141	19881	2803221	11·874342	5·20482	141
142	20164	2863288	11·916375	5·21710	142
143	20449	2924207	11·958260	5·22932	143
144	20736	2985984	12·0	5·24148	144
145	21025	3048625	12·041594	5·25358	145
146	21316	3112136	12·083046	5·26563	146
147	21609	3176523	12·124355	5·27763	147
148	21904	3241792	12·165525	5·28957	148
149	22201	3307949	12·206555	5·30145	149
150	22500	3375000	12·247448	5·31329	150
151	22801	3442951	12·288205	5·32507	151
152	23104	3511808	12·328828	5·33680	152
153	23409	3581577	12·369316	5·34848	153
154	23716	3652264	12·409673	5·36010	154
155	24025	3723875	12·449899	5·37168	155
156	24336	3796416	12·489996	5·38321	156
157	24649	3869893	12·529964	5·39469	157
158	24964	3944312	12·569805	5·40612	158
159	25281	4019679	12·609520	5·41750	159
160	25600	4096000	12·649110	5·42883	160
161	25921	4173281	12·688577	5·44012	161
162	26244	4251528	12·727922	5·45136	162
163	26569	4330747	12·767145	5·46255	163
164	26896	4410944	12·806248	5·47370	164
165	27225	4492125	12·845232	5·48480	165
166	27556	4574296	12·884098	5·49586	166
167	27889	4657463	12·922848	5·50687	167
168	28224	4741632	12·961481	5·51784	168
169	28561	4826809	13·0	5·52877	169
170	28900	4913000	13·038404	5·53965	170
171	29241	5000211	13·076696	5·55049	171
172	29584	5088448	13·114877	5·56129	172
173	29929	5177717	13·152946	5·57205	173
174	30276	5268024	13·190906	5·58277	174
175	30625	5359375	13·228756	5·59344	175
176	30976	5451776	13·266499	5·60407	176
177	31329	5545233	13·304134	5·61467	177
178	31684	5639752	13·341664	5·62522	178
179	32041	5735339	13·379088	5·63574	179
180	32400	5832000	13·416407	5·64621	180
181	32761	5929741	13·453624	5·65665	181

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
182	33124	6028568	13·490737	5·66705	182
183	33489	6128487	13·527749	5·67741	183
184	33856	6229504	13·564660	5·68773	184
185	34225	6331625	13·601470	5·69801	185
186	34596	6434856	13·638181	5·70826	186
187	34969	6539203	13·674794	5·71847	187
188	35344	6644672	13·711309	5·72865	188
189	35721	6751269	13·747727	5·73879	189
190	36100	6859000	13·784048	5·74889	190
191	36481	6967871	13·820275	5·75896	191
192	36864	7077888	13·856406	5·76899	192
193	37249	7189057	13·892444	5·77899	193
194	37636	7301384	13·928388	5·78896	194
195	38025	7414875	13·964240	5·79889	195
196	38416	7529536	14·0	5·80878	196
197	38809	7645373	14·035668	5·81864	197
198	39204	7762392	14·071247	5·82847	198
199	39601	7880599	14·106736	5·83827	199
200	40000	8000000	14·142135	5·84803	200
201	40401	8120601	14·177446	5·85776	201
202	40804	8242408	14·212670	5·86746	202
203	41209	8365427	14·247806	5·87713	203
204	41616	8489664	14·282856	5·88676	204
205	42025	8615125	14·317821	5·89636	205
206	42436	8741816	14·352700	5·90594	206
207	42849	8869743	14·387494	5·91548	207
208	43264	8998912	14·422205	5·92499	208
209	43681	9129329	14·456832	5·93447	209
210	44100	9261000	14·491376	5·94392	210
211	44521	9393931	14·525839	5·95334	211
212	44944	9528128	14·560219	5·96273	212
213	45369	9663597	14·594519	5·97209	213
214	45796	9800344	14·628738	5·98142	214
215	46225	9938375	14·662878	5·99072	215
216	46656	10077696	14·696938	6·0	216
217	47089	10218313	14·730919	6·00924	217
218	47524	10360232	14·764823	6·01846	218
219	47961	10503459	14·798648	6·02765	219
220	48400	10648000	14·832397	6·03681	220
221	48841	10793861	14·866068	6·04594	221
222	49284	10941048	14·899664	6·05504	222
223	49729	11089567	14·933184	6·06412	223
224	50176	11239424	14·966629	6·07317	224
225	50625	11390625	15·0	6·08220	225
226	51076	11543176	15·033296	6·09119	226
227	51529	11697083	15·066519	6·10017	227
228	51984	11852352	15·099668	6·10911	228

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
229	52441	12008989	15·132746	6·11803	229
230	52900	12167000	15·165750	6·12692	230
231	53361	12326391	15·198684	6·13579	231
232	53824	12487168	15·231546	6·14463	232
233	54289	12649337	15·264337	6·15344	233
234	54756	12812904	15·297058	6·16224	234
235	55225	12977875	15·329709	6·17100	235
236	55696	13144256	15·362291	6·17974	236
237	56169	13312053	15·394804	6·18846	237
238	56644	13481272	15·427248	6·19715	238
239	57121	13651919	15·459624	6·20582	239
240	57600	13824000	15·491933	6·21446	240
241	58081	13997521	15·524174	6·22308	241
242	58564	14172488	15·556349	6·23167	242
243	59049	14348907	15·588457	6·24025	243
244	59536	14526784	15·620499	6·24879	244
245	60025	14706125	15·652475	6·25732	245
246	60516	14886936	15·684387	6·26582	246
247	61009	15069223	15·716233	6·27430	247
248	61504	15252992	15·748015	6·28276	248
249	62001	15438249	15·779733	6·29119	249
250	62500	15625000	15·811388	6·29960	250
251	63001	15813251	15·842979	6·30799	251
252	63504	16003008	15·874507	6·31635	252
253	64009	16194277	15·905973	6·32470	253
254	64516	16387064	15·937377	6·33302	254
255	65025	16581375	15·968719	6·34132	255
256	65536	16777216	16·	6·34960	256
257	66049	16974593	16·031219	6·35786	257
258	66564	17173512	16·062378	6·36609	258
259	67081	17373979	16·093476	6·37431	259
260	67600	17576000	16·124515	6·38250	260
261	68121	17779581	16·155494	6·39067	261
262	68644	17984728	16·186414	6·39882	262
263	69169	18191447	16·217274	6·40695	263
264	69696	18399744	16·248076	6·41506	264
265	70225	18609625	16·278820	6·42315	265
266	70756	18821096	16·309506	6·43122	266
267	71289	19034163	16·340134	6·43927	267
268	71824	19248832	16·370705	6·44730	268
269	72361	19465109	16·401219	6·45531	269
270	72900	19683000	16·431676	6·46330	270
271	73441	19902511	16·462077	6·47127	271
272	73984	20123648	16·492422	6·47922	272
273	74529	20346417	16·522711	6·48715	273
274	75076	20570824	16·552945	6·49506	274
275	75625	20796875	16·583124	6·50295	275

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
276	76176	21024576	16·613247	6·51083	276
277	76729	21253933	16·643317	6·51868	277
278	77284	21484952	16·673332	6·52651	278
279	77841	21717639	16·703293	6·53433	279
280	78400	21952000	16·733200	6·54213	280
281	78961	22188041	16·763054	6·54991	281
282	79524	22425768	16·792855	6·55767	282
283	80089	22665187	16·822603	6·56541	283
284	80656	22906304	16·852299	6·57313	284
285	81225	23149125	16·881943	6·58084	285
286	81796	23393656	16·911534	6·58853	286
287	82369	23639903	16·941074	6·59620	287
288	82944	23887872	16·970562	6·60385	288
289	83521	24137569	17·0	6·61148	289
290	84100	24389000	17·029386	6·61910	290
291	84681	24642171	17·058722	6·62670	291
292	85264	24897088	17·088007	6·63428	292
293	85849	25153757	17·117242	6·64185	293
294	83436	25412184	17·146428	6·64939	294
295	87025	25672375	17·175564	6·65693	295
296	87616	25934336	17·204650	6·66444	296
297	88209	26198073	17·233687	6·67194	297
298	88804	26463592	17·262676	6·67942	298
299	89401	26730899	17·291616	6·68688	299
300	90000	27000000	17·320508	6·69432	300
301	90601	27270901	17·349351	6·70175	301
302	91204	27543608	17·378147	6·70917	302
303	91809	27818127	17·406895	6·71657	303
304	92416	28094464	17·435595	6·72395	304
305	93025	28372625	17·464249	6·73131	305
306	93636	28652616	17·492855	6·73866	306
307	94249	28934443	17·521415	6·74599	307
308	94864	29218112	17·549928	6·75331	308
309	95481	29503629	17·578395	6·76061	309
310	96100	29791000	17·606816	6·76789	310
311	96721	30080231	17·635192	6·77516	311
312	97344	30371328	17·663521	6·78242	312
313	97969	30664297	17·691806	6·78966	313
314	98596	30959144	17·720045	6·79688	314
315	99225	31255875	17·748239	6·80409	315
316	99856	31554496	17·776388	6·81128	316
317	100489	31855013	17·804493	6·81846	317
318	101124	32157432	17·832554	6·82562	318
319	101761	32461759	17·860571	6·83277	319
320	102400	32768000	17·888543	6·83990	320
321	103041	33076161	17·916472	6·84702	321
322	103684	33386248	17·944358	6·85412	322

Number.	Square.	Cube.	Square Root.	Cube Root	Number
323	104329	33698267	17·972200	6·86121	323
324	104976	34012224	18·0	6·86828	324
325	105625	34328125	18·027756	6·87534	325
326	106276	34645976	18·055470	6·88238	326
327	106929	34965783	18·083141	6·88941	327
328	107584	35287552	18·110770	6·89648	328
329	108241	35611289	18·138357	6·90348	329
330	108900	35937000	18·165902	6·91042	330
331	109561	36264691	18·193405	6·91739	331
332	110224	36594368	18·220867	6·92435	332
333	110889	36926037	18·248287	6·93130	333
334	111556	37259704	18·275666	6·93823	334
335	112225	37595375	18·303005	6·94514	335
336	112896	37933056	18·330302	6·95205	336
337	113569	38272753	18·357559	6·95894	337
338	114244	38614472	18·384776	6·96581	338
339	114921	38958219	18·411952	6·97268	339
340	115600	39304000	18·439088	6·97953	340
341	116281	39651821	18·466185	6·98636	341
342	116964	40001688	18·493242	6·99319	342
343	117649	40353607	18·520259	7·0	343
344	118336	40707584	18·547237	7·00679	344
345	119025	41063625	18·574175	7·01357	345
346	119716	41421736	18·601075	7·02034	346
347	120409	41781923	18·627936	7·02710	347
348	121104	42144192	18·654758	7·03384	348
349	121801	42508549	18·681541	7·04058	349
350	122500	42875000	18·708286	7·04729	350
351	123201	43243551	18·734994	7·05400	351
352	123904	43614208	18·761663	7·06069	352
353	124609	43986977	18·788294	7·06737	353
354	125316	44361864	18·814887	7·07404	354
355	126025	44738875	18·841443	7·08069	355
356	126736	45118016	18·867962	7·08734	356
357	127449	45499293	18·894443	7·09397	357
358	128164	45882712	18·920887	7·10058	358
359	128881	46268279	18·947295	7·10719	359
360	129600	46656000	18·973666	7·11378	360
361	130321	47045881	19·0	7·12036	361
362	131044	47437928	19·026297	7·12693	362
363	131769	47832147	19·052558	7·13349	363
364	132496	48228544	19·078784	7·14003	364
365	133225	48627125	19·104973	7·14656	365
366	133956	49027896	19·131126	7·15309	366
367	134689	49430863	19·157244	7·15959	367
368	135424	49836032	19·183326	7·16609	368
369	136161	50243409	19·209372	7·17258	369

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
370	136900	50653000	19·235384	7·17905	370
371	137641	51064811	19·261360	7·18551	371
372	138384	51478848	19·287301	7·19196	372
373	139129	51895117	19·313207	7·19840	373
374	139876	52313624	19·339079	7·20483	374
375	140625	52734375	19·364916	7·21124	375
376	141376	53157376	19·390719	7·21765	376
377	142129	53582633	19·416487	7·22404	377
378	142884	54010152	19·442222	7·23042	378
379	143641	54439939	19·467922	7·23679	379
380	144400	54872000	19·493588	7·24315	380
381	145161	55306341	19·519221	7·24950	381
382	145924	55742968	19·544820	7·25584	382
383	146689	56181887	19·570385	7·26216	383
384	147456	56623104	19·595917	7·26848	384
385	148225	57066625	19·621416	7·27478	385
386	148996	57512456	19·646882	7·28107	386
387	149769	57960603	19·672315	7·28736	387
388	150544	58411072	19·697715	7·29363	388
389	151321	58863859	19·723082	7·29989	389
390	152100	59319000	19·748417	7·30614	390
391	152881	59776471	19·773719	7·21238	391
392	153664	60236288	19·798989	7·31861	392
393	154449	60698457	19·824227	7·32482	393
394	155236	61162984	19·849433	7·33103	394
395	156025	61629875	19·874606	7·33723	395
396	156816	62099136	19·899748	7·34342	396
397	157609	62570773	19·924858	7·34959	397
398	158404	63044792	19·949937	7·35576	398
399	159201	63521199	19·974984	7·36191	399
400	160000	64000000	20·0	7·36806	400
401	160801	64481201	20·024984	7·37419	401
402	161604	64964808	20·049937	7·38032	402
403	162409	65450827	20·074859	7·38643	403
404	163216	65939264	20·099751	7·39254	404
405	164025	66430125	20·124611	7·39863	405
406	164836	66923416	20·149441	7·40472	406
407	165649	67419143	20·174241	7·41079	407
408	166464	67917312	29·199009	7·41685	408
409	167281	68417929	20·223748	7·42291	409
410	168100	68921000	20·248456	7·42895	410
411	168921	69426531	20·273134	7·43499	411
412	169744	69934528	20·297783	7·44101	412
413	170569	70444997	20·322401	7·44703	413
414	171396	70957944	20·346989	7·45303	414
415	172225	71473375	20·371548	7·45903	415
416	173056	71991296	20·396078	7·46502	416

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
417	173889	72511713	20·420577	7·47099	417
418	174724	73034632	20·445048	7·47696	418
419	175561	73560059	20·469489	7·48292	419
420	176400	74088000	20·493901	7·48887	420
421	177241	74618461	20·518284	7·49481	421
422	178084	75151448	20·542638	7·50074	422
423	178929	75686967	20·566963	7·50666	423
424	179776	76225024	20·591260	7·51257	424
425	180625	76765625	20·615528	7·51847	425
426	181476	77308776	20·639767	7·52436	426
427	182329	77854483	20·663978	7·53024	427
428	183184	78402752	20·688160	7·53612	428
429	184041	78953589	20·712315	7·54198	429
430	184900	79507000	20·736441	7·54784	430
431	185761	80062991	20·760539	7·55368	431
432	186624	80621568	20·784609	7·55952	432
433	187489	81182737	20·808652	7·56535	433
434	188356	81746504	20·832666	7·57117	434
435	189225	82312875	20·856653	7·57698	435
436	190096	82881856	20·880613	7·58278	436
437	190969	83453453	20·904545	7·58857	437
438	191844	84027672	20·928449	7·59436	438
439	192721	84604519	20·952326	7·60013	439
440	193600	85184000	20·976177	7·60590	440
441	194481	85766121	21·0	7·61166	441
442	195364	86350888	21·023796	7·61741	442
443	196249	86938307	21·047565	7·62315	443
444	197136	87528384	21·071307	7·62888	444
445	198025	88121125	21·095023	7·63460	445
446	198916	88716536	21·118712	7·64032	446
447	199809	89314623	21·142374	7·64602	447
448	200704	89915392	21·166010	7·65172	448
449	201601	90518849	21·189620	7.65741	449
450	202500	91125000	21·213203	7·66309	450
451	203401	91733851	21·236760	7·66876	451
452	204304	92345408	21·260291	7·67443	452
453	205209	92959677	21·283796	7·68008	453
454	206116	93576664	21·307275	7·68573	454
455	207025	94196375	21·330729	7·69137	455
456	207936	94818816	21·354156	7·69700	456
457	208849	95443993	21·377558	7·70262	457
458	209764	96071912	21·400934	7·70823	458
459	210681	96702579	21·424285	7·71384	459
460	211600	97336000	21·447610	7·71944	460
461	212521	97972181	21·470910	7·72503	461
462	213444	98611128	21·494185	7·73061	462
463	214369	99252847	21·517434	7·73618	463

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
464	215296	99897344	21·540659	7·74175	464
465	216225	100544625	21·563858	7·74731	465
466	217156	101194696	21·587033	7·75286	466
467	218089	101847563	21·610182	7·75840	467
468	219024	102503232	21·633307	7·76393	468
469	219961	103161709	21·656407	7·76946	469
470	220900	103823000	21·679483	7·77498	470
471	221841	104487111	21·702534	7·78049	471
472	222784	105154048	21·725561	7 78599	472
473	223729	105823817	21·748563	7·79148	473
474	224676	106496424	21·771541	7·79697	474
475	225625	107171875	21·794494	7·80245	475
476	226576	107850176	21·817424	7·80792	476
477	227529	108531333	21·840329	7·81338	477
478	228484	109215352	21·863211	7·81884	478
479	229441	109902239	21·886068	7·82429	479
480	230400	110592000	21·908902	7·82973	480
481	231361	111284641	21·931712	7·83516	481
482	232324	111980168	21·954498	7·84059	482
483	233289	112678587	21·977261	7·84601	483
484	234256	113379904	22·0	7·85142	484
485	235225	114084125	22·022715	7·85682	485
486	236196	114791256	22·045407	7·86222	486
487	237169	115501303	22·068076	7·86761	487
488	238144	116214272	22·090722	7·87299	488
489	239121	116930169	22·113344	7·87836	489
490	240100	117649000	22·135943	7·88373	490
491	241081	118370771	22·158519	7·88909	491
492	242064	119095488	22·181073	7·89444	492
493	243049	119823157	22·203603	7·89979	493
494	244036	120553784	22·226110	7 90512	494
495	245025	121287375	22·248595	7·91045	495
496	246016	122023936	22·271057	7·91578	496
497	247009	122763473	22·293496	7·92109	497
498	248004	123505992	22·315913	7·92640	498
499	249001	124251499	22·338307	7·93171	499
500	250000	125000000	22 360679	7·93700	500
501	251001	125751501	22·383029	7·94229	501
502	252004	126506008	22·405356	7·94757	502
503	253009	127263527	22·427661	7·95284	503
504	254016	128024064	22·449944	7·95811	504
505	255025	128787625	22·472205	7·96337	505
506	256036	129554216	22·494443	7·96862	506
507	257049	130323843	22·516660	7·97387	507
508	258064	131096512	22·538855	7·97911	508
509	259081	131872229	22·561028	7·98434	509
510	260100	132651000	22·583179	7·98956	510

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
511	261121	133432831	22·605309	7·99478	511
512	262144	134217728	22·627417	8·0	512
513	263169	135005697	22·649503	8·00520	513
514	264196	135796744	22·671568	8·01040	514
515	265225	136590875	22·693611	8·01559	515
516	266256	137388096	22·715633	8·02077	516
517	267289	138188413	22·737634	8·02595	517
518	268324	138991832	22·759613	8·03112	518
519	269361	139798359	22·781571	8·03629	519
520	270400	140608000	22·803508	8·04145	520
521	271441	141420761	22·825424	8·04660	521
522	272484	142236648	22·847319	8·05174	522
523	273529	143055667	22·869193	8·05688	523
524	274576	143877824	22·891046	8·06201	524
525	275625	144703125	22·912878	8·06714	525
526	276676	145531576	22·934689	8·07226	526
527	277729	146363183	22·956480	8·07737	527
528	278784	147197952	22·978250	8·08248	528
529	279841	148035889	23·0	8·08757	529
530	280900	148877000	23·021728	8·09267	530
531	281961	149721291	23·043437	8·09775	531
532	283024	150568768	23·065125	8·10283	532
533	284089	151419437	23·086792	8·10791	533
534	285156	152273304	23·108440	8·11298	534
535	286225	153130375	23·130067	8·11804	535
536	287296	153990656	23·151673	8·12309	536
537	288369	154854153	23·173260	8·12814	537
538	289444	155720872	23·194827	8·13318	538
539	290521	156590819	23·216373	8·13822	539
540	291600	157464000	23·237900	8·14325	540
541	292681	158340421	23·259406	8·14827	541
542	293764	159220088	23·280893	8·15329	542
543	294849	160103007	23·302360	8·15830	543
544	295936	160989184	23·323807	8·16331	544
545	297025	161878625	23·345235	8·16830	545
546	298116	162771336	23·366642	8·17330	546
547	299209	163667323	23·388031	8·17828	547
548	300304	164566592	23·409399	8·18326	548
549	301401	165969149	23·430749	8·18824	549
550	302500	166375000	23·452078	8·19321	550
551	303601	167284151	23·473389	8·19817	551
552	304704	168196608	23·494680	8·20313	552
553	305809	169112377	23·515952	8·20808	553
554	306916	170031464	23·537204	8·21302	554
555	308025	170953875	23·558438	8·21796	555
556	309136	171879616	23·579652	8·22289	556
557	310249	172808693	23·600847	8·22782	557

Number	Square	Cube	Square Root.	Cube Root.	Number.
558	311364	173741112	23·622023	8·23274	558
559	312481	174676879	23·643180	8·23766	559
560	313600	175616000	23·664319	8·24257	560
561	314721	176558481	23·685438	8·24747	561
562	315844	177504328	23·706539	8·25237	562
563	316969	178453547	23·727621	8·25726	563
564	318096	179406144	23·748684	8·26214	564
565	319225	180362125	23·769728	8·26702	565
566	320356	181321496	23·790754	8·27190	566
567	321489	182284263	23·811761	8·27677	567
568	322624	183250432	23·832750	8·28163	568
569	323761	184220009	23·853720	8·28649	569
570	324900	185193000	23·874672	8·29134	570
571	326041	186169411	23·895606	8·29619	571
572	327184	187149248	23·916521	8·30103	572
573	328329	188132517	23·937418	8·30586	573
574	329476	189119224	23·958297	8·31069	574
575	330625	190109375	23·979157	8·31551	575
576	331776	191102976	24·0	8·32033	576
577	332929	192100033	24·020824	8·32514	577
578	334084	193100552	24·041630	8·32995	578
579	335241	194104539	24·062418	8·33475	579
580	336400	195112000	24·083189	8·33955	580
581	337561	196122941	24·103941	8·34434	581
582	338724	197137368	24·124676	8·34912	582
583	339889	198155287	24·145392	8·35390	583
584	341056	199176704	24·166091	8·35867	584
585	342225	200201625	24·186773	8·36344	585
586	343396	201230056	24·207436	8·36820	586
587	344569	202262003	24·228082	8·37296	587
588	345744	203297472	24·248711	8·37771	588
589	346921	204336469	24·269322	8·38246	589
590	348100	205379000	24·289915	8·38720	590
591	349281	206425071	24·310491	8·39194	591
592	350464	207474688	24·331050	8·39667	592
593	351649	208527857	24·351591	8·40139	593
594	352836	209584584	24·372115	8·40611	594
595	354025	210644875	24·392621	8·41083	595
596	355216	211708736	24·413111	8·41554	596
597	356409	212776173	24·433583	8·42024	597
598	357604	213847192	24·454038	8·42494	598
599	358801	214921799	24·474476	8·42963	599
600	360000	216000000	24·494897	8·43432	600
601	361201	217081801	24·515301	8·43900	601
602	362404	218167208	24·535688	8·44368	602
603	363609	219256227	24·556058	8·44836	603
604	364816	220348864	24·576411	8·45302	604

Number.	Square.	Cube.	Square Root.	Cube Root.	Number
605	366025	221445125	24·596747	8·45769	605
606	367236	222545016	24·617067	8·46234	606
607	368449	223648543	24·637370	8·46700	607
608	369664	224755712	24·657656	8·47164	608
609	370881	225866529	24·677925	8·47628	609
610	372100	226981000	24·698178	8·48092	610
611	373321	228099131	24·718414	8·48555	611
612	374544	229220928	24·738633	8·49018	612
613	375769	230346397	24·758836	8·49480	613
614	376996	231475544	24·779023	8·49942	614
615	378225	232608375	24·799193	8·50403	615
616	379456	233744896	24·819347	8·50864	616
617	380689	234885113	24·839484	8·51324	617
618	381924	236029032	24·859605	8·51784	618
619	383161	237176659	24·879710	8·52243	619
620	384400	238328000	24·899799	8·52701	620
621	385641	239483061	24·919871	8·53160	621
622	386884	240641848	24·939927	8·53617	622
623	388129	241804367	24·959967	8·54075	623
624	389376	242970624	24·979992	8·54531	624
625	390625	244140625	25·0	8·54987	625
626	391876	245314376	25·019992	8·55443	626
627	393129	246491883	25·039968	8·55898	627
628	394384	247673152	25·059928	8·56353	628
629	395641	248858189	25·079872	8·56808	629
630	396900	250047000	25·099800	8·57261	630
631	398161	251239591	25·119713	8·57715	631
632	399424	252435968	25·139610	8·58168	632
633	400689	253636137	25·159491	8·58620	633
634	401956	254840104	25·179356	8·59072	634
635	403225	256047875	25·199206	8·59523	635
636	404496	257259456	25·219040	8·59974	636
637	405769	258474853	25·238858	8·60425	637
638	407044	259694072	25·258661	8·60875	638
639	408321	260917119	25·278449	8·61324	639
640	409600	262144000	25·298221	8·61773	640
641	410881	263374721	25·317977	8·62222	641
642	412164	264609288	25·337718	8·62670	642
643	413449	265847707	25·357444	8·63118	643
644	414736	267089984	25·377155	8·63565	644
645	416025	268336125	25·396850	8·64012	645
646	417316	269586136	25·416530	8·64458	646
647	418609	270840023	25·436194	8·64904	647
648	419904	272097792	25·455844	8·65349	648
649	421201	273359449	25·475478	8·65794	649
650	422500	274625000	25·495097	8·66239	650
651	423801	275894451	25·514701	8·66683	651

umber.	Square.	Cube.	Square Root.	Cube Root.	Number.
652	425104	277167808	25·534290	8·67126	652
653	426409	278445077	25·553864	8·67569	653
654	427716	279726264	25·573423	8·68012	654
655	429025	281011375	25·592967	8·68454	655
656	430336	282300416	25·612496	8·68896	656
657	431649	283593393	25·632011	8·69337	657
658	432964	284890312	25·651510	8·69778	658
659	434281	286191179	25·670995	8·70218	659
660	435600	287496000	25·690465	8·70658	660
661	436921	288804781	25·709920	8·71098	661
662	438244	290117528	25·729360	8·71537	662
663	439569	291434247	25·748786	8·71975	663
664	440896	292754944	25·768197	8·72414	664
665	442225	294079625	25·787593	8·72851	665
666	443556	295408296	25·806975	8·73289	666
667	444889	296740963	25·826343	8·73726	667
668	446224	298077632	25·845696	8·74162	668
669	447561	299418309	25·865034	8·74598	669
670	448900	300763000	25·884358	8·75034	670
671	450241	302111711	25·903667	8·75469	671
672	451584	303464448	25·922962	8·75903	672
673	452929	304821217	25·942243	8·76338	673
674	454276	306182024	25·961510	8·76771	674
675	455625	307546875	25·980762	8·77205	675
676	456976	308915776	26·0	8·77638	676
677	458329	310288733	26·019223	8·78070	677
678	459684	311665752	26·038438	8·78502	678
679	461041	313046839	26·057628	8·78934	679
680	462400	314432000	26·076809	8·79365	680
681	463761	315821241	26·095976	8·79796	681
682	465124	317214568	26·115129	8·80227	682
683	466489	318611987	26·134268	8·80657	683
684	467856	320013504	26·153393	8·81086	684
685	469225	321419125	26·172504	8·81515	685
686	470596	322828856	26·191601	8·81944	686
687	471969	324242703	26·210684	8·82373	687
688	473344	325660672	26·229754	8·82800	688
689	474721	327082769	26·248809	8·83228	689
690	476100	328509000	26·267851	8·83655	690
691	477481	329939371	26·286878	8·84082	691
692	478864	331373888	26·305892	8·84508	692
693	480249	332812557	26·324893	8·84934	693
694	481636	334255384	26·343879	8·85359	694
695	483025	335702375	26·362852	8·85784	695
696	484416	337153536	26·381811	8·86209	696
697	485809	338608873	26·400757	8·86633	697
698	487204	340068392	26·419689	8·87057	698

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
699	488601	341532099	26·438608	8·87480	699
700	490000	343000000	26·457513	8·87904	700
701	491401	344472101	26·476404	8·88326	701
702	492804	345948408	26·495282	8·88748	702
703	494209	347428927	26·514147	8·89170	703
704	495616	348913664	26·532998	8·89592	704
705	497025	350402625	26·551836	8·90013	705
706	498436	351895816	26·570660	8·90433	706
707	499849	353393243	26·589471	8·90853	707
708	501264	354894912	26·608269	8·91273	708
709	502681	356400829	26·627053	8·91693	709
710	504100	357911000	26·645825	8·92112	710
711	505521	359425431	26·664583	8·92530	711
712	507944	360944128	26·683328	8·92949	712
713	508369	362467097	26·702059	8·93366	713
714	509796	363994344	26·720778	8·93784	714
715	511225	365525875	26·739483	8·94201	715
716	512656	367061696	26·758176	8·94618	716
717	514089	368601813	26·776855	8·95034	717
718	515524	370146232	26·795522	8·95450	718
719	516961	371694959	26·814175	8·95865	719
720	518400	373248000	26·832815	8·96280	720
721	519841	374805361	26·851443	8·96695	721
722	521284	376367048	26·870057	8·97110	722
723	522729	377933067	26·888659	8·97524	723
724	524176	379503424	26·907248	8·97937	724
725	525625	381078125	26·925824	8·98350	725
726	527076	382657176	26·944387	8·98763	726
727	528529	384240583	26·962937	8·99176	727
728	529984	385828352	26·981475	8·99588	728
729	531441	387420489	27·0	9·0	729
730	532900	389017000	27·018512	9·00411	730
731	534361	390617891	27·037011	9·00822	731
732	535824	392223168	27·055498	9·01232	732
733	537289	393832837	27·073972	9·01643	733
734	538756	395446904	27·092434	9·02052	734
735	540225	397065375	27·110883	9·02462	735
736	541696	398688256	27·129319	9·02871	736
737	543169	400315553	27·147743	9·03280	737
738	544644	401947272	27·166155	9·03688	738
739	546121	403583419	27·184554	9·04096	739
740	547600	405224000	27·202941	9·04504	740
741	549081	406869021	27·221315	9·04911	741
742	550564	408518488	27·239676	9·05318	742
743	552049	410172407	27·258026	9·05724	743
744	553536	411830784	27·276363	9·06130	744
745	555025	413493625	27·294688	9·06536	745

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
746	556516	415160936	27·313000	9·06942	746
747	558009	416832723	27·331300	9·07347	747
748	559504	418508992	27·349588	9·07751	748
749	561001	420189749	27·367864	9·08156	749
750	562500	421875000	27·386127	9·08560	750
751	564001	423564751	27·404379	9·08963	751
752	565504	425259008	27·422618	9·09367	752
753	567009	426957777	27·440845	9·09770	753
754	568516	428661064	27·459060	9·10172	754
755	570025	430368875	27·477263	9·10574	755
756	571536	432081216	27·495454	9·10976	756
757	573049	433798093	27·513633	9·11378	757
758	574564	435519512	27·531799	9·11779	758
759	576081	437245479	27·549954	9·12180	759
760	577600	438976000	27·568097	9·12580	760
761	579121	440711081	27·586228	9·12980	761
762	580644	442450728	27·604347	9·13380	762
763	582169	444194947	27·622454	9·13779	763
764	583696	445943744	27·640549	9·14178	764
765	585225	447697125	27·658633	9·14577	765
766	586756	449455096	27·676705	9·14975	766
767	588289	451217663	27·694764	9·15373	767
768	589824	452984832	27·712812	9·15771	768
769	591361	454756609	27·730849	9·16168	769
770	592900	456533000	27·748873	9·16565	770
771	594441	458314011	27·766886	9·16962	771
772	595984	460099648	27·784888	9·17358	772
773	597529	461889917	27·802877	9·17754	773
774	599076	463684824	27·820855	9·18150	774
775	600625	465484375	27·838821	9·18545	775
776	602176	467288576	27·856776	9·18940	776
777	603729	469097433	27·874719	9·19334	777
778	605284	470910952	27·892651	9·19728	778
779	606841	472729139	27·910571	9·20122	779
780	608400	474552000	27·928480	9·20516	780
781	609961	476379541	27·946377	9·20909	781
782	611524	478211768	27·964262	9·21302	782
783	613089	480048687	27·982137	9·21695	783
784	614656	481890304	28·0	9·22087	784
785	616225	483736625	28·017851	9·22479	785
786	617796	485587656	28·035691	9·22870	786
787	619369	487443403	28·053520	9·23261	787
788	620944	489303872	28·071337	9·23652	788
789	622521	491169069	28·089143	9·24043	789
790	624100	493039000	28·106938	9·24433	790
791	625681	494913671	28·124722	9·24823	791
792	627264	496793088	28·142494	9·52213	792

Number	Square.	Cube.	Square Root.	Cube Root.	Number.
793	628849	498677257	28·160255	9·25602	793
794	630436	500566184	28·178005	9·25991	794
795	632025	502459875	28·195744	9·26379	795
796	633616	504358336	28·213472	9·26767	796
797	635209	506261573	28·231188	9·27155	797
798	636804	508169592	28·248893	9·27543	798
799	638401	510082399	28·266588	9·27930	799
800	640000	512000000	28·284271	9·28317	800
801	641601	513922401	28·301943	9·28704	801
802	643204	515849608	28·319604	9·29090	802
803	644809	517781627	28·337254	9·29476	803
804	646416	519718464	28·354893	9·29862	804
805	648025	521660125	28·372521	9·30247	805
806	649636	523606616	28·390139	9·30632	806
807	651249	525557943	28·407745	9·31017	807
808	652864	527514112	28·425340	9·31401	808
809	654481	529475129	28·442925	9·31785	809
810	656100	531441000	28·460498	9·32169	810
811	657721	533411731	28·478061	9·32553	811
812	659344	535387328	28·495613	9·32936	812
813	660969	537367797	28·513154	9·33319	813
814	662596	539353144	28·530685	9·33701	814
815	664225	541343375	28·548204	9·34083	815
816	665856	543338496	28·565713	9·34465	816
817	667489	545338513	28·583211	9·34847	817
818	669124	547343432	28·600699	9·35228	818
819	670761	549353259	28·618176	9·35609	819
820	672400	551368000	28·635642	9·35990	820
821	674041	553387661	28·653097	9·36370	821
822	675684	555412248	28·670542	9·36750	822
823	677329	557441767	28·687976	9·37130	823
824	678976	559476224	28·705400	9·37509	824
825	680625	561515625	28·722813	9·37888	825
826	682276	563559976	28·740215	9·38267	826
827	683929	565609283	28·757607	9·38646	827
828	685584	567663552	28·774989	9·39024	828
829	687241	569722789	28·792360	9·39402	829
830	688900	571787000	28·809720	9·39779	830
831	690561	573856191	28·827070	9·40156	831
832	692224	575930368	28·844410	9·40533	832
833	693889	578009537	28·861739	9·40910	833
834	695556	580093704	28·879058	9·41286	834
835	697225	582182875	28·896366	9·41662	835
836	698896	584277056	28·913664	9·42038	836
837	700569	586376253	28·930952	9·42414	837
838	702244	588480472	28·948229	9·42789	838
839	703921	590589719	28·965496	9·43164	839

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
840	705600	592704000	28·982753	9·43538	840
841	707281	594823321	29·0	9·43913	841
842	708964	596947688	29·017236	9·44287	842
843	710649	599077107	29·034462	9·44660	843
844	712336	601211584	29·051678	9·45034	844
845	714025	603351125	29·068883	9·45407	845
846	715716	605495736	29·086079	9·45779	846
847	717409	607645423	29·103264	9·46152	847
848	719104	609800192	29·120439	9·46524	848
849	720801	611960049	29·137604	9·46896	849
850	722500	614125000	29·154759	9·47268	850
851	724201	616295051	29·171904	9·47639	851
852	725904	618470208	29·189039	9·48010	852
853	727609	620650477	29·206163	9·48381	853
854	729316	622835864	29·223278	9·48751	854
855	731025	625026375	29·240380	9·49122	855
856	732736	627222016	29·257477	9·49491	856
857	734449	629422793	29·274562	9·49861	857
858	736164	631628712	29·291637	9·50230	858
859	737881	633839779	29·308701	9·50599	859
860	739600	636056000	29·325756	9·50968	860
861	741321	638277381	29·342801	9·51336	861
862	743044	640503928	29·359836	9·51705	862
863	744769	642735647	29·376861	9·52073	863
864	746496	644972544	29·393876	9·52440	864
865	748225	647214625	29·410882	9·52807	865
866	749956	649461896	29·427877	9·53174	866
867	751689	651714363	29·444863	9·53541	867
868	753424	653972032	29·461839	9·53908	868
869	755161	656234909	29·478805	9·54274	869
870	756900	658503000	29·495762	9·54640	870
871	758641	660776311	29·512709	9·55005	871
872	760384	663054848	29·529646	9·55371	872
873	762129	665338617	29·546573	9·55736	873
874	763876	667627624	29·563491	9·56101	874
875	765625	669921875	29·580398	9·56465	875
876	767376	672221376	29·597297	9·56829	876
877	769129	674526133	29·614185	9·57193	877
878	770884	676836152	29·631064	9·57557	878
879	772641	679151439	29·647934	9·57920	879
880	774400	681472000	29·664793	9·58283	880
881	776161	683797841	29·681644	9·58646	881
882	777924	686128968	29·698484	9·59009	882
883	779689	688465387	29·715315	9·59371	883
884	781456	690807104	29·732137	9·59733	884
885	783225	693154125	29·748949	9·60095	885
886	784996	695506456	29·765752	9·60456	886

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
887	786769	697864103	29·782545	9·60818	887
888	788544	700227072	29·799328	9·61179	888
889	790321	702595369	29·816103	9·61539	889
890	792100	704969000	29·832867	9·61900	890
891	793881	707347971	29·849623	9·62260	891
892	795664	709732288	29·866369	9·62620	892
893	797449	712121957	29·883105	9·62979	893
894	799236	714516984	29·899832	9·63339	894
895	801025	716917375	29·916550	9·63698	895
896	802816	719323136	29·933259	9·64056	896
897	804609	721734273	29·949958	9·64415	897
898	806404	724150792	29·966648	9·64773	898
899	808201	726572699	29·983328	9·65131	899
900	810000	729000000	30·0	9·65489	900
901	811801	731432701	30·016662	9·65846	901
902	813604	733870808	30·033314	9·66204	902
903	815409	736314327	30·049958	9·66560	903
904	817216	738763264	30·066592	9·66917	904
905	819025	741217625	30·083217	9·67274	905
906	820836	743677416	30·099833	9·67630	906
907	822649	746142643	30·116440	9·67986	907
908	824464	748613312	30·133038	9·68341	908
909	826281	751089429	30·149626	9·68697	909
910	828100	753571000	30·166206	9·69052	910
911	829921	756058031	30·182776	9·69406	911
912	831744	758550528	30·199337	9·69761	912
913	833569	761048497	30·215889	9·70115	913
914	835396	763551944	30·232432	9·70469	914
915	837225	766060875	30·248966	9·70823	915
916	839056	768575296	30·265491	9·71177	916
917	840889	771095213	30·282007	9·71530	917
918	842724	773620632	30·298514	9·71883	918
919	844561	776151559	30·315012	9·72236	919
920	846400	778688000	30·331501	9·72588	920
921	848241	781229961	30·347981	9·72941	921
922	850084	783777448	30·364452	9·73293	922
923	851929	786330467	30·380915	9·73644	923
924	853776	788889024	30·397368	9·73996	924
925	855625	791453125	30·413812	9·74347	925
926	857476	794022776	30·430248	9·74698	926
927	859329	796597983	30·446674	9·75049	927
928	861184	799178752	30·463092	9·75399	928
929	863041	801765089	30·479501	9·75750	929
930	864900	804357000	30·495901	9·76100	930
931	866761	806954491	30·512292	9·76449	931
932	868624	809557568	30·528675	9·76799	932
933	870489	812166237	30·545048	9·77148	933

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
934	872356	814780504	30·561413	9·77497	934
935	874225	817400375	30·577769	9·77846	935
936	876096	820025856	30·594117	9·78194	936
937	877969	822656953	30·610455	9·78542	937
938	879844	825293672	30·626785	9·78890	938
939	881721	827936019	30·643106	9·79238	939
940	883600	830584000	30·659419	9·79586	940
941	885481	833237621	30·675723	9·79933	941
942	887364	835896888	30·692018	9·80280	942
943	889249	838561807	30·708305	9·80627	943
944	891136	841232384	30·724583	9·80973	944
945	893025	843908625	30·740852	9·81319	945
946	894916	846590536	30·757113	9·81665	946
947	896809	849278123	30·773365	9·82011	947
948	898704	851971392	30·789608	9·82357	948
949	900601	854670349	30·805843	9·82702	949
950	902500	857375000	30·822070	9·83047	950
951	904401	860085351	30·838287	9·83392	951
952	906304	862801408	30·854497	9·83736	952
953	908209	865523177	30·870698	9·84081	953
954	910116	868250664	30·886890	9·84425	954
955	912025	870983875	30·903074	9·84769	955
956	913936	873722816	30·919249	9·85112	956
957	915849	876467493	30·935416	9·85456	957
958	917764	879217912	30·951575	9·85799	958
959	916681	881974079	30·967725	9·86142	959
960	921600	884736000	30·983866	9·86484	960
961	923521	887503681	31·0	9·86827	961
962	925444	890277128	31·016124	9·87169	962
963	927369	893056347	31·032241	9·87511	963
964	929296	895841344	31·048349	9·87853	964
965	931225	898632125	31·064449	9·88194	965
966	933156	901428696	31·080540	9·88535	966
967	935089	904231063	31·996623	9·88876	967
968	937024	907039232	31·112698	9·89217	968
969	938961	909853209	31·128764	9·89558	969
970	940900	912673000	31·144823	9·89698	970
971	942841	915498611	31·160872	9·90238	971
972	944784	918330048	31·176914	9·90578	972
973	946729	921167317	31·192947	9·90917	973
974	948676	924010424	31·208973	9·91257	974
975	950625	926859375	31·224990	9·91596	975
976	952576	929714176	31·240998	9·91935	976
977	954529	932574833	31·256999	9·92273	977
978	956484	935441352	31·272991	9·92612	978
979	958441	938313739	31·288975	9·92950	979
980	960700	951192000	31·304951	9·93288	980

Number.	Square.	Cube.	Square Root.	Cube Root.	Number.
981	962361	944076141	31·320919	9·93626	981
982	964324	946966168	31·336879	9·93963	982
983	966289	949862087	31·352830	9·94300	983
984	968256	952763904	31·368774	9·94637	984
985	970225	955671625	31·384709	9·94974	985
986	972196	958585256	31·400636	9·95311	986
987	974169	961504803	31·416556	9·95647	987
988	976144	964430272	31·432467	9·95983	988
989	978121	967361669	31·448370	9·96319	989
990	980100	970299000	31·464265	9·96655	990
991	982081	973242271	31·480152	9·96990	991
992	984064	976191488	31·496031	9·97326	992
993	986049	979146657	31·511902	9·97661	993
994	988036	982107784	31·527765	9·97995	994
995	990025	985074875	31·543620	9·98330	995
996	992016	988047936	31·559467	9·98664	996
997	994009	991026973	31·575306	9·98999	997
998	996004	994011992	31·591138	9·99332	998
999	998001	997002999	31·606961	9·99666	999
1000	1000000	1000000000	31·623776	10·	1000

SILVER, TO PURIFY AND REDUCE.—Silver, as used in the arts and coinage, is alloyed with a portion of copper. To purify it, dissolve the metal in nitric acid slightly diluted, and add common salt, which throws down the whole of the silver in the form of chloride. To reduce it into a metallic state several methods are used: 1. The chloride must be repeatedly washed with distilled water, and placed in a zinc cup; a little diluted sulphuric acid being added, the chloride is soon reduced. The silver when thoroughly washed is quite pure. In the absence of a zinc cup, a porcelain cup containing a zinc plate may be used. The process is expedited by warming the cup.

2. Digest the washed chloride with pure copper and ammonia. The quantity of ammonia need not be sufficient to dissolve the chloride. Leave the mixture for a day, then wash the silver thoroughly.

3. Boil the washed and moist chloride in solution of pure potash, adding a little sugar: when washed it is quite pure.

WELDING COMPOSITION.—Mix borax with $\frac{1}{10}$th of sal ammoniac, fuse the mixture, and pour it on an iron plate. When cold, pulverise it, and mix it with an equal weight of quick lime, sprinkle it on iron, which is heated to redness, and replace it in the fire. It may be welded below the usual heat.

BLACKING RECIPES.

Liquid Blacking, for Boots and Shoes.—1. Ivory black, 8 oz.; molasses, 2 oz.; sweet oil, ½ oz. Mix to form a paste. Add gradually ½ oz. of oil of vitriol, and then half a pint of vinegar, and 1½ pint of water, or sour beer. Some prefer mixing the oil of vitriol with the sweet oil.

2. Ivory black, 2 lbs.; molasses, 2 lbs.; sweet oil, ½ lb. Mix, and add ½ lb. oil of vitriol, and enough beer or vinegar to make up a gallon.

3. Ivory black, 3 lbs.; molasses, 4 lbs.; vinegar, 1 pint; oil of vitriol, 8 oz.; water, 1 gallon.

4. Ivory black, 2 lbs.; neat's-foot oil, 4 oz. Mix, and add 3 quarts of sour beer or vinegar, and a spoonful of any kind of spirits; stir till smooth, and add 2 oz. of oil of vitriol, and sprinkle on it ½ drachm of powdered resin. Then boil together 3 pints of sour ale with a little logwood, and ½ oz. of Prussian blue, 8 oz. of honey, and 8 oz. of molasses. Mix, but do not bottle it for two or three days.

5. Ivory black, 8 oz.; brown sugar, or molasses, 8 oz.; sweet oil, 1 oz.; oil of vitriol, ½ oz.; vinegar, two quarts. Mix the oil with the molasses, then add the oil of vitriol and vinegar, and lastly the ivory black.

Blacking for Dress Boots.—1. Gum, 8 oz.; molasses, 2 oz.; ink, 1 pint; vinegar, 2 oz.; spirit of wine, 2 oz. Dissolve the gum and molasses in the ink and vinegar, strain, and add the spirit.

2. To the above add 1 oz. of sweet oil, and ½ oz. of lampblack. [These are applied with a sponge, and allowed to dry out of the dust. They will not bear the wet.]

3. Beat together the whites of 2 eggs, a table-spoonful of spirit of wine, a lump of sugar, and a little finely powdered ivory black to thicken.

Blacking, without Polishing.—Molasses, 4 oz.; lampblack, ½ oz.; yeast, a table-spoonful; 2 eggs; a tea-spoonful of olive oil; a tea-spoonful of turpentine. Mix well. To be applied with a sponge, without brushing.

India Rubber Blacking.—Ivory black, 60 lbs.; molasses, 45 lbs.; vinegar (No. 24), 20 gallons; powdered gum, 1 lb; India rubber oil, 9 lbs. (The latter is made by dissolving, by heat, 18 oz. of India rubber in 9 lbs. of rape oil.) Grind the whole smooth in a paint mill. Then add, by small quantities at a time, 12 lbs. of oil of vitriol, stirring it strongly for half an hour a day for a fortnight.

Paste Blacking.—1. Oil of vitriol, 2 parts; sweet oil, 1 part; molasses, 3 parts; ivory black, 4 parts. Mix.

2. This may be made with the ingredients of liquid blacking, using sufficient vinegar, in which a little gum has been dissolved, to form a paste. Make it into cakes, and dry it.

3. (Bailey's Blacking Balls.) Bruised gum tragacanth, 1 oz. water, 4 oz. Mix, and add 2 oz of neat's-foot oil, 2 oz. of fine ivory black, 2 oz. of Prussian blue. Mix, and evaporate to a proper consistence.

Blacking for Harness.—1. Isinglass or gelatine, ¼ oz.; powdered indigo, ¼ oz; soft soap, 4 oz.; logwood, 4 oz.; glue, 5 oz. Boil together in 2 pints of vinegar, till the glue is dissolved; then strain through a cloth, and bottle for use.

2. Melt 8 oz. of beeswax in an earthen pipkin, and stir into it 2 oz. of ivory black, 1 oz. of Prussian blue ground in oil, 1 oz. of oil of turpentine, and ½ oz. of copal varnish. Make it into balls. To be applied with a brush, and polished with an old handkerchief.

3. Molasses ½ lb.; lampblack, 1 oz.; yeast, 1 spoonful; of sugar candy, olive oil, gum tragacanth, and isinglass, 1 oz. each; a cow's gall. Mix all together with 2 pints of stale beer, and let it stand before the fire for an hour.

Heel Balls.—1. Melt together 4 oz. of mutton suet, 1 oz. of beeswax, 1 oz. of sweet oil, ½ oz. oil of turpentine, and stir in 1 oz. of powdered gum arabic, and ½ oz. of fine lampblack.

2. Beeswax, 8 oz.; tallow, 1 oz.; powdered gum, 1 oz.; lampblack, q. s.

Heel balls are used not merely by the shoemaker, but to copy inscriptions, raised patterns, &c., by rubbing the ball on paper laid over the article to be copied.

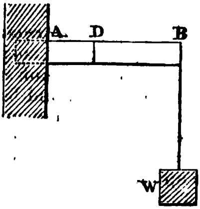
BLACKLEAD PENCILS.—The easiest way of producing, not only blacklead, but all sorts of pencils, is by the following process, which at once combines simplicity, cheapness, and the finest quality.

Take white or pipe-clay: put it into a tub of clean water, to soak for twelve hours, then agitate the whole, until it resembles milk, let it rest two or three minutes, and pour off the supernatant milky liquor into a second vessel, allow it to settle, pour off the clear, and dry the residue on a filter. Then add blacklead, any quantity. Powder it, and calcine it at a white heat in a loosely covered crucible, cool, and carefully pulverize, then add prepared clay, prepared plumbago, equal parts. Water to mix. Make them into a paste, and put it into oiled moulds of the size required, dry very gradually, and apply sufficient heat to give the required degree of hardness; lastly, the pieces should be taken carefully from the moulds, and placed in the grooves of the cedar. The more clay and heat employed the harder the crayon; less clay and heat of course produces a contrary effect. The shade of black may also be varied in the same way. Each mould must be made of four pieces of wood, nicely fitted together.

BLACK FOR MINIATURE PAINTERS.—Take camphor, and set it on the fire, and collect the soot by means of a saucer or paper funnel inverted over it.

STRAIN AND STRESS OF MATERIALS.

Fig. 1.



Let $A\ B$ be a beam of timber, firmly fixed in a wall at A, and a weight, W, measured in pounds avoirdupois, acting at the extremity B, at right angles to $A\ B$.

If $A\ B$ be one foot, and the weight W be one pound, then the strain produced at A is called a *unit of strain.*

If the beam $A\ B$ be (l) feet long, and the weight be (W) pounds, then the *units of strain* produced at A, by the weight acting at B, will be $l\ W$. And the *units of strain* which the weight W produces at any other part of the beam D, are measured by $W.\ B\ D$.

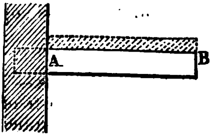
Let $A\ B = 10$ feet, and the weight W be equal to 112 lbs., and $B\ D = 7$ feet.

The *units of strain* at $A = 112 \times 10 = 1120$.

The *units of strain* at $D = 112 \times 7 = 784$.

The greatest strain on the beam is at A, at which place the beam would break if it was equally strong throughout.

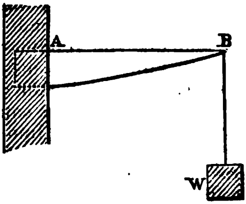
Fig. 2



If the weight W be uniformly distributed over the whole length of the beam $A\ B$, as in fig. 2, the *units of strain* at A will be only one-half as great as that produced by the weight W acting as in fig. 1.

The *units of strain* at A, which are produced by the beam itself, are equal to the weight of the beam multiplied by half its length.

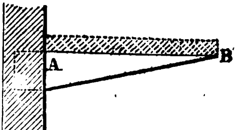
Fig. 3.



The beam $A\ B$, fig. 3, is equally strong between the points A and B, when the underside of it is a common parabola.

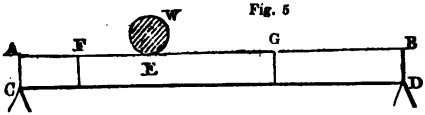
Hence, from a square beam, one-third part of it may be cut off without diminishing its strength.

Fig. 4.



If the weight W be uniformly distributed over the whole length of the beam $A\ B$, as in fig. 4; then the beam is equally strong when the underside of it is a straight line. In this case, one half the beam may be cut away without diminishing its strength.

Fig. 5



Let the weight W (fig. 5) be sustained by a beam $A\ B$, which rests on two props at C and D.

The pressure on the prop at C is equal to $W.\ B\ E : A\ B$.

The pressure on the prop at D is equal to $W.\ A\ E : A\ B$.

The *units of strain* at E are equal to $W.\ A\ E.\ B\ E : A\ B$.

The *units of strain* at G are equal to $W.\ A\ E.\ B\ G : A\ B$.

The *units of strain* at F are equal to $W.\ B\ E.\ A\ F : A\ B$.

The greatest strain, which is produced by the weight W, is at E

The *units of strain* at the middle of the beam, produced by the weight W acting at E, are equal to $\frac{W.\ A\ E.}{2}$.

Let $A\ B = 18$ feet, and a weight of 112 lbs. be placed at E which is 8 feet from A.

Apply these numbers to the above formulæ and their results.

The pressure on the prop at C is equal to $\frac{10 \times 112}{18} = 62.5$ lbs.

The pressure on the prop at D is equal to $\frac{8 \times 112}{18} = 49.8$ lbs.

The *units of strain* at E are equal to $\frac{10 \times 8 \times 112}{18} = 497.77$.

The *units of strain* on the middle are equal to $\frac{8 \times 112}{2} = 448$.

When the weight W is laid on the middle of the beam $A\ B$, the *units of strain* on the middle are equal to $\frac{W.\ A\ B}{4}$.

If the weight W be uniformly distributed along the beam $A\ B$, the *units of strain* on the middle of it will be equal to $\frac{W.\ A\ B}{8}$ which is only one half the strain that is produced by the weight having been laid on the middle.

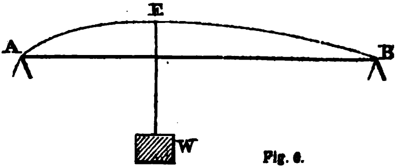
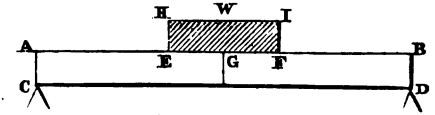


Fig. 6.

When the beam $A\ B$ (fig. 6), supports a weight W, at E, it is equally strong between the points A and B, if the upper sides, $A\ E$, $B\ E$, be two parabolas whose vertex is A and B respectively.

Fig. 7.



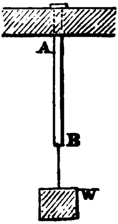
Let the weight W have a bearing $E\ F$ (fig. 7), equal on both sides of the centre G, and also let the weight be equally distributed on the bearing $E\ F$.

The *units of strain* at G are equal to $\dfrac{W.A\,B}{4} - \dfrac{W.E\,F}{8}$

Now, if the weight W were a sphere, and were laid on the middle of the beam at G, the *units of strain* at G would be equal to $\dfrac{W.A\,B}{4}$.

If the same weight be formed into a cube, whose side is $E\ F$, the *units of strain* at the centre G will be less than in the case of the sphere by $\dfrac{W.E\,F}{8}$.

Fig. 8.



Let $A\ B$ be any beam suspended vertically from the point A (fig. 8): and let the sectional area be constant from A to B, where a weight W lbs. is acting to extend the beam.

Put a = area of the section of the beam in square inches.

l = length of the beam in feet before the weight is applied to elongate it.

e = the elongation produced by the weight W.

E = weight which would be necessary to make e equal to l. The quantity E is called the modulus of elasticity of the material of which the beam is composed.

In the case of the beam being compressed by the weight W acting in the opposite direction,

Put c = compression produced by the weight W.

C = force which is necessary to make c equal to half of (l). The quantity C is called the modulus of elasticity of the material, when it is subject to compression.

$$E = \frac{W\,l}{a\,e} \text{ and } C = \frac{W\,l}{a\,c}$$

Units of work done to elongate the beam e feet $= \dfrac{W\,e}{2}$.

Units of work done to compress the beam c feet $= \frac{Wc}{2}$.

Mean results of experiments on four different kinds of Cast-iron bars, 10 feet long and 1 square inch in section.

Weight laid on bar per square inch $= W$.	Extension of bar in inches $= 12\,e$.	Set of bar in inches.	The value of $\frac{12\,W}{e}$.
lbs.	inches.		
1054	.009	....	117085
1581	.0137	.00022	115131
2108	.0186	.00055	113308
3161	.0287	.00107	110150
4215	.0391	.00175	107802
5269	.0500	.00265	105377
6323	.0613	.00372	103142
7376	.0734	.00517	100496
8430	.0859	.00664	98139
9484	.0995	.00844	95316
10538	.1136	.01062	92762
11591	.1283	.01306	90347
12645	.1448	.01609	87329
13700	.1668	.02097	82133
14793	.1859	.02410	79576

Hence, the breaking weight per square inch of section is 14793 lbs. = 6.6 tons nearly; and the ultimate extension is .1859 inches, or $\frac{1}{645}$ of the whole length, 10 feet.

If we deduct the set ·0209 from ·1859, we shall have ·165 inches for the elongation produced by the weight 14793 lbs.

$$\therefore E = \text{modulus of elasticity} = \frac{14793 \times 10 \times 12}{.165} = 10758545.$$

$\therefore$ **Breaking weight = 6.6 tons × area of section in square inches.**

If the weight 5269 be taken, the modulus of elasticity will be considerably increased. Deduct .00175 the set from .05, leaving .04825 inches for the elongation due to the weight 5269 lbs.

$$\therefore E = \text{modulus of elasticity} = \frac{5269 \times 10 \times 12}{.04825} = 13104249.$$

This difference in the modulus of elasticity arises from the circumstance of the law of elasticity not being proportional to the weight.

TABLE
Of the Tensile Strength of Wrought Iron.
The Bar was 10 feet long and 1 square inch section.

Weight laid on the Bar W.	Extension of the Bar or value of $12\,e$.	Set of Bar.	The value of $\frac{12\,W}{e}$
lbs.	inches.	inches.	
1262	·00520		242665
3785	·01690	·0005	223998
6809	·02772	·0005	227608
8833	·03790	·0005	233061
11856	·04854	·0005	233966
13880	·05950	·0007	233285
16404	·06980	·0007	235016
18928	·08170	·00130	231675
21452	·09310	·00270	230415
23975	·10570	·00410	226824
26499	·12040	·00680	220092
29023	·14500	·0120	200157
30284	·19910	·0120	
	·23660	·1082	after bearing the weight 17 hours.
31546	·24200	·1083	130357
ditto	·24490	·1111	after five minutes.
35332	2·04	1·874	17320

The bar broke with a weight of 24 tons per square inch of section. Hence the tensile force of wrought iron is nearly four times as great as the tensile force of cast iron.

TABLE
Of the Compressive Strength of Wrought Iron.
The Bar was 10 feet long and 1 square inch section.

Weight laid on the Bar, or (W).	Decrement of length, or the value of $12\,c$.	Weight laid on the bar, or (W).	Decrement of length, or the value of $12\,c$.
lbs.	inches.	lbs.	inches.
5098	·028	23018	·119
9578	·052	25258	·130
14058	·073	27498	·142
16298	·085	29738	·154
18538	·096	31978	·174
20778	·107	34218	·214

The crushing force of wrought iron is 12 tons per square inch. It is a curious fact, that cast iron is decreased in length nearly double what wrought iron is, by the same weight; but the wrought iron bar will sink to any degree with little more than 12 tons per square inch, whilst cast iron will bear 43·56 tons to produce the same effect.

A wrought bar will bear a compression of $\frac{1}{565}$ of its length, without its utility being destroyed.

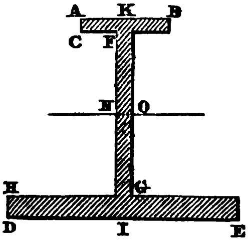
Compression of Cast Iron.

Mean results of experiments on four different kinds of Cast Iron, 10 feet long, and 1 square inch in section.

Weight laid on the bar (W).	Decrement of length, or the value of 12 c.	Set of bar in inches.	The value of $\frac{12\,W}{c}$.
lbs.	inches.	inches.	
2065	·01875	·00047	110119
4129	·03878	·00226	106485
6194	·05978	·00400	103617
8259	·07879	·00645	104822
10324	·09944	·00847	103819
12388	·12030	·010875	102980
14453	·14163	·01405	102049
16518	·16338	·01712	101101
18583	·18505	·02051	100420
20464	·20624	·02484	100114
24777	·24961	·03220	99263
28906	·29699	·04300	97331
33031	·35341	·06096	93463

The crushing or compressive force of cast iron per square inch is 43·56 tons, which has been obtained from eleven kinds of cast iron. But the tensile force of cast iron is 6·6 tons; therefore the compressive force is equal to the square of the tensile force, or $(6{\cdot}6)^2$.

Transverse Strength of Beams.



To find the neutral line, forces of extension, forces of compression, moments of extension, and moments of compression of a beam subject to transverse flexure.

Let the form of the section of the beam be that of the figure $A\,B\,D\,E$, where BC, HE, represent sections of the top and bottom ribs, FG that of the vertical one connecting them, and $N\,O$ pass through the neutral line.

Put a, $a' = N\,I$, $N\,K$, respectively.

$c, c' = D\,H,\ A\,C$, respectively.
$b, b' = D\,E,\ A\,B$, do.
$\beta =$ the thickness of the vertical rib.

$f, f' =$ tensile and compressive forces of the material, in a square inch of section, as exerted at a distance (a) on opposite sides of the neutral line.

For the determination of the neutral line

$$f\left\{ba^2 - (b-\beta)\,(a-c)^2\right\} = f'\left\{b'\,a'^2 - (b'-\beta)\,(a'-c')^2\right\}$$

And $a + a' = D$, where D is the whole depth of the beam.

For moderate strains per square inch $f = f'$

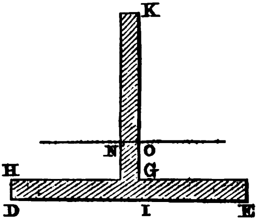
$$\therefore\ ba^2 - (b-\beta)\,(a-c)^2 = b'\,(D-a)^2 - (b'-\beta)\,(D-a-c')^2$$

$$\text{Moments of extension} = \frac{f}{3\,a}\left\{ba^3 - (b-\beta)\,(a-c)^3\right\}$$

$$\text{Moments of compression} = \frac{f}{3\,a}\left\{b'\,a'^3 - (b'-\beta)\,(a'-c')^3\right\}$$

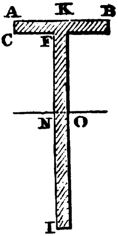
If W be the weight laid on the middle, and l equal length between supports,

$$\therefore\ \frac{W\,l}{4} = \frac{f}{3\,a}\left\{ba^3 + b'a'^3 - (b-\beta)\,(a-c)^3 - (b'-\beta)\,(a'-c')^3\right\}$$



If the form of the section be this,
Then $b' = \beta$
Therefore, for the neutral line
$$b\,a^2 - (b-\beta)\,(a-c)^2 = \beta\,(D-a)^2.$$
Moment of extension
$$= \frac{f}{3\,a}\left\{b\,a^3 - (b-\beta)\,(a-c)^3\right\}$$
$$\text{Moment of compression} = \frac{f\,\beta\,a'^3}{3\,a}$$

$$\text{And } \frac{W\,l}{4} = \frac{f}{3\,a}\left\{ba^3 + \beta\,a'^3 - (b-\beta)\,(a-c)^3\right\}$$



If the form of the section be this,
Then $b = \beta$
Therefore, for the neutral line
$$\beta\,a^2 = b'\,(D-a)^2 - (b'-\beta)\,(D-a-c)^2$$

$$\text{Moment of extension} = \frac{f\,\beta\,a^3}{3}$$

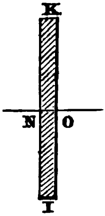
Moment of compression

$$= \frac{f}{3a}\left\{ b' a'^3 - (b' - \beta)(a' - c')^3 \right\}$$

$$\text{And } \frac{Wl}{4} = \frac{f}{3a}\left\{ \beta a^3 + b' a'^3 - (b' - \beta)(a' - c')^3 \right\}$$

If the form of the section be this,

Then, $b = \beta$ and $b' = \beta$



Therefore, for the neutral line

$$2a = D$$

or the neutral line is in the middle of the section.

$$\text{Moment of extension} = \frac{f\beta D^2}{12}$$

$$\text{Moment of compression} = \frac{f\beta D^2}{12}$$

$$\therefore Wl = \frac{2 f\beta D^2}{3}$$

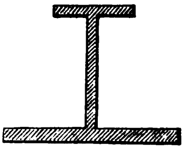
Transverse Strength of Cast-Iron Bars.

Length of Bar between supports, with its dimensions.	Breaking weight laid on middle.	Ultimate deflexion in inches.	Mean of experiments.
	lbs.		
$4\frac{1}{2}$ feet, with 1 inch square	440	1·779	3
9 feet, with 2 inches square	1338	3·0035	6
$13\frac{1}{2}$ feet, with 3 inches square	2861	4·667	5
$6\frac{3}{4}$ feet, with 3 inches square	6117	1·2916	3

From the three last experiments we find $\frac{2f}{3} = 1490.$

$$\therefore W = 1490 \times \frac{\beta D^2}{l}$$

For a cast-iron beam, where W is the breaking weight in lbs., β is the breadth of the beam measured in inches, D the depth of the beam measured in inches, and l the length of beam between supports measured in feet.



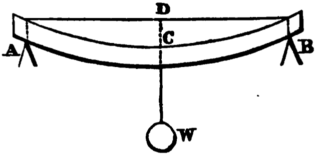
The best dimensions of a beam, whose section is given in the figure, are when the bottom flange contains six times as much area as the top flange. And the breaking weight of such beams may be found by the following admirable rule:

Multiply the sectional area of the bottom flange in square inches, by the depth of the beam in inches, and divide the product by the distance between the supports, measured in feet, then 2·14 times the quotient will give the breaking weight in tons.

A cast-iron bar is not weakened by passing half the breaking weight over it 96,000 times, with a velocity of 81 feet per minute.

Deflection of Beams.

Let the beam be supported at A and B, and weight W applied at the middle C.



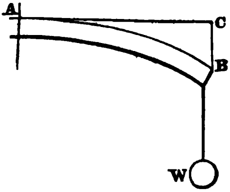
$$\therefore DC = \frac{W l^3}{4 E \beta D^3}$$

$E =$ the modulus of elasticity of the material.

$\beta =$ breadth of beam in in.

$D =$ depth of beam in inches.

l = length of beam in inches.



Let the beam be supported at A, and a weight W applied at the other extremity.

$$\therefore BC = \frac{4 W l^3}{E \beta D^3}$$

Rule for finding the ultimate deflexion of a cast-iron beam:

Ultimate deflexion DC in inches $= \dfrac{3 l^2}{40 \cdot D}$ for first figure.

Ultimate deflexion BC in inches $= \dfrac{6 l^2}{5 D}$ for second figure,

where l is measured in feet and D in inches.

These values for the ultimate deflexion are independent of the breadth of the beam.

Find the ultimate deflexion of a cast-iron bar, the distance between the supports being 24 feet, and depth $4\frac{1}{2}$ inches.

$$\text{Ultimate deflexion} = \frac{3}{40}\,\frac{l^2}{D^2} = \frac{3 \times 24^2}{40 \times 4\frac{1}{2}} = 9{\cdot}6 \text{ inches.}$$

If the weight W be uniformly distributed along the beam, deflexion will be in all cases $\frac{5}{8}$ of the deflexion which is produ by the weight acting on the middle, or in the case of having o one support, acting at the extremity.

Transverse Flexure of a Wrought-Iron Bar by Pressure acting Horizontally.

Length of bar 14 feet $7\frac{1}{2}$ inches, depth of bar in direction pressure 1·515 inches, breadth 5·523 inches, distance betw supports 13 feet 6 inches. The experiment was continued to limit of perfect elasticity, or to that point at which the elastic was sensibly injured.

Weight applied, acting horizontally.	Deflexions after five minutes.	Sets after five minutes.	Ratio of weights to deflexions.
lbs.	inches.	inches.	
28	·051	·0	549
56	·112	·0	500
112	·232	·0	483
168	·344	·001	488
224	·458	·002	489
280	·571	·003	490
336	·684	·003	491
392	·800	·004	490
448	·916	·006	489
504	1·005	·007	501
560	1·124	·008	498
616	1·222	·010	504
672	1·332	·011	504
728	1·434	·017	508
784	1·547	·019	507
840	1·693	·019	496
896	1·823	·019	492
952	1·933	·020	493
1008	2·044	·021	493
1064	2·165	·022	491

Mean 498.

To find the weight which a wrought-iron beam is capable of bearing without injuring its elasticity.

$$W = \frac{1078\,\beta\,D^2}{l}\text{ lbs.} = \frac{\beta\,D^2}{2\,l}\text{ tons, nearly.}$$

β and D are measured in inches, and l in feet, being the distance between the supports.

What is the weight that can be laid on a wrought-iron bar, 20 feet long, 3 inches broad, and 6 inches deep, without injuring its elasticity?

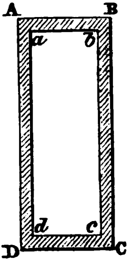
$$\therefore W = \frac{3 \times 36}{40} = \frac{108}{40} = 2{\cdot}7 \text{ tons.}$$

The deflexion of a wrought-iron beam, supported at each end, and loaded in the middle, when the elastic limit is obtained.

$$\text{Deflexion in inches} = {\cdot}0167 \times \frac{l^2}{D}.$$

The length, l, is measured in feet, and D, the depth, in inches.
Taking the bar given in the last example,

$$\text{Deflexion} = {\cdot}0167 \times \frac{400}{6} = 1{\cdot}11 \text{ inches.}$$



Hollow Rectangular Beams.

Let $A\,B\,C\,D$ be the section of a hollow rectangular beam.

Let $A\,D = D$, and $a\,d = d$
$A\,B = B$, and $a\,b = b$

$$\therefore W\,l = \frac{2\,f}{3\,D}\left\{B\,D^3 - b\,d^3\right\}$$

where W is the weight applied at the middle between the supports, and f is a constant depending on the nature of the material.

FLUID FOR ETCHING ON COPPER.—Verdigris 4 parts; salt 4; sal ammoniac 4; alum 1; water 16; strong vinegar 12. Dissolve with heat.

ACID FOR ETCHING ON STEEL.—Pyroligneous acid 5 parts; alcohol 1; nitric acid 1. Mix the first two, then add the nitric acid.

TABLE

Of Experiments on the Transverse Strength of Rectangular Tubes of Wrought-Iron, supported at each end, and the weight laid on the middle.

Distance between the supports.	Weight of tubes between the supports.	Breaking weights, exclusive of the weights of the tubes.	External depth of the tubes.	External breadth of the tubes.	Thickness of the plates of the tubes.
Feet.		Tons.	Inches.	Inches.	Inches.
30·0	42·62 cwt.	57·5	24	16	·525
7·5	72·36 lbs.	4·454	6	4	·1325
30·0	23·09 cwt.	22·84	24	16	·272
7·5	35·53 lbs.	1·409	6	4	·065
3·75	9·65 "	1·1	3	2	·061
3·75	4·34 "	·3	3	2	·03
45·0	130·36 cwt.	114·76	36	24	·75
3·75	9·65 lbs.	1·1	3	2	·061
30·0	39 cwt.	54·3	24	16	·50

In several of these experiments the tubes gave way by the metal at the top becoming wrinkled.

In similar tubes the strength, and consequently the breaking weight, is proportional to (1·9) power of the lineal dimensions.

From these experiments the breaking weight may be obtained as follows:

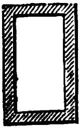
$$W = \frac{3}{4\,l\,D}\left\{ B\,D^3 - b\,d^3 \right\} \text{ in tons.}$$

The breadths and depths are measured in inches, and the length in feet.

If the thickness of the metal be equal to t inches completely round the section,

Then, $W = \frac{3}{4\,D\,l}\left\{ B\,D^3 - (B - 2\,t)(D - 2\,t)^3 \right\}$

the breaking weight in tons for a wrought-iron tube, whose form of section is . . . .

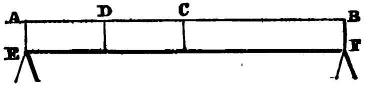


What is the breaking weight of a rectangular tube 40 feet long, depth 2 feet 6 inches, thickness of plate $\frac{1}{4}$ inch, and breadth 18 inches?

$$W = \frac{3}{4800}\left\{ 18 \times 30^3 - 17{\cdot}5 \times 29{\cdot}5^3 \right\}$$

$$= \frac{1}{1600} \left\{ 486000 - 449267 \right\} = 22 \cdot 96 \text{ tons.}$$

From a great number of well arranged experiments, on the strength of iron beams and tubes, it follows that they may be safely reduced in strength from the middle towards the extremities in the ratio indicated by theory.



Let $A\,B$ be a beam supported at its extremities E and F, and put F equal to the necessary strength at the middle of the beam.

Then, the necessary strength at $D = F \times \dfrac{A\,C^2 - C\,D^2}{A\,C^2}$

The tensile force of wrought iron is to its compressive force as 2 to 1.

Hence, the plate on the upper side of hollow wrought-iron tubes should contain an area twice as great as the plate on the under side.

Strength of Cast-Iron Pillars.

The breaking weight of solid cylindrical cast-iron pillars.

In solid pillars, with their ends rounded, and moveable,

$$\text{Breaking weight in tons} = 14 \cdot 9 \times \frac{d^{3 \cdot 6}}{l^{1 \cdot 7}} \quad . \quad . \quad . \quad (1)$$

In solid pillars, with their ends flat, and incapable of motion,

$$\text{Breaking weight in tons} = 44 \cdot 16 \times \frac{d^{3 \cdot 6}}{l^{1 \cdot 7}} \quad . \quad . \quad . \quad (2)^*$$

where l is the length of pillar in feet, and d the diameter in inches.

In hollow pillars of cast-iron, where D, d are the external, internal diameters, and l the length: both ends of the pillar were moveable.

$$\text{Breaking weight in tons} = 13 \times \frac{D^{3 \cdot 6} - d^{3 \cdot 6}}{l^{1.7}}$$

In hollow cast-iron beams, whose ends were flat and firmly fixed,

$$\text{Breaking weight in tons} = 44 \cdot 3\, \frac{D^{3 \cdot 6} - d^{3 \cdot 6}}{l^{1 \cdot 7}}$$

Of three cylindrical pillars of steel, wrought and cast iron, and wood, all of the same length and diameter, the first having its ends

* Formula (1) was obtained from the mean result of eighteen pillars, varying in length from 121 times the diameter down to 15 times. The formula (2) was derived from eleven pillars, with flat ends, varying in length from 78 to 28 times the diameter.

rounded, the second with one end round and the other end flat, and the third with both ends flat, the strengths are as 1, 2, and 3.

These formula and results were obtained from experiments on pillars, varying in length from 121 times the diameter down to 15 times.

Effects of Temperature upon the Strength of Cast-Iron.

The strength of cast-iron is not reduced when its temperature is raised to 600°, which is nearly that of melting lead; and it does not differ very widely whatever the temperature may be, provided the bar be not heated so as to be red hot.

EXAMPLE.

Find the strength of a hollow cylindrical cast-iron pillar, 14 feet long, 6·2 inches external diameter, and 4·1 inches internal; the pillar being flat and well supported at the ends.

$$14^{1\cdot7} = 88\cdot801 \qquad 6\cdot2^{3\cdot6} = 712\cdot22 \quad \text{and} \quad 4\cdot1^{3\cdot6} = 160\cdot7$$

$$\therefore \text{ Breaking weight in tons} = 44\cdot3 \times \frac{D^{3\cdot6} - d^{3\cdot6}}{l^{1\cdot7}}$$

$$= 44\cdot3 \times \frac{712\cdot22 - 160\cdot7}{88\cdot801}$$

$$= 275$$

Comparative Strength of Long Pillars.

If the strength of cast-iron pillars be 1000, then wrought-iron will be 1745, cast-steel 2518, Dantzic oak 108·8, and red deal 78·5.

The strength of similar pillars is as the square of their linear dimensions.

Resistance to Torsion.

Let l = length of prism from the fixed end to the point of application of the lever used to twist it.

r = radius of prism, if round.

b, d = breadth and thickness, if rectangular.

W = the weight acting by means of the lever to twist the prism.

L = length of the lever to which the weight W is applied.

θ = angle of torsion.

R = resistance to torsion at the time of fracture.

C = constant for each species of body.

$$\pi = 3\cdot14159, \text{ \&c.}$$

For a cylinder,

$$2\,L\,l\,W = C\,\pi\,\theta\,r^4 \text{ and } 2\,W\,L = \pi\,R\,r.$$

For a square,

$$6\,L\,l\,W = C\,\theta\,d^4 \text{ and } 6\,W\,L = \sqrt{2}\cdot R\,d^3.$$

For a rectangle,

$$3\,L\,l\,W(b^2 + d^2) = C\,\theta\,b^3\,d^3 \text{ and } 3\,W\,L\,\sqrt{b^2 + d^2} = R\,b^2\,d^2$$

The Ultimate Resistance of a Cast-iron Beam to Torsion.

In a cylinder, $WL = 51055\ r^3$.

In a square prism, $WL = 7660\ d^3$.

In a rectangular prism, $W\ L = 10834\ \dfrac{b^2\ d^2}{\sqrt{b^2 + d^2}}$.

All the dimensions are taken in inches.

Strength of Ropes.

The cohesion of hempen fibres is 6400 lbs. for every square inch of section.

$$\text{Breaking weight in tons} = \frac{\text{circumference squared}}{4}$$

the circumference being measured in inches.

Ex.—Find the breaking weight of a rope 6 inches in circumference.

$$\text{Breaking weight} = \frac{36}{4} = 9 \text{ tons.}$$

For a common cable,

$$\text{Breaking weight in tons} = \frac{\text{circumference squared}}{5}$$

These are practical rules and easy of application.

PROCESSES FOR STAINING WOODS.

Mahogany Color (Dark).—Boil ½ lb. of madder and 2 oz. of logwood in a gallon of water; then brush the wood well over with the hot liquid. When dry, go over the whole with a solution of 2 drachms of pearlash in a quart of water.

Mahogany Color (Light).—Brush over the surface with diluted nitrous acid, and when dry apply the following, with a soft brush: Dragon's blood, 4 oz.; common soda, 1 oz.; spirit of wine, 3 pints. Let it stand in a warm place, shake it frequently, and then strain. Repeat the application until the proper color is obtained.

To Stain Maple a Mahogany Color.—Dragon's blood, ½ oz; alkanet, ¼ oz.; aloes, 1 dr.; spirit of wine, 16 oz. Apply it with a sponge or brush.

Rosewood.—Boil 8 oz. of logwood in 3 pints of water until reduced to half; apply it, boiling hot, two or three times, letting it dry between each. Afterwards put in the streaks, with a camel's hair pencil, dipped in a solution of copperas and verdigris in a decoction of logwood.

Ebony.—Wash the wood repeatedly with a solution of sulphat of iron; let it dry, then apply a hot decoction of logwood and nut galls for two or three times. When dry, wipe it with a wet sponge and when dry again, polish with linseed oil.

Red.—1. Take a pound of Brazil wood and mix it with a gallo of stale urine. Pour over the wood while boiling hot. Before i dries it should be laid over with alum water. 2. A fine red may als be obtained by a solution of dragon's blood in spirits of wine.

Yellow.—Nitric acid, lightly diluted, will produce a fine yello on wood. Sometimes, if the wood is not in proper condition, it wil create a brown. Care must be taken that the acid used be not to strong, or it will render the wood nearly black.

Blue.—Take of alum 4 parts; water 85 parts. Boil.

Purple.—To produce this color, take of logwood 11 parts; alun 3 parts; water 29 parts. Boil.

Mahogany.—1. Linseed oil 2 pounds; alkanet 3 ounces. Hea them together and macerate for six hours, then add resin 2 ounces beeswax 2 ounces. Boiled oil may be advantageously used insteas of the linseed oil.

2. Brazil-wood (ground); water sufficient; add a little alum an potash. Boil.

3. Logwood 1 part; water 8 parts. Make a decoction, and appl it to the wood; when dry, give it two or three coats of the follow ing varnish: dragon's blood 1 part; spirits of wine 20 parts. Mix

To take Stains out of Mahogany.—Spirits of salts 6 parts; sal of lemons 1 part. Mix, then drop a little on the stains, and rul them until they disappear.

To Stain Musical Instruments.—Crimson: Boil one pound o ground Brazil wood in three quarts of water for an hour; strain it and add half an ounce of cochineal; boil it again for half an hou gently, and it will be fit for use.

Purple: Boil a pound of chip logwood in three quarts of wate for an hour; then add four ounces of alum.

LOGARITHMS.

Logarithms literally signify *ratios of numbers;* hence Logarithmic Tables may be various, but those in common use for the facilitating of arithmetical operations generally are of the following corresponding progressions, viz.:—

Arithmetical, 0, 1, 2, 3, &c., or series of logarithms.
Geometrical, 1, 10, 100, 1000, &c., or ratio of numbers.

And thus it may be perceived, that if the log of 10 be 1, the log. of any number less than 10 must consist wholly of decimals, because increasing by a decimal ratio. Again; if the log. of 100

be 2, the log. of any intermediate number between 10 and 100 must be 1, with so many decimals annexed; and in like manner, the log. of any intermediate number between 100 and 1000, must be 2, with decimals annexed proportionally, as before.

Application and Utility of Common Logarithmic Tables.

The whole numbers of the series of logarithms, as 1, 2, 3, &c., are called the indices, or characteristics of the logarithm, and which must be added to the logarithm obtained by the table, in proportion to the number of figures contained in the given sum. Thus suppose the logarithm be required for a sum of only two figures, the index is 1; if of three figures, the index is 2; and if of four figures, the index is 3, &c.; being always a number less by unity than the number of figures the given sum contains.

EXAMPLES.

The index of 8 is 0, because it is less than 10.
The index of 80 is 1, because it is less than 100.
The index of 800 is 2, because it is less than 1000.
The index of 8000 is 3, because it is less than 10,000, &c.

The index of a decimal is always the number which denotes the significant figure from the decimal point, and is marked with the sign, thus, —, to distinguish it from a whole number

EXAMPLES.

The index of ·32549 is — 1, because the first significant figure is the first decimal.

The index of ·032549 is — 2, because the first significant figure is the second decimal.

The index of ·0032549 is — 3, because the first significant figure is the third decimal, &c., of any other sum.

If the given sum for which the logarithm is required contains or consists of both integers and decimals, the index is determined by the integer part, without having any regard to the other.

1. *To find the logarithm of any whole number under* 100.

Look for the number under N in the first page of any Logarithmic Table; then immediately on the right of it is the logarithm required, with its proper index. Thus the log. of 64 is 1·806180, and the log. of 72 is 1·857332.

2 *To find the logarithm of any number between* 100 *and* 1000, *or any sum not exceeding* 4 *figures.*

Find the first three figures in the left-hand column of the page under N, in which the number is situated, and the fourth figure, at the top or bottom of the page; then the logarithm directly under the fourth figure, and in a line with the three figures in the column on the left, with its proper index, is the logarithm required. Thus, the log. of 450 is 2·653213, and the log. of 7464 is 3·872972. Or, the log. of 378·5 is 2·578066, and that of ·7854 is — 1·895091.

3. *To find the number indicated by a given logarithm.*

Look for the decimal part of the given logarithm in the different columns, and if it cannot be found exactly, take the next less. Then under N in the left-hand column, and in a line with the logarithm found, are three figures of the number required, and on the top of the column in which the found logarithm stands is one figure more; place the decimal point as indicated by the logarithmic index, which determines the sum, properly valued, as required.

If the logarithm cannot be found exactly in the tables, subtract from it the next less that can be found, and divide the remainder by the tabular difference; the quotient will be the rest of the figures of the given number, which, being annexed to the tabular number already found, is the proper number required.

Ex. Required, the number answering to the logarithm 3·233568.

Given logarithm . . . . . = 3·233568
Next less is the log. of 1712 = 3·233504

Remainder 64

Tab. Diff. = 253, and $\frac{64}{253} = 2{\cdot}5$

Hence the number required = 1712·25.

For practical purposes in mechanics, logarithms are seldom resorted to, unless for the raising of the powers of numbers or extraction of their roots. These operations, when tables are at hand, they very much facilitate; involution or the raising of powers, being performed simply by multiplication, and evolution, or the extraction of roots, by division, as in simple arithmetic.

Ex. 1. Required, the square or second power of 25·791.

Log. of 25·791 = 1·411468
Multiplied by 2 the power required.

Logarithm 2·822936 indicated number or square required = 665·175.

Ex. 2. What is the cube of 30·7146?

Logarithm = 1·487345
Multiplied by 3 the power required.

Logarithm 4·462035 indicated number or cube required = 28975·7.

Ex. 3. Required, the square root of 365.

$$\text{Log.} = \frac{2{\cdot}562293}{2} = 1{\cdot}281146 \text{ indicated number or root} = 19{\cdot}105.$$

Ex. 4. Find the cube root of 12345.

$$\text{Log.} = \frac{4\cdot091491}{3} = 1\cdot363830 \text{ indicated number or root} = 23\cdot116.$$

For TABLE OF LOGARITHMS, see p. 483.

ENGRAVING IN ALTO-RELIEVO.—In the common operation of engraving, the desired effect is produced by making incisions upon the copper-plate with a steel instrument of an angular shape, which incisions are filled with printing-ink, and transferred to the paper by means of a roller, which is passed over its surface. There is another mode of producing these lines or incisions, by means of diluted nitrous acid, in which the impression is taken in the same way. Another method of engraving is done upon a principle exactly the reverse, for instead of the subject being cut into the copper, it is the interstice between the lines which is removed by diluted aquafortis, and the lines are left as the surface, from which the impression is taken by means of a common type-printing press, instead of a copper-plate press.

This is effected by drawing with common turpentine varnish, covered with lampblack, whatever is required upon the plate; and when the varnish is thoroughly dry, the acid is poured upon it, and the interstice of course removed by its action upon the uncovered part of the copper. If the subject is very full of dark shadows, this operation will be performed with little risk of accident, and with the removal of very little of the interstice between the lines; but if the distance between the lines is great, the risk and difficulty is very much increased, and it will be requisite to cut away the parts which surround the lines with a graver, in order to prevent the dabber with the printing-ink from reaching the bottom, and thus producing a blurred impression. It is obvious, therefore, that the more the plate is covered with work, the less risk there will be in the preparation of it with the acid, after the subject is drawn, and the less trouble will there be in removing the interstice, if any, from those places where there is little shading.

GLASS, SOLUBLE.—Mix ten parts of carbonate of potash, fifteen of quartz (or of sand free from iron of alumina), and one part of charcoal. Fuse together. The mass is soluble in four or five parts of water; and the filtered solution evaporated to dryness yields a transparent glass, permanent in the air.

TABLE

By which to Determine the various Distances of the Movable Points in a Parallel Motion.

Radius of beams in feet.	Parallel bars in feet.	Length of radius rods in feet and inches.	
4 feet.	2	2	0
	2¼	1	4⅜
	2½	0	10⅞
	2¾	0	6⅞
	3	0	4
4½ feet.	2	3	1½
	2¼	2	3
	2½	1	7¼
	2¾	1	1⅜
	3	0	9
	3¼	0	5¾
5 feet.	2	4	6
	2¼	3	4⅜
	2½	2	6
	2¾	1	10⅞
	3	1	4
	3¼	0	11¼
	3½	0	7¾
5½ feet.	2¼	4	8⅜
	2½	3	7¼
	2¾	2	9
	3	2	1
	3¼	1	6⅜
	3½	1	1¾
	3¾	0	9⅞
6 feet.	2¾	3	10
	3	3	0
	3¼	2	3⅞
	3½	1	9½
	3¾	1	4¼
	4	1	0
	4½	0	8½

Radius of beams in feet.	Parallel bars in feet.	Length of radius rods in feet and inches.	
6½ feet.	3	4	1⅛
	3¼	3	3
	3½	2	6⅞
	3¾	2	0¼
	4	1	6¾
	4¼	1	2¼
	4½	0	10½
7 feet.	3	5	4
	3¼	4	4
	3½	3	6
	3¾	3	2⅞
	4	2	3
	4¼	1	9
	4½	1	4⅜
	4¾	1	0⅞
	5	0	9⅜
7½ feet.	3½	4	6⅞
	3¾	3	9
	4	3	0¾
	4¼	2	6
	4½	2	0
	4¾	1	7
	5	1	3
	5¼	0	11½
8 feet.	3½	5	9½
	3¾	4	9¾
	4	4	0
	4¼	3	3¾
	4½	2	8⅝
	4¾	2	2¾
	5	1	9⅝
	5¼	1	5¼

Radius of beams in feet.	Parallel bars in feet.	Length of radius rods in feet and inches.	
8½ feet.	3¾	6	0¼
	4	5	6¾
	4¼	4	3
	4½	3	6⅜
	4¾	2	11½
	5	2	5⅜
	5¼	2	0⅛
	5½	1	7⅝
9 feet.	4	6	3
	4¼	5	3¾
	4½	4	6
	4¾	3	9⅜
	5	3	2½
	5¼	2	8⅛
	5½	2	2¾
	5¾	1	10
	6	1	6
9½ feet.	4½	5	6⅝
	4¾	4	9
	5	4	0⅝
	5¼	3	5⅜
	5½	2	10⅞
	5¾	2	5¼
	6	2	0½
10 feet.	4¾	5	9½
	5	5	0
	5¼	4	3½
	5½	3	8⅛
	5¾	3	1¾
	6	2	8
	6¼	2	3
	6½	1	10½

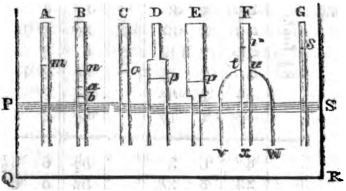
Radius of beams in feet.	Parallel bars in feet.	Length of radius rods in feet and inches.	
10½ feet.	5	6	0½
	5¼	5	3
	5½	4	6½
	5¾	3	11
	6	3	3¼
	6¼	2	10¾
	6½	2	5⅜
	6¾	2	1
11 feet.	5¼	6	3½
	5½	5	6
	5¾	4	9½
	6	4	2
	6¼	3	7⅜
	6½	3	1⅜
	6¾	2	8⅛
11½ feet.	5½	6	6½
	5¾	5	9
	6	5	0½
	6¼	4	5
	6½	3	10½
	6¾	3	4
	7	2	10⅝
	7¼	2	6
12 feet.	5¾	6	8¼
	6	6	0
	6¼	5	3½
	6½	4	7⅞
	6¾	4	1
	7	3	6⅞
	7¼	3	1⅜
	7½	2	8½
	7¾	2	4

CAPILLARY ATTRACTION.

If a number of glass tubes, open at both ends, be immersed, the water will rise to the same height in each tube, so long as the diameter of the tube exceeds the fifteenth of an inch; in all tubes less than this, the water will rise higher in the tube whose diameter is the least. Such tubes, whose diameters are less than one fifteenth of an inch, are called *capillary tubes*, from the Latin word *capillus*, signifying a hair.

Phenomena of Capillary Attraction.

Let $P\,Q\,R\,S$ be a vessel containing water to the line $P\,S$. The water will rise in the capillary tubes $A\,B\,C$ to the heights $m\,n\,o$, which are inversely proportional to their diameter. If B be broken at a, the water will not rise to the top of it, but will stand at b, a little below the top, whatever be the length or diameter of the tube. And, if the tube be taken out of the water and laid horizontally, the water will recede from the end that was immersed.



If a tube D be composed of two different bores, the water will rise to the height p; and if another tube, E, of the same form and size, be immersed, with its smaller end downwards, the water will rise in it to the same height p.

If the vessel $F\,v\,w$ be plunged into water, and by exhaustion the water is raised to the capillary tube $F\,t\,w$, it will afterwards ascend to the height r, which is just the same as in a capillary tube G of the same bore as $F\,t\,u$, and length $F\,x$.

In tubes of the same matter, immersed in the same fluid, the product of the elevations by the diameter is a constant quantity.

In a glass tube, immersed in water, this constant has been found by Muschenbrock, ·039; by Weitbrecht, ·0428; by Monge, ·042; by Atwood ·053.

From these numbers, the diameter of a tube may be found, in which the water will rise, by capillary attraction, the height 7 inches.

$$\text{Diameter} = \frac{\cdot 039}{7} = \cdot 0056 \text{ inches, nearly.}$$

The constant quantity, here referred to, is called the modulus of capillary attraction.

The following moduli are from Brewster; they were obtained

with a glass tube of ·0561 of an inch diameter, by means of an improved apparatus:

Name of Fluid.	Modulus.	Name of Fluid.	Modulus.
Water, ...	·0327	Oil of hyssop, . . .	·0195
Very hot water, . .	·0301	Oil of rosemary, . .	·0193
Muriatic acid, . . .	·0248	Oil of bergamot, . .	·0192
Oil of boxwood, . .	·0240	Oil of amber, . . .	·0192
Oil of cassia, . . .	·0236	Oil of anise seeds, . .	·0192
Nitrous acid, . . .	·0232	Oil of Barbadoes tar,	·0191
Oil of rapeseed, . .	·0227	Laudanum, ...	·0191
Castor oil, ...	·0226	Oil of cloves, . . .	·0187
Nitric acid, ...	·0223	Oil of turpentine, . .	·0187
Oil of spermaceti, . .	·0220	Oil of lemon, . . .	·0187
Oil of almonds, . . .	·0217	Oil of lavender, . .	·0184
Oil of olives, ...	·0215	Oil of camomile, . .	·0184
Balsam of Peru, . .	·0212	Oil of peppermint, .	·0184
Muriate of antimony,	·0209	Oil of sassafras, . .	·0184
Oil of rhodium, . .	·0205	Highland whisky, . .	·0184
Oil of pimento, . .	·0203	Brandy, ...	·0183
Cajeput oil, ...	·0200	Oil of wormwood, . .	·0183
Balsam of capivi, . .	·0200	Oil of dill seed, . . .	·0182
Oil of thyme, . . .	·0199	Oil of ambergris, . .	·0181
Oil of bricks distilled from spermaceti oil,	·0193	Oil of juniper, . . .	·0180
		Oil of nutmeg, . . .	·0180
Oil of caraway seeds,	·0198	Alcohol, ...	·0178
Oil of rue, ...	·0198	Oil of savine, . . .	·0174
Oil of spearmint, . .	·0197	Ether, ...	·0160
Balsam of sulphur, .	·0196	Oil of wine, ...	·0153
Oil of sweet fennel seeds, ...	·0195	Sulphuric acid, . . .	·0112

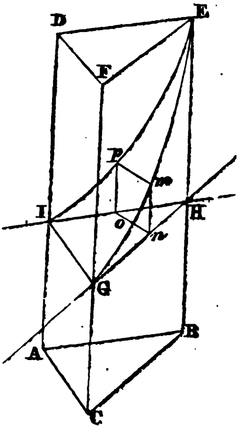
These experiments were made with a tube, carefully cleaned and dried after each experiment. A dry tube will raise the water to a less height than a wet one.

When capillary tubes are plunged into mercury, it falls instead of rising, as is the case with other fluids; and its fall is such, that when it is multiplied by the diameter of the tube, the product is a constant quantity ·015 (Cavendish).

When water is made to pass through a capillary tube of such a bore that the fluid is discharged only by successive drops, the tube, when electrified, will furnish a constant and accelerated stream; and the acceleration is proportional to the smallness of the bore. A jet of warm water will rise to a much greater height than a jet of cold water, though the water in both cases moved through the

same aperture, and was influenced by the same pressure. A syphon which discharges cold water only by drops, will furnish warm water in an uninterrupted stream.

Let *CEEB*, *ADEB*, be two plates of glass, having their sides *EB* joined together with wax, and their surfaces smooth and clean; and also their sides, *AD*, *CF*, separated slightly so as to form the angle *ABC*. If this apparatus be plunged in a vessel, so that *IHG* represent the water's surface, then the water will rise between the plates of glass, by capillary attraction, to the height *IEG*, so that the boundary of the water on the planes *FEBC*, *DEBA*, will be the hyperbolas *GE* and *IE*, having for their asymptotes the surface of the fluid and the line *EH*.



The height, *n m*, to which the water will rise, is regulated entirely by the same laws which prevail in the case of the tubes; calling the distance, *n o*, between the plates the diameter of the tube.

Hence the height, *n m*, is equal to the height in a tube whose diameter is equal to *n o*; and so on for any other point.

All phenomena of capillary attraction are exhibited equally both in air and in vacuo, and they are entirely independent of the thickness of the material composing the tubes and plates.

The elevation and depression is not proportional to the density of the liquid; water stands much higher in a glass tube than alcohol.

WOODS.

How to Polish Wood.

Take a piece of pumice-stone and water, and pass repeatedly over the work until the rising of the grain is cut down. Then take powdered tripoli and boiled linseed oil, and polish the work to a bright surface.

To Gather and Preserve Woods.

Woods should be gathered and exposed in a dry situation, to a heat of from 90° to 150° Fah., until sufficiently dry. The larger kinds are more easily chipped before drying.

To Preserve Woodwork.

Take boiled oil and finely powdered charcoal; mix to the consistence of paint, and give the woodwork two or three coats with it. This composition is well adapted for casks, water-spouts, &c.

To produce Figures on Wood.

Slack some lime in stale wine. Dip a brush in it, and form on the wood figures to suit your fancy. When dry, rub it well with a rind of pork.

STEAM-ENGINE.

To Estimate, by means of an Indicator, the Amount of Effective Power produced by a Steam-Engine.

Rule. Multiply the area of the piston in square inches by the average force of the steam in lbs., and by the velocity of the piston in feet per minute; divide the product by 33,000, and $\frac{7}{10}$ths of the quotient equal the effective power.

Ex. Suppose an engine with a cylinder of $37\frac{1}{2}$ inches diameter, a stroke of 7 feet, and making 17 revolutions per minute, or 238 feet velocity, and the average indicated pressure of the steam 16·73 lbs. per square inch; required the effective power.

$$\frac{\text{Area} = 1104{\cdot}4687 \text{ inches} \times 16{\cdot}73 \text{ lbs.}, \times 238 \text{ feet.}}{33000}$$

$$= \frac{133{\cdot}26 \times 7}{10} \; 93{\cdot}282 \text{ horse power.}$$

To determine the proper Velocity for the Piston of a Steam-Engine.

Rule. Multiply the logarithm of the nth part of the stroke at which the steam is cut off by 2·3, and to the product of this add ·7. Multiply the sum by the distance in feet the piston has travelled when the steam is cut off, and 120 times the square root of the product will equal the proper velocity for the piston in feet per minute.

Ex. Let the steam be cut off in an 8-feet stroke when the piston has travelled $\frac{1}{4}$th of the length; required its proper velocity.

Logarithm of 4 =	0·60206
Multiplied by	2·3
	1·384738
To which add	·7
	2·084738
	2

$\sqrt{4{\cdot}169476} = 2{\cdot}04 \times 120 = 245$ feet, velocity per minute.

TABLE
Of Approximate Velocities for the Pistons of Steam-Engines.

CONDENSING ENGINES.			NON-CONDENSING ENGINES.		
Length of stroke in feet.	Velocity in feet per minute.	Number of revolutions per minute.	Length of stroke in feet.	Velocity in feet per minute.	Number of revolutions per minute.
2	160	40	1½	186	62
2½	177½	35½	2	200	50
3	192	32	2½	212½	42½
3½	203	29	2¾	217½	39½
4	214	26¾	3	222	37
4½	220½	24½	3½	231	33
5	230	23	4	236	29½
5½	236½	21½	4½	243	27
6	240	20	5	247½	24¾
7	245	17½	5½	253	23
8	256	16	6	264	22

Of the Parallel Motion in a Steam-Engine.

When the power from the piston is communicated by means of a beam or lever moving upon an axis, the parallel motion becomes a very important portion of the machine; for then it forms the link of connexion, and by its properties renders the action of alternate circular motion, and reciprocating vertical motion, mutually agreeable, thereby properly insuring to the piston-rod a truly direct line to that of the cylinder; but to effect this, the greatest degree of exactitude of the various parts is required, otherwise extra friction is created, and the effective power of the engine proportionately diminished.

THE PROPERTIES AND MISCELLANEOUS EFFECTS OF HEAT.

Linear Expansion of Metals from 32° to 212°.—FARADAY.

Zinc,	1 part in	322	Gold,	1 part in	682
Lead,	"	351	Bismuth,	"	719
Tin, pure,	"	403	Iron,	"	812
Tin, impure,	"	500	Antimony,	"	923
Silver,	"	524	Palladium,	"	1000
Copper,	"	581	Platinum,	"	1100
Brass,	"	584	Flint Glass,	"	1248

TABLE

Of the Expansion of Water by Heat.—By DALTON.

Temperature.	Expansion.	Temperature.	Expansion.
12° Fahrenheit.	100236	122° Fahrenheit.	101116
22	100090	132	101367
32	100022	142	101638
42	100000	152	101934
52	100021	162	102245
62	100083	172	102575
72	100180	182	102916
82	100312	192	103265
92	100477	202	103634
102	100672	212	104012
112	100880		

TABLE

Of the Heating Power of various Combustible Substances, exhibiting the utmost quantity of Water evaporated by the given Weights, and the smallest quantity of Air capable of producing total Combustion. DR. URE.

Species of Combustible.	Pounds of water which a pound can heat from 0° to 212°.	Pounds of boiling water evaporated by 1 pound.	Weight of atmospheric air at 32° to burn 1 pound.
			Smallest quantity.
Perfectly dry wood, . .	35·00	6·36	5·96
Wood in its ordinary state,	26·00	4·72	4·47
Wood charcoal, ...	73·00	13·27	11·46
Pit coal, ...	60·00	10·90	9·26
Coke, ...	65·00	11·81	11·46
Turf, ...	30·00	5·45	4·60
Turf charcoal, ...	64·00	11·63	9·86
Carburetted hydrogen gas,	76·00	13·81	14·58
Oil, ... } Wax, ... } Tallow, ... }	78·00	14·18	15·00
Alcohol of the shops, . .	52·60	9·56	11·60

TABLE

Of boiling points of water holding various proportions of salt in solution.

	Parts of Salt.	Degrees of Fahrenheit.	Degrees of Reaumer.	Degrees of Centigrade.
Saturated solution . . .	36·37	226·6	86·2	107·8
" " . . .	33·34	224·9	85·7	107·2
" " . . .	30·30	223·7	85·2	106·5
" " . . .	27·28	222·5	84·7	105 8
" " . . .	24·25	221·4	84·1	105·2
" " . . .	21·22	220·2	83·6	104·6
" " . . .	18·18	219	83	103·9
" " . . .	15·15	217·9	82·6	103·3
" " . . .	12·12	216·7	82·1	102·6
" " . . .	9·09	215·5	81 6	102
" " . . .	6 06	214·4	81·1	101·3
Sea-water	3·03	213·2	80·5	100·7
Common water	0·00	212	80	100

Expansion of Liquids in Volume from 32° to 212° Fahrenheit.

1000	parts of	water	become	1046
"	"	oil	"	1080
"	"	mercury	"	1018
"	"	spirits of wine	"	1110
"	"	air	"	1373

Of the Linear Dilatation of Solids by Heat. Dimensions which a bar takes at 212°, whose length at 32° is 1·000000.

Cast iron, . . .	1·00111111	Cast brass, . . .	1·0018750
Steel (rod), . . .	1·00114470	Silver, ...	1·0018900
Steel, not tempered, . . .	1·00107875	Tin, ...	1·0028400
		Lead, ...	1·00284836
Ditto, tempered yellow, .	1·00136900	Zinc, ...	1·00294200
		Glass from 32° to 212°, . .	1·00086130
Ditto, at a higher rate, . . .	1·00123956		
		Glass from 212° to 392°, . .	1·00091827
Iron, ...	1·00118203		
Soft iron, forged, .	1·00122045	Glass from 392° to 572°, . .	1·00101114
Gold, ...	1·00150000		
Copper, ...	1·00191000		

Of Capacities of Bodies for Heat referred to Water as the Standard.

Water, ...	1·0000	Iron, ...	·1300
Olive oil, ...	·7100	Hardened steel, ...	·1230
Linseed oil, ...	·5280	Steel softened by fire, .	·1200
Oil of turpentine, . . .	·4720	Soft bar iron, ...	·1190
Quicksilver, ...	·0330	Brass, ...	·1160
Ice, ...	·9000	Copper, ...	·1140
Pit coal, ...	·2777	Zinc, ...	·1000
Chalk, ...	·2700	Ashes of charcoal, . . .	·0909
Sea salt, ...	·2300	Silver, ...	·0820
Sulphur, ...	·1900	Tin, ...	·0704
Ashes of cinders, . . .	·1855	White lead, ...	·0670
Black lead, ...	·1880	Gold, ...	·0500
Ashes of elm wood, . .	·1402	Lead, ...	·0420

TABLE

Of the Expansion of Atmospheric Air by Heat.

Degrees of Fahrenheit.	Bulk.	Degrees of Fahrenheit.	Bulk.	Degrees of Fahrenheit.	Bulk.
32°	1000	65°	1077	100°	1152
35	1007	70	1089	120	1194
40	1021	75	1099	140	1235
45	1032	80	1110	160	1275
50	1043	85	1121	180	1315
55	1055	90	1132	200	1364
60	1066	95	1142	212	1376

The pressure or gravity of the atmosphere, being equal to a column of water 34 feet in height, is the means or principle on which rests the utility of the common pump, also of the syphon, and all other such hydraulic applications. In the pump, the internal pressure on the surface of the liquid is removed by the action of the bucket; and as by degrees the density becomes lessened, so the water rises by the external pressure to the above-named height; and at such height it will remain, unless by some derangement of construction taking place, the atmospheric fluid is allowed to enter and displace the liquid column. But observe, if the temperature of the water or other liquid be so elevated that steam or vapor arise through it, then, according to the vapor's accumulation of density, may the action of the pump be partially or wholly destroyed; and the only means of evasion in such cases is to place the working bucket beneath the surface of the liquid which is required to be raised.

TABLE

Of the Degrees of the three Thermometrical Scales,

Above Boiling Point of Water.

Fahrenheit.	Centigrade.	Reaumur.	Fahrenheit.	Centigrade.	Reaumur.	Fahrenheit.	Centigrade.	Reaumur.	Fahrenheit.	Centigrade.	Reaumur.	Fahrenheit.	Centigrade.	Reaumur.
392	200	160	356	180	144	320	160	128	284	140	112	248	120	96
391			355			319			283			247		
390	199		354	179		318	159		282	139		246	119	
389		159	353		143	317		127	281		111	245		95
388	198		352	178		316	158		280	138		244	118	
387		158	351		142	315		126	279		110	243		94
386	197		350	177		314	157		278	137		242	117	
385	196	157	349	176	141	313	156	125	277	136	109	241	116	93
384			348			312			276			240		
383	195	156	347	175	140	311	155	124	275	135	108	239	115	92
382			346			310			274			238		
381	194		345	174		309	154		273	134		237	114	
380		155	344		139	308		123	272		107	236		91
379	193		343	173		307	153		271	133		235	113	
378		154	342		138	306		122	270		106	234		90
377	192		341	172		305	152		269	132		233	112	
376	191	153	340	171	137	304	151	121	268	131	105	232	111	89
375			339			303			267			231		
374	190	152	338	170	136	302	150	120	266	130	104	230	110	88
373			337			301			265			229		
372	189		336	169		300	149		264	129		228	109	
371		151	335		135	299		119	263		103	227		87
370	188		334	168		298	148		262	128		226	108	
369		150	333		134	297		118	261		102	225		86
368	187		332	167		296	147		260	127		224	107	
367	186	149	331	166	133	295	146	117	259	126	101	223	106	85
366			330			294			258			222		
365	185	148	329	165	132	293	145	116	257	125	100	221	105	84
364			328			292			256			220		
363	184		327	164		291	144		255	124		219	104	
362		147	326		131	290		115	254		99	218		83
361	183		325	163		289	143		253	123		217	103	
360		146	324		130	288		114	252		98	216		82
359	182		323	162		287	142		251	122		215	102	
358		145	322		129	286		113	250		97	214		81
357	181		321	161		285	141		249	121		213	101	

To convert the Degrees in the three Scales into each other.

To convert Centigrade or Reaumur's into Fahrenheit's Degrees.—Multiply the number of degrees by 9, divide the product by 5 for Centigrade, or by 4 for Reaumur's; add 32 to the quotient, and the sum will be degrees of Fahrenheit.

To convert Fahrenheit's into Centigrade or Reaumur's Degrees.—Subtract 32 from the number of degrees, and divide the remainder by 9; multiply the quotient by 5 for Centigrade, or 4 for Reaumur's; the products will be the required degrees respectively.

Comparative Table of the Degrees of the three Thermometrical Scales.

Fahr't	Cent.	Rea.	Fahr't	Cent.	Rea.	Fahr't	Cent.	Rea.	Fahr't	Cent.	Rea.	Fahr't	Cent.	Rea.
212	100	80	167	75	60	122	50	40	77	25	20	32	0	0
211			166			121			76			31		
210	99	79	165	74	59	120	49	39	75	24	19	30	-1	-1
209			164			119			74			29		
208	98		163	73		118	48		73	23		28	-2	
207	97	78	162	72	58	117	47	38	72	22	18	27	-3	-2
206			161			116			71			26		
205	96	77	160	71	57	115	46	37	70	21	17	25	-4	-3
204			159			114			69			24		
203	95	76	158	70	56	113	45	36	68	20	16	23	-5	-4
202			157			112			67			22		
201	94	75	156	69	55	111	44	35	66	19	15	21	-6	-5
200			155			110			65			20		
199	93		154	68		109	43		64	18		19	-7	
198	92	74	153	67	54	108	42	34	63	17	14	18	-8	-6
197			152			107			62			17		
196	91	73	151	66	53	106	41	33	61	16	13	16	-9	-7
195			150			105			60			15		
194	90	72	149	65	52	104	40	32	59	15	12	14	-10	-8
193			148			103			58			13		
192	89	71	147	64	51	102	39	31	57	14	11	12	-11	-9
191			146			101			56			11		
190	88		145	63		100	38		55	13		10	-12	
189	87	70	144	62	50	99	37	30	54	12	10	9	-13	-10
188			143			98			53			8		
187	86	69	142	61	49	97	36	29	52	11	9	7	-14	-11
186			141			96			51			6		
185	85	68	140	60	48	95	35	28	50	10	8	5	-15	12
184			139			94			49			4		
183	84	67	138	59	47	93	34	27	48	9	7	3	-16	-13
182			137			92			47			2		
181	83		136	58		91	33		46	8		1	-17	
180	82	66	135	57	46	90	32	26	45	7	6	0	-18	-14
179			134			89			44			-1		
178	81	65	133	56	45	88	31	25	43	6	5	-2	-19	-15
177			132			87			42			-3		
176	80	64	131	55	44	86	30	24	41	5	4	-4	-20	-16
175			130			85			40			-5		
174	79	63	129	54	43	84	29	23	39	4	3	-6	-21	-17
173			128			83			38			-7		
172	78		127	53		82	28		37	3		-8	-22	
171	77	62	126	52	42	81	27	22	36	2	2	-9	-23	-18
170			125			80			35			-10		
169	76	61	124	51	41	79	26	21	34	1	1	-11	-24	-19
168			123			78			33			-12		
												-13	-25	-20

Table *of the Weight of Substances of Construction, showing the weight of a cubic inch, and a cubic foot, in ounces and pounds avoirdupois, and also the number of cubic inches in one pound, of the substances most used in construction.*

Names of Bodies.	Weight of a cubic foot.		Weight of a cubic inch.		Number of cubic inches in a pound.
	in oz.	in lbs.	in oz.	in lbs.	
Copper, cast, .	8788	549·25	5·086	·3178	3·146
Copper, sheet, .	8915	557·18	5·159	·3225	3·103
Brass, cast, . .	8396	524·75	4 852	·3037	3·293
Iron, cast, . .	7271	445·43	4·208	·263	3·802
Iron, bar, . .	7631	476·93	4·410	·276	3·623
Lead, ...	11344	709·00	6·456	·4103	2·437
Steel, soft, . .	7833	489·56	4·527	·2833	3·530
Steel, hard, . .	7816	488·50	4·517	·2827	3·537
Zinc, cast, . .	7190	449·37	4·156	·26	3·845
Tin, cast, . . .	7292	455·75	4·215	·2636	3·790
Bismuth, . . .	9880	619·50	5·710	·3585	2·789
Gun-metal, . .	8784	549·00	5·0075	·3177	3·147
Sand, ...	1520	95·00	·8787	·055	18·190
Coal, ...	1250	78·12	·7225	·0452	22·120
Brick, ...	2000	125·00	1·156	·0723	13·824
Stone, paving, .	2416	151·00	1·396	·0873	11·443
Slate, ...	2672	167·00	1·544	·0967	10·347
Marble, . . .	2742	171·37	1·585	·0991	10·083
White lead, . .	3160	197·50	1·826	·1143	8·750
Glass, ...	2880	180·00	1·664	·1042	9·600
Tallow, . . .	945	59·06	·5462	·0087	29·258
Cork, ...	240	15·00	·138	·0197	115·200
Larch, ...	544	34·00	·315	·0201	50·823
Elm, ...	556	34·75	·321	·0201	49·726
Pine, pitch, . .	660	41·25	·382	·024	41·890
Beech, . . .	696	43·50	·403	·0252	39·724
Teak, ...	745	46·56	·431	·027	37·113
Ash, ...	760	47·50	·440	·0275	36·370
Mahogany, . .	852	53·25	·493	·0308	32·449
Oak, ...	970	60·62	·561	·0351	28·505
Oil of turpentine,	870	54·37	·503	·0315	31·771
Olive oil, . . .	915	57·18	·529	·0331	30·220
Linseed oil, . .	932	58·25	·539	·0337	29·655
Spirits, proof, .	927	57·93	·536	·03352	29·288
Water, distilled,	1000	60 50	·578	·03617	27·648
" sea, . .	1028	64·25	·594	·0372	26·894
Tar, ...	1015	63·43	·587	·0367	27·242
Vinegar, . . .	1026	64·12	·593	·037	26·949
Mercury, . . .	13568	848·00	7·851	·4908	2·037

Conducting Power of Materials used in the Construction of Houses.

As observed by Mr. Hutchinson.

Slate,	100	Oak wood,	33·66
Keene's cement,	19·01	Asphalt,	45·19
Plaster and sand,	18·70	Chalk (soft),	56·38
Plaster of Paris,	20·26	Stock brick,	60·14
Roman cement,	20·80	Bathstone,	61·08
Beech wood,	22·44	Fire brick,	61·70
Lath and plaster,	25 55	Lead,	521·34
Fir wood,	27·60		

Air and gases are very imperfect conductors. Heat appears to be propagated through them almost entirely by conveyance, the heated portions of air becoming lighter, and diffusing the heat through the mass in their ascent as in liquids. Hence, in heating a room with hot air, the hot air should be introduced at the lowest part. The advantage of double windows for warmth depends, in a great measure, on the sheet of air confined between them through which heat is very slowly transmitted.

Capacity of Bodies for Transmitting Heat.

The capacity which bodies possess of transmitting heat does not depend upon their transparency; or bodies are not all transparent to heat in the same proportion that they are transparent to light. The following plates of an equal thickness of ·1031 inches allowed very different proportions of heat to pass through them.

Of 100 rays transmitted from an Argand oil lamp there were:

Rock salt,	92	Emerald,	29
Mirror glass,	62	Gypsum,	20
Rock crystal,	62	Fluor spar,	15
Iceland spar,	62	Citric acid,	15
Rock crystal, smoky & brown	57	Rochelle salt,	12
Carbonate of lead,	52	Alum,	12
Sulphate of barytes,	33	Sulphate of copper,	0

SOLDERS.

For Lead.—Melt one part of block tin, and, when in a state of fusion, add 2 parts of lead. Resin should be used with this solder.

For Tin.—Pewter, 4 parts; tin, 1; bismuth, 1. Melt them together and run them into slips. Resin is also used with this solder.

For Gold.—Pure gold, 12 parts; silver, 2; copper, 4.

For Brass.—Brass, 2 parts; zinc, 1.

For Iron.—Good tough brass, with a small quantity of borax.

For Pewter.—Bismuth, 2 parts; lead, 1; tin, 2.

For Copper.—Copper, 2 parts; zinc, 1.

For Silver.—Silver, 5 parts; brass, 6; zinc, 2.

Hard Solder.—Copper, 2 parts; zinc, 1.

Soft Solder.—Tin, 2 parts; lead, 1 part.

TABLE

Of proportions for making Shafting with Half-lap Couplings, showing length of Neck and sizes of Coupling-box. (Manchester Rules.)

Diameter of Neck.	Length of Neck.	Diameter of Coupling.	Length of Lap.	Length of Box.	Diameter of Box.
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
2	4	3¼	2¼	5½	5½
2¼	4½	3½	2¾	6	6
2½	5	4	3	6½	6¾
2¾	5½	4¼	3¼	7	7½
3	6	4¾	3½	7½	7¾
3¼	6¼	5	3¾	8	8¼
3½	6½	—		—	—
4	7	6	4	8½	9½
4½	7½	6½	4½	9	10½
5	8	7¼	5	9½	11¼
5½	8½	8½	5½	11	12¼
6	9	9	6	12	13½
6½	9½	9¾	6½	13	14¾
7	10¼	10½	7½	14	16
7½	11¼	—		—	—
8	12	12	8	16½	18
8½	12½	12½	8½	17	19
9	13½	13	9	18	20
9½	14	—	—	—	—
10	14½	14	10	18½	22
11	15	16	11	20	24
12	16	17½	12	21	26

Gradations of Temperature.

The following are interesting facts in the range of temperature:

Below Zero (Fah.)

166° Greatest artificial cold. (Faraday.)
150 Liquid nitrous oxide freezes.
122 Liquid sulphuretted hydrogen freezes.
105 Liquid sulphurous acid freezes.
91 Greatest artificial cold measured by Walker.
56 Greatest natural cold observed by a "verified" thermometer. (Sabine.)
70 Greatest natural cold observed at Fort Reliance by Back. (Doubtful.)
58 Estimated temperature of planetary space. (Fourier.)
47 Sulphuric ether freezes.
39 Mercury freezes.
30 Liquid cyanogen freezes. (Faraday.)
13 Mean temperature at the Pole. (Arago.)
11 A mixture of two parts alcohol and one part water freezes.
7 A mixture of equal parts alcohol and water freezes.

Above Zero (Fah.)

20°	Strong wine freezes.
28	Vinegar freezes.
30	Milk freezes.
32	Ice melts.
41	Mean temperature at Edinburgh.
50·7	Mean temperature of London.
60	Mean temperature at Rome.
81·5	Mean temperature at the equator.
98	Heat of the human blood.
98	Ether boils.
100	Phosphorus melts.
173	Alcohol boils
117	Highest natural temperature observed of a hot wind in Upper Egypt. (Burckhardt.)
133	Wood-spirit boils.
142	Spermaceti melts.
151·34	Beeswax melts.
212	Water boils.
226	Sulphur melts.
242	Nitric acid boils.
283	A compound of equal parts of tin and bismuth melts.
442	Tin melts.
460	The surface of polished steel acquires a pale straw color.
476	Bismuth melts.
554	Phosphorus boils.
560	Oil of turpentine boils.
580	The surface of polished steel acquires a uniform deep blue.
590	Sulphuric acid boils. (Dalton.)
594	Lead melts.
600	Linseed oil boils.
635	Lowest ignition of iron in the dark.
662	Mercury boils.
700	Zinc melts.
752	Iron bright red in the dark.
810	Antimony melts.
884	Iron red hot in the twilight.
1077	Red heat fully visible in the daylight.
1141	Heat of a common fire. (Daniell.)
1869	Brass melts.
1873	Silver melts.
1996	Copper melts.
2016	Gold melts.
2500	Steel melts.
2786	Cast-iron melts.
3080	Platinum melts.

The line of perpetual congelation has a variable altitude in different climates.

At the equator it is		14760	feet.
At the Alps	"	8120	"
In Iceland	"	3084	"

At the polar regions ice is perpetually observed at the surface of the earth.

PROPERTIES OF NUMBERS.

1. A *Prime Number* is that which can only be measured by 1 or unity.

2. A *Composite Number* is that which can be measured (or divided without a remainder) by some number greater than unity.

3. A *Perfect Number* is that which is equal to the sum of all its divisors, or aliquot parts: thus $6 = \frac{6}{2} + \frac{6}{3} + \frac{6}{6}$.

4. If an *odd* number divides an *even* number, it will also divide the half of it.

5. If the last digit of any number be divisible by 2, the whole number is divisible by 2.

6. If the two last digits be divisible by 4, the whole number is divisible by 4.

7. If the three last digits be divisible by 8, the whole number is divisible by 8.

8. If a number terminate with 5, it is divisible by 5; and if it terminate with 0, it is divisible by either 10 or 5.

9. If the sum of the digits constituting any number be divisible by 3 or 9, the whole is divisible by 3 or 9; and if also the last digit is even, the whole number is divisible by 18.

10. If the sum of the digits of any number be divisible by 6, and the right hand digit by 2, the whole is divisible by 6.

11. If the sum of the 1st, 3d, 5th, &c., digits of any number be equal to that of the 2d, 4th, 6th, &c., that number is divisible by 11. Thus 327943 contains 11 = 29813 times exactly.

12. If a square number be either multiplied or divided by a square, the product or quotient is a square; and conversely, if a square number be either multiplied or divided by a number that is not a square, the product or quotient is not a square.

13. The product arising from two different prime numbers cannot be a square number.

14. The product of no two different numbers prime to each other (that is, 1 being the common measure) can make a square, unless each of those numbers be a square.

15. The square root of an integral number, that is not a *complete* square, can neither be expressed by an integer nor by any rational fraction; so with the cube root of an integer.

16. Every prime number greater than *two*, is made up of 4 times some number, + 1 or − 1; that is, of one of the forms $4n + 1$, or $4n - 1$.

17. Any prime number greater than 3, divided by 6, will leave a remainder of 1 or 5: that is, every number greater than 3, is one of the forms $6n + 1$, or $6n - 1$.

18. The number of prime numbers is infinite.

19. A square number cannot terminate with an *odd* number of cyphers.

20. If a square number terminate with 4, the last figure but one will be an *even* number.

21. If a square number terminate with 5, it will terminate with 25.

22. No square number can terminate with two equal digits, except two *cyphers*, or two *fours*.

23. No number whose last digit is 2, 3, 7, or 8, is a square number.

24. If a cube number be divisible by 7, it is also divisible by the cube of 7.

25. The difference between any integral cube and its root is always divisible by 6.

26. Neither the sum nor the difference of two cubes can be a cube.

27. A cube number may end with any of the natural numbers.

28. All the powers of any number that end with 6, will terminate with 6: so with the numeral 5.

TABLE

Of the first Nine Powers of the first Nine Numbers.

1st	2d	3d	4th	5th	6th	7th	8th	9th
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

TABLE

Of Useful Numbers.

π	= 3·1415927	$\sqrt{2}$	= 1·4142136
Log. π	0·4971499	$\frac{1}{\sqrt{2}}$	0·7071068
Log. $_\epsilon \pi$	1·1447299	$\pi\sqrt{2}$	4·4428829
$\frac{1}{\pi}$	0·3183099	$\frac{\pi}{\sqrt{2}}$	2·2214415
π^2	9·8696044	$\frac{\sqrt{2}}{\pi}$	0·4501582
$\frac{1}{\pi^2}$	0·1013212	$\sqrt{\frac{\pi}{2}}$	1·2533141
$\sqrt{\pi}$	1·7724538	$\sqrt{\frac{2}{\pi}}$	0·7978846
$\frac{1}{\sqrt{\pi}}$	0·5641896		

ϵ	= 2·7182818
Log. ϵ	0·4342945
Modulus of common logarithms	·434294482
Log. of ditto	9·6377843
g	32·19084
$\sqrt{g}$	5·67363
Log. g	1·5077222
Inches in a French mètre	39·37079
Log. of ditto	1·5951741
Feet in ditto	3·2808992
Log. of ditto	0·5159929
Square feet in the square mètre	10 764297
Acres in the Are	0·024711
Lbs. in a kilogramme	2·20548
Log. of ditto	0·3435031
Imperial gallons in a litre	0·2200967
Lbs. per square inch in 1 kilogramme per square millimetre	1422
Cwts ditto, ditto	12·7
Volume of a sphere whose diameter is 1	0 5235988
Arc of 1° to rad 1	0·017453293
Arc of 1′ to rad. 1	0·000290888
Arc of 1″ to rad 1	0·000004848
Degrees in an arc whose length is 1	57·295780°
Grains in 1 oz. avoirdupois	437½

Grains in 1 lb. ditto	7000
Grains in a cubic inch of distilled water, Bar. 30 in., Th. 62°	252·458
Cubic inches in an ounce of water	1·73298
Cubic inches in the imperial gallon	277·276
Feet in a geographical mile	6075·6
Log. of ditto	3·7835892
Feet in a statute mile	5280
Log. of ditto	3·7226339
Length of seconds' pendulum in inches	39·19084
Cubic inches in 1 cwt. of cast iron	430·25
" " Bar iron	397·60
" " Cast brass	368·88
" " Cast copper	352·41
" " Cast lead	272·80
Cubic feet in 1 ton of paving stone	14·835
" " Granite	13·505
" " Marble	13·070
" " Chalk	12·874
" " Limestone	11·273
" " Elm	64·460
" " Honduras mahogany	64·000
" " Mar Forest fir	51·650
" " Beech	51·494
" " Riga fir	47·762
" " Ash and Dantzic oak	47·158
" " Spanish mahogany	42·066
" " English oak	36·205
To find the weight in lbs. of 1 foot of common rope, multiply the square of its circumference in inches by	·044 to ·046
Ditto for a cable	·027

TABLE

Surface of Boilers' Tubes of Different Lengths and Diameters.

Diameter.	Length.	Surface.	Diameter.	Length.	Surface.
In.	Ft. in.	Sq. ft.	In.	Ft. in.	Sq. ft.
2½	5 0	3·27	3	6 6	5·1
"	5 3	3·42	"	6 8	5 2
"	5 6	3·6	"	7 0	5·5
"	5 9	3·75	"	7 6	5·89
"	6 0	3·9	"	8 0	6·28
3	6 0	4·7	"	8 6	6·67
"	6 3	4·9			

RECIPES FOR MAKING DIFFERENT KINDS OF GLASS.

1. *Bottle Glass.*—1. Dry glauber salts, 11 pounds; soaper salts, 12 pounds; half a bushel of waste soap ashes; sand, 56 pounds; glass skimmings, 22 pounds; green broken glass, 1 cwt.; basalt, 25 pounds. This mixture affords a dark-green glass.

2. Yellow or white sand, 100 parts; kelp, 30 to 40; lixiviated wood ashes, from 160 to 170 parts; fresh wood ashes, 30 to 40 parts; potter's clay, 80 to 100 parts; cullet, or broken glass, 100. If basalt be used, the proportion of kelp may be diminished.

2. *Green Window, or Broad Glass.*—Dry glauber salts, 11 pounds; soaper salts, 10 pounds; half a bushel of lixiviated soap waste; 50 pounds of sand; 22 pounds of glass pot skimmings; 1 cwt. of broken green glass.

3. *Crown Glass.*—300 parts of fine sand; 200 of good soda ash; 33 of lime; from 250 to 300 of broken glass; 60 of white sand; 30 of purified potash; 15 of saltpetre; (1 of borax;) ½ of arsenious acid.

4. *Nearly White Table Glass.*—1. 20 pounds of potashes; 11 pounds of dry glauber salts; 16 of soaper salt; 55 of sand; 140 of cullet of the same kind.

2. 100 parts of sand; 235 of kelp; 60 of wood ashes; 1½ of manganese; 100 of broken glass.

5. *White Table Glass.*—1. 40 pounds of potashes; 11 of chalk; 76 of sand; ½ of manganese; 95 of white cullet.

2. 50 of purified potashes; 100 of sand; 20 of chalk, and 2 of saltpetre.

6. *Crystal Glass.*—1. 60 parts of purified potashes; 120 of sand; 24 of chalk; 2 of saltpetre; 2 of arsenious acid; $\frac{1}{16}$ of manganese.

2. Purified pearlashes, 70 parts; white sand, 120; saltpetre, 10; ½ of arsenious acid; ⅛ of manganese.

3. 67 of sand: 23 of purified pearlashes; 10 of sifted slaked lime; ½ of manganese; 5 to 8 of red lead.

4. 120 of white sand; 50 of red lead; 40 of purified pearlash; 20 of saltpetre; ⅛ of manganese.

5. 120 of white sand; 40 of pearlash purified; 35 of red lead; 13 of saltpetre; $\frac{1}{12}$ of manganese.

6. 30 of the finest sand; 20 of red lead; 8 of pearlash purified; 2 of saltpetre; a little arsenious acid and manganese.

7. 100 of sand; 45 of red lead; 35 of purified pearlashes; $\frac{1}{7}$ of manganese; $\frac{1}{5}$ of arsenious acid.

7. *Plate Glass.*—1. Very white sand, 300 parts; dry purified soda, 100 parts; carbonate of lime, 43 parts; manganese, 1; cullet, 300.

2. Finest sand, 720 parts; purified soda, 450; quicklime, 80; saltpetre, 25; cullet, 425.

A little borax has also been prescribed; much of it communicates an exfoliating property to glass.

TABLE

Of Prime Numbers to 5000.

2	197	461	751	1051	1381	1697
3	199	463	757	1061	1399	1699
5	211	467	761	1063	1409	1709
7	223	479	769	1069	1423	1721
11	227	487	773	1087	1427	1723
13	229	491	787	1091	1429	1733
17	233	499	797	1093	1433	1741
19	239	503	809	1097	1439	1747
23	241	509	811	1103	1447	1753
29	251	521	821	1109	1451	1759
31	257	523	823	1117	1453	1777
37	263	541	827	1123	1459	1783
41	269	547	829	1129	1471	1787
43	271	557	839	1151	1481	1789
47	277	563	853	1153	1483	1801
53	281	569	857	1163	1487	1811
59	283	571	859	1171	1489	1823
61	293	577	863	1181	1493	1831
67	307	587	877	1187	1499	1847
71	311	593	881	1193	1511	1861
73	313	599	883	1201	1523	1867
79	317	601	887	1213	1531	1871
83	331	607	907	1217	1543	1873
89	337	613	911	1223	1549	1877
97	347	617	919	1229	1553	1879
101	349	619	929	1231	1559	1889
103	353	631	937	1237	1567	1901
107	359	641	941	1249	1571	1907
109	367	643	947	1259	1579	1913
113	373	647	953	1277	1583	1931
127	379	653	967	1279	1597	1933
131	383	659	971	1283	1601	1949
137	389	661	977	1289	1607	1951
139	397	673	983	1291	1609	1973
149	401	677	991	1297	1613	1979
151	409	683	997	1301	1619	1987
157	419	691	1009	1303	1621	1993
163	421	701	1013	1307	1627	1997
167	431	709	1019	1319	1637	1999
173	433	719	1021	1321	1657	2003
179	439	727	1031	1327	1663	2011
181	443	733	1033	1361	1667	2017
191	449	739	1039	1367	1669	2027
193	457	743	1049	1373	1693	2029

2039	2399	2789	3203	3581	3967	4371
2053	2411	2791	3209	3583	3989	4391
2063	2417	2797	3217	3593	4001	4397
2069	2423	2801	3221	3607	4003	4409
2081	2437	2803	3229	3613	4007	4421
2083	2441	2819	3251	3617	4013	4423
2087	2447	2833	3253	3623	4019	4441
2089	2459	2837	3257	3631	4021	4447
2099	2467	2843	3259	3637	4027	4451
2111	2473	2851	3271	3643	4049	4457
2113	2477	2857	3299	3659	4051	4463
2129	2503	2861	3301	3671	4057	4481
2131	2521	2879	3307	3673	4073	4483
2137	2531	2887	3313	3677	4079	4493
2141	2539	2897	3319	3691	4091	4507
2143	2543	2903	3323	3697	4093	4513
2153	2549	2909	3329	3701	4099	4517
2161	2551	2917	3331	3709	4111	4519
2179	2557	2927	3343	3719	4127	4523
2203	2579	2939	3347	3727	4129	4547
2207	2591	2953	3359	3733	4133	4549
2213	2593	2957	3361	3739	4139	4561
2221	2609	2963	3371	3761	4153	4567
2237	2617	2969	3373	3767	4157	4583
2239	2621	2971	3389	3769	4159	4591
2243	2633	2999	3391	3779	4177	4597
2251	2647	3001	3407	3793	4201	4603
2267	2657	3011	3413	3797	4211	4621
2269	2659	3019	3433	3803	4217	4637
2273	2663	3023	3449	3821	4219	4639
2281	2671	3037	3457	3823	4229	4643
2287	2677	3041	3461	3833	4231	4649
2293	2683	3049	3463	3847	4241	4651
2297	2687	3061	3467	3851	4243	4657
2309	2689	3067	3469	3853	4253	4663
2311	2693	3079	3491	3863	4259	4673
2333	2699	3083	3499	3877	4261	4679
2339	2707	3089	3511	3881	4271	4691
2341	2711	3109	3517	3889	4273	4703
2347	2713	3119	3527	3907	4283	4721
2351	2719	3121	3529	3911	4289	4723
2357	2729	3137	3533	3917	4297	4729
2371	2731	3163	3539	3919	4327	4733
2377	2741	3167	3541	3923	4337	4751
2381	2749	3169	3547	3929	4339	4759
2383	2753	3181	3557	3931	4349	4783
2389	2767	3187	3559	3943	4357	4787
2393	2777	3191	3571	3947	4363	4789

4793	4817	4877	4919	4943	4969	4999
4799	4831	4889	4931	4951	4973	5003
4801	4861	4903	4933	4957	4987	5009
4813	4871	4909	4937	4967	4993	

TABLE
Of Solid Inches and Solid Feet.

Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.
1=	1728	26=	44928	51=	88128	76=	131328
2	3456	27	46656	52	89856	77	133056
3	5184	28	48384	53	91584	78	134784
4	6912	29	50112	54	93312	79	136512
5	8640	30	51840	55	95040	80	138240
6	10368	31	53568	56	96768	81	139968
7	12096	32	55296	57	98496	82	141696
8	13824	33	57024	58	100224	83	143424
9	15552	34	58752	59	101952	84	145152
10	17280	35	60480	60	103680	85	146880
11	19008	36	62208	61	105408	86	148608
12	20736	37	63936	62	107136	87	150336
13	22464	38	65664	63	108864	88	152064
14	24192	39	67392	64	110592	89	153792
15	25920	40	69120	65	112320	90	155520
16	27648	41	70848	66	114048	91	157248
17	29376	42	72576	67	115776	92	158976
18	31104	43	74304	68	117504	93	160704
19	32832	44	76032	69	119232	94	162432
20	34560	45	77760	70	120960	95	164160
21	36288	46	79488	71	122688	96	165888
22	38016	47	81216	72	124416	97	167616
23	39744	48	82944	73	126144	98	169344
24	41472	49	84672	74	127872	99	171072
25	43200	50	86400	75	129600	100	172800

TABLE
Showing the Weight of Cast-Iron Plates, 12 inches square, and from $\frac{1}{8}$ of an inch to 1 inch thick.

Width in Inches.	$\frac{1}{8}$ ·125		$\frac{1}{4}$ ·25		$\frac{3}{8}$ 375		$\frac{1}{2}$ ·5		$\frac{5}{8}$ ·625		$\frac{3}{4}$ ·75		$\frac{7}{8}$ ·875		One Inch.	
	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz	lbs.	oz	lbs.	oz.	lbs.	oz.
12	4	$13\frac{3}{8}$	9	$10\frac{5}{8}$	14	8	19	$5\frac{3}{8}$	24	$2\frac{3}{4}$	29	0	33	$13\frac{3}{8}$	38	$10\frac{3}{4}$

To find the Horse Power that a Cast-Iron Wheel is capable of transmitting.

Multiply the breadth of the teeth or face of the wheel in inches by the square of the thickness of one tooth, and divide by the length of the teeth, for the strength at a velocity of 136 feet per minute.

Thus a wheel with the breadth of teeth = 7½ inches, thickness = 1·4, and length = 2, ought to transmit 7·35 horse power. For

$$1{\cdot}4^2 = 1{\cdot}96, \text{ and } \frac{7{\cdot}5 \times 1{\cdot}96}{2} = 7{\cdot}35.$$

The strength at any other velocity is found by multiplying the power so obtained by any other required velocity, and by ·0044.

Thus, the wheel as above, at the velocity of 320 feet per minute, would be capable of transmitting 10·3488 horse power.

TABLE

Of the Dimensions of Wheels in Actual Use.

Pitch in inches.	Character of Wheel.	Number of teeth.	Bre'dth in inches.	No. of revolutions per minute.	Horse Power. Actual.	Horse Power. Calculated.
1½	Spur Wheel, . .	72	4½	120	8	7·5
2¼	Spur Wheel, . .	95	6	25	1½	1·676
3¼	Bevil Wheel, . .	40	7	30½	20	24·34
2⅝	Cog Wheel, . . .	60	6	40	12	15·82
5½	Bevil Wheel, . .	70	12	10	60	67·396
2½	Spur Wheel, . .	90	8	12	6	9·72
3¾	Internal, ...	80	9	20	41	48·8
3	Cog Spur Wheel, .	60	8	30	121	·177
6	Spur Wheel, . .	30	14	7	21	·26½
4	Spur Wheel, . .	100	10	8	25	29·6
2⅞	Spur Wheel, . .	33	7	55	23	·25
2¾	Spur Wheel, . .	108	7	20	25	·26
2½	Internal, . . .	100	7	10	87	90·4
5	Internal, ...	60	12	12	55	53·5
5	Spur, ...	41	10	20	61	·50
476	Spur, ...	50	12	23	65	71·3
3¾	Bevil Wheel, . .	35	10	24	26	25·6
4	Cog Bevil Wheel, .	50	10	28	33	32·6
4	Cog Spur Wheel, .	35	9	20	18	16·3
6	On Water Wheel, .	112	14	12	110	·168
4¾	Spur Wheel, . .	55	10	16	56	54·56

TABLE

Showing the Circumference of a Rope equal to a Chain made of Iron of a given Diameter, and the Weight in Tons that each is proved to carry; also the weight of a Foot of Chain made from Iron of that dimension.

Rope's circumference in inches.	Chain Diameter in inches.	Proved to carry in tons.	Weight of a linear foot in lbs. avoir.
3	¼ & $\frac{1}{16}$	1	1·08
4	⅜	2	1·5
4¾	⅜ & $\frac{1}{16}$	3	2
5¼	½	4	2·7
6	½ & $\frac{1}{16}$	5	3·3
6½	⅝	6	4
7	⅝ & $\frac{1}{16}$	8	4·6
7½	¾	9½	5·5
8	¾ & $\frac{1}{16}$	11½	6·1
9	⅞	13	7·2
9½	⅞ & $\frac{1}{16}$	15	8·4
10½	1 inch.	18	9·4

The Transverse Strength of a body is that power which it exerts in opposing any force acting in a perpendicular direction to its length, as in the case of beams, levers, &c., it is inversely as their lengths, and directly as their breadths, and the square of their depths. But, if cylindrical, as the cubes of their diameters.

That is, if a beam 5 feet long, 2 inches broad, and 3 inches deep, can carry 1798 lbs., another beam of the same material, 10 feet long, 2 inches broad, and 3 inches deep, will only carry 899 lbs., being inversely as their lengths.

Again, if a beam 5 feet long, 2 inches broad, and 3 inches deep, can support 1798 lbs., another beam of the same material, 4 inches broad, and 3 inches deep, will support double that weight, being directly as their breadths.

A beam of the same material, 5 feet long, 2 inches broad, and 6 inches deep, will sustain 7192 lbs., being as the square of their depths.

TABLE

Showing the Equivalents and Specific Gravities of sixty-two Simple Substances.

Name of Substance.	Symbol.	Equivalent or Atomic Weight.	Specific Gravity.	Name of Substance.	Symbol.	Equivalent or Atomic Weight.	Specific Gravity.
Hydrogen,	H.	1	·0689	**METALS**			
Oxygen,	Oor.	8	1·026	*Continued.*			
Nitrogen,	N.	14·2	1·529				
Chlorine,	Cl.	35·5	2·444	Chromium,	Cr.	28·19	5·9
Carbon,	C.	6·12	44·1	Mercury,	Hg.	203	13·5
Iodine,	I.	126·5	4·948	Silver,	Ag.	108·3	10·5
Sulphur,	S.	16·1	1·99	Gold,	Au.	200	19·3
Phosphorus,	P.	15·7	1·7	Platinum,	Pt.	98·84	21·5
Fluorine,	F.	18·7		Tin,	Sn.	58·9	7·29
Bromine,	Br.	78·4	3·	Cobalt,	Co.	29·5	7·83
Boron,	B.	11		Manganese,	Mn.	27·7	8·0
Selenium,	Se.	40	4·5	Nickel,	Ni.	29·5	8·8
				Antimony,	Sb.	64·6	6·7
				Arsenic,	As.	37·7	5·7
METALS.				Palladium,	Pd.	53·35	11·5
				Rhodium,	R.	52·2	11
Potassium,	K.	39·2	·865	Asmium,	Os.	99·7	10
Sodium,	Na.	23·5	·972	Iridium,	Ir.	99·8	18·68
Lithium,	L.	10		Cadmium,	Cd.	55·8	8·6
Calcium,	Ca.	20·5		Molybde-			
Magnesium,	Mg.	12·7		num,	Mo.	47·9	8·6
Silicon,	Si.	22		Tungsten, or			
Aluminum,	Al.	13·7		Wolfram,	W.	94·8	17
Iron,	Fe.	28	7·7	Vanadium,	V.	68·5	
Lead,	Pb.	103·7	11·35	Uranium,	U.	217·2	
Copper,	Cu	31·7	8·8	Titanium,	Ti.	24·5	
Columbium,	Cm.	184·8		Cerium,	Ce.	46	
Glucinum,	G.	26		Niobium,	Nr.		
Yltrium,	Y.	32		Pelopium,	Pe.		
Zirconium,	Zr	34		Norium,	No.		
Thorinum,	Th.	60		Didymium,	D.		
Strontium,	Sr.	43·8		Lantanum,	Ln.	48	
Barium,	Ba.	68·6		Jerbium,	Tb.		
Bismuth,	Bi.	71·5		Erbium,	E.		
Tellurium,	Te.	64·2		Rutnheium,	Ru.	52	
Zinc,	Z.	32·3	From 6·8 to 7·1				

The Feeding Properties of different Vegetables.

In comparison with 10 lbs. of hay.

Hay,	10	Carrots,	35
Clover hay,	8	Cabbage,	30 to 40
Vetch hay,	4	Pease and beans,	2 to 3
Wheat straw,	52	Wheat,	5
Barley straw,	52	Barley,	6
Oat straw,	55	Oats,	5
Pea straw,	6	Rye,	5
Potatoes,	28	Indian corn,	6
Old potatoes,	40	Bran,	5
Turnips,	60	Oil-cake,	2

Thus 2 lbs. of oil-cake is worth as much as 55 lbs. of oat straw.

PENDULUMS.

A pendulum that vibrates seconds, or 60 in the latitude of London, is 39·1393 inches long; and $\sqrt{39 \cdot 1393} \times 60 = 375 \cdot 36$, which serves as a constant number for other pendulums; thus, 375·36 divided by the square root of the pendulum's length, gives the number of vibrations per minute; and divided by the vibrations per minute, gives the square root of the length of pendulums.

Example 1.—Required the number of vibrations a pendulum of 25 inches long will make per minute.

$$\frac{375 \cdot 36}{\sqrt{25}} = 75 \cdot 072 \text{ vibrations per minute.}$$

Example 2.—Required the length of a pendulum to make 80 vibrations per minute.

$$\frac{375 \cdot 36}{80} = 4 \cdot 692^2 = 22 \cdot 014864 \text{ inches long.}$$

Table *containing the Length of Pendulums to vibrate Seconds in various parts of the World.*

	Inches.		Inches.
At Sierra Leone,	39·01954	At New York,	39·10153
" Trinidad,	39·01879	" Bordeaux,	39 11282
" Madras,	39·02630	" Paris,	39 12843
" Jamaica,	39·03508	" Edinburgh,	39 15540
" Rio Janeiro,	39·01206	" Greenland,	39·20328

A pendulum vibrating half seconds in the latitude of London is 9·8 inches in length; and for quarter seconds, 2·5 inches.

TABLE

Showing the Symbols and Equivalents of Binary Compounds.

Name of Compound.	Symbol.	Equivalent.	Remarks.
Water,	$O\,H$	9	
Binoxide of hydrogen,	$O^2\,H$	17	Easily decomposed by the metals and metallic oxides.
Protoxide of nitrogen,	$O\,N$	22·2	Supports combustion; its taste is sweet and pleasant.
Binoxide of nitrogen,	$O^2\,N$	30·2	Transparent and colorless, produces orange red vapors in the atmospheric air and oxygen.
Hyponitrous acid,	$O^3\,N$	38·2	It is colorless at 0 degrees, but green at common temperatures.
Bicarburet of nitrogen,	$C^2\,N$	26·44	Called cyanogen, cannot support combustion.
Nitrous acid,	$O^4\,N$	46·2	Its vapor is a deep red color, and is rapidly absorbed by water.
Nitric acid,	$O^5\,N$	54·2	Extremely acid and caustic, emits suffocating fumes.
Ammonia,	$H^3\,N$	17	Sometimes called spirits of hartshorn, or volatile alkali.
Sulphurous acid,	$O^2\,S$	32·1	Does not support respiration or combustion. Used in bleaching and diseases of the skin.
Sulphuric acid,	$O^3\,S$	40·1	Sometimes called oil of vitriol. Very acid and corrosive.
Protoxide of carbon,	$C\,O$	14·12	Inflammable, transparent, colorless. Burns with a blue flame.
Carbonic acid,	$C\,O^2$	22·12	Non-supporter of combustion or respiration. Transparent and colorless.
Hydruret of carbon,	$H\,C$	7·12	Sometimes called olefiant gas. Burns with a rich yellow flame.
Bihydruret of carbon,	$H^2\,C$	8·12	Fire-damp, which causes the explosions in coal mines.
Bisulphuret of carbon,	$S^2\,C$	38·32	Very volatile. Evaporating rapidly at natural temperature.

Bicarburetted hydrogen,	C^2H		
Boracic acid,	BO^3	35	Easily fused. Much used with soda as a flux.
Chlorous acid,	O^4Cl	67·5	Explodes at a low temperature, dangerous to obtain. The fumes with phosphorus to be carefully avoided.
Chloric acid,	O^5Cl	75·5	Dissolves zinc and iron.
Hydrochloric acid,	HCl	36·5	Muriatic acid. Great affinity for water. Possesses an acrid, pungent, suffocating odor.
Quadrochlorine of nitrogen,	Cl^4O	156·2	Detonates with violence when exposed to heat. Its odor is penetrating and insupportable.
Nitro-muriatic acid,			Composed of chlorine 1, water 1, and nitrous acid 1. Known by the name of Aqua Regia, from its power of dissolving gold.
Iodic acid,	IO^5	166·5	Obtained from iodine and nitric acid.
Teriodide of nitrogen,	I^3N	393·7	Detonates by a slight pressure.
Hydriodic acid,	HI	127·5	Acts powerfully upon mercury.
Hydrofluoric acid,	HF	19·7	Its vapors highly irritating. Produces ulceration on the skin.
Phosphoric acid,	P^2O^5	71·4	
Phosphorous acid,	P^2O^3	55·4	Powerful taste, and a disagreeable fœtid smell. It is a powerful deoxidating agent. Precipitates gold, silver, mercury, and platinum in the metallic form.
Phosphuretted hydrogen,	H^3P^2	34·4	Transparent and colorless. It detonates with oxygen when heated to 300°, or when the electric spark passes through it.
Selenious acid,	O^2Se	56	
Selenic acid,	O^3Se	64	Bears a great resemblance to sulphuric acid.
Selenureted hydrogen,	HSe	41	
Protoxide of iron,	OFe	36	
Peroxide of iron,	O^3Fe^2	80	Or sesquioxide. The brown rust of iron consists of this oxide. The color is red.

TABLE

Showing the Symbols and Equivalents of Binary Compounds. (*Continued.*)

Name of Compound.	Symbol.	Equivalent.	Remarks.
Black oxide of iron, . .	$O^4 Fe^3$	116	This compound is formed when iron is oxidated in the air, or in contact with water at a high temperature.
Protoxide of lead, ...	$O Pb$	111·7	Commonly called *litharge.* Used in flint glass.
Dinoxide of lead, ...	$O Pb^2$	215·4	
Quadrotisoxide of lead, ..	$O^4 Pb^3$	343·1	Called red lead. Much employed as a pigment.
Binoxide of lead, ...	$O^2 Pb$	119·7	
Dinoxide of copper, .. .	$O Cu^2$	71·4	Called red oxide of copper. Native production. Found in copper mines in crystals of a red color.
Protoxide of copper, . .	$O Cu$	39·7	Called black oxide of copper, or copper black.
Binoxide of copper, . .	$O^2 Cu$	47·7	
Protoxide of zinc, ...	$O Z$	40·3	The only combination of oxygen and zinc we know.
Sesquioxide of antimony,	$O^3 Sb^2$	153·2	Occurs native; commonly called oxide of Antimony.
Antimonious acid, ...	$O^4 Sb^2$	161·2	It combines with alkalies by fusing them together.
Antimonic acid, ...	$O^5 Sb^2$	169·2	
Protoxide of tin, or stannum, ...	$O Sn$	66·9	Sometimes called black oxide of tin; great attraction for oxygen.
Binoxide of tin, ...	$O^2 Sn$	74·9	Occurs native, generally associated with oxide of iron.
Bisulphuret of tin, ...	$S^2 Sn$	91·1	Formerly called mosaic gold. Used in the arts to furnish a bronze, termed bronze powder.

Chloride of tin, ...	$Cl\,Sn$	94·4	Powerful deoxidating agent. Used in calico printing, and as a mordant fixing colors.
Bichloride of tin, . . .	$Cl^2\,Sn$	129·9	Called permuriate. Used in dyeing and calico printing.
Protoxide of bismuth, .	$O\,Bi$	79·5	
Chloride of bismuth, . .	$Cl\,Bi$	107	It was formerly called butter of bismuth.
Protoxide of manganese, .	$O\,Mn$	35·7	
Sesquioxide of manganese,	$O^3\,Mn^2$	79·4	It occurs native; pure, and as a hydrate.
Red oxide of manganese,	$O^4\,Mn^3$	115·1	
Binoxide of manganese, .	$O^2\,Mn$	43·7	Used in the preparation of oxygen and chlorine. It is used to give a dark coating to earthenware.
Protoxide of cobalt, . .	$O\,Co$	37·5	Commonly called smalt when combined with a little silica and potassa. In this state it is much employed in coloring glass and glazing of earthenware.
Arsenious acid, ...	$O^3\,As^2$	99·4	Extremely poisonous, either internally or externally.
Arsenic acid, ...	$O^5\,As^2$	115·4	Considered as noxious as arsenious acid, or more so.
Sesquioxide of arsenic, .	$H^3\,As^2$	78·4	Deleterious. Killed Gehlen in 1815. It has an offensive odor. Burns with a blue flame.
Protosulphuret of arsenic,	$S\,As$	53·8	It occurs native. Called realgar; used as a pigment.
Sesquisulphuret of arsenic,	$S\,As^2$	123·7	It occurs native; a brilliant yellow. Used as a pigment, known as "King's yellow." Used in calico printing to deoxidate indigo.
Sulphuret of mercury, .	$S\,Hg$	219·1	Artificial Cinnabar. When powdered it is vermillion.
Bisulphuret of mercury, .	$S^2\,Hg$	235·2	Called calomel.
Chloride of mercury, . .	$Cl\,Hg$	238·5	
Protoxide of silver, . .	$O\,Ag$	116·3	
Prochloride of silver, . .	$Cl\,Ag$	143·8	Exposed to the sun becomes purple. It occurs native, and much formed in chemical operations.
Protoxide of gold, . . .	$O\,Au$	208	
Binoxide of gold, . . .	$O^2\,Au$	216	It has a dark green color.
Teroxide of gold, . . .	$O^3\,Au$	224	Sometimes called auric acid.

TABLE

***Showing the Symbols and Equivalents of Binary Compounds.* (*Continued.*)**

VEGETABLE ACIDS AND SALTS.

Name of Compound.	Symbol.	Equivalent.	Remarks.
Acetic acid, ...	$O^3 C^4 H^3$	51·48	Pungent and agreeable odor; crystallizes at a low temperature; blisters the skin.
Tartaric acid, ...	$O^5 C^4 H^2$	66·48	Solution in water very sour; crystallizes in prisms.
Citric acid, ...	$O^4 C^4 H^2$	58·48	Nearly like tartaric acid.
Oxalic acid, ...	$O^3 C^2$	36·24	Powerful poison; two or three drachms produce death. It is like Epsom salts in appearance.
Benzoic acid, ...	$O^3 C^{14} H^5$	114·68	Is very white; its odor is fragrant and peculiar. Burns with a yellow flame.
Gallic acid, ...	$O^5 C^7 H^3$	85·84	Employed as a re-agent. It takes fire when exposed to heat, and produces salts of iron, the basis of black ink.
Hydrocyanic acid, . . .	$H C^2 N$	27·44	Prussic acid; dangerous poison.
Cyanic acid, ...	$O C^2 N$	34·44	It is liquid, volatile, and poisonous.
Ferrocyanic acid, ...	$C^6 H^2 N Fe$	109·32	Made into Prussian blue. Not poisonous in small doses.
Ethers, ...			Produced by the action of different acids on alcohol.
Sulphuric ether, ...	$O C^4 H^5$	37·48	Used to produce artificial cold; is very inflammable.
Nitrate of iron, ...	$O^5 N + O Fe$	90·2	Deliquescent, and attracts oxygen from the air.
Sulphate of iron, ...	$O^3 S + O Fe$	76·1	Made from iron pyrites. Copperas and green vitriol are procured from this salt.
Carbonate of iron, . . .	$O^2 C + O Fe$	58·12	Attracts oxygen from the air, and assumes the appearance of the rust of iron.

Acetate of iron, ...			Much employed in dyeing and calico printing.
Nitrate of lead, ...	$O^5 N + O Pb$	165·9	Used in calico printing.
Sulphate of lead, . . .	$O^3 N + O Pb$	151·8	
Phosphate of lead, . . .			Insoluble.
Carbonate of lead, . . .	$O^2 C + O Pb$	133·82	Usually called white lead.
Acetate of lead, ...	$A + O Pb$	163·18	Sugar of lead, used in dyeing and calico printing.
Subacetate of lead, . . .	$A + O^2 Pb^3$	274·88	Called Goulard's extract.
Chloride of lead, . . .			Patent yellow; is a mixture of chloride and oxide of lead.
Nitrate of copper. . . .	$O^5 N + O Cu$	93·9	Deliquescent, and kept in close vessels.
Sulphate of copper, . .	$O^3 N + O Cu$	79·8	Blue vitriol, employed as an escharotic.
Acetate of copper, . . .	$A + O Cu$	91·18	Crystals of a bluish green color.
Nitrate of zinc, ...	$O^5 N + O Z$	94·5	Crystallizes in four-sided prisms. Deliquescent.
Sulphate of zinc, . . .	$O^3 N + O Z$	80·4	White vitriol, rhombic prisms colorless.
Carbonate of zinc, . . .	$O^2 N + O Z$	62·42	
Acetate of zinc, ...	$A + O Z$	91·78	Crystallizes in rhomboidal prisms with shining lustre.
Nitrate of mercury, . .	$O^5 N + O Hg$	265 2	
Sulphate of mercury, . .	$O^3 N + O Hg$	251·1	
Bichloride of mercury, .	$Cl^2 Hg$	274	Corrosive sublimate—dangerous.
Nitrate of silver, . . .	$O^5 N + O Ag$	170·5	Darkens when exposed to light. Common marking ink is composed of this and a little mucilage.
Sulphate of silver, . . .	$O^3 N + O Ag$	152·54	
Phosphate of silver, . .	$O^5 P^2 + O Ag$		
Alcohol, ...	$O C^2 H^3$	23·24	
Common sugar, ...	$O^{11} C^{12} H^{11}$	172·44	
Starch sugar, ...	$O^4 C^{12} H^{12}$	199·44	
Uric acid, ...	$O^3 C^6 H N^2$	91·12	
Urea, ...	$2 (H^2 O C N)$	60·64	

RECIPE FOR DYEING HATS.

The bath for dyeing hats, employed by the London manufacturers, consists, for 12 dozen, of

144	Pounds of	logwood;
12	"	green sulphate of iron or copperas,
$7\frac{1}{2}$	"	verdigris.

The copper is made of a semi-cylindrical shape, and should be surrounded with an iron jacket, or case, into which steam may be admitted, so as to raise the temperature of the interior bath to 190° Fah., but no higher; otherwise the heat is apt to affect the stiffening varnish, called the gum, with which the body of the hat has been imbued. The logwood having been introduced and digested for some time, the copperas and verdigris are added in successive quantities, and in the above proportions, along with every successive two or three dozen of hats suspended upon the dipping machine. Each set of hats, after being exposed to the bath, with occasional airings, during 40 minutes, is taken off the pegs, and laid out upon the ground to be more completely blackened by the peroxydizement of the iron with the atmospheric oxygen. In 3 or 4 hours the dyeing is completed. When fully dyed, the hats are well washed in running water.

A skilful operator furnishes the following valuable information relative to the *stiffening* of hats. He says:

All the solutions of gums which I have hitherto seen prepared by hatters, have not been perfect, but in a certain degree a mixture, more or less, of the gums, which are merely suspended, owing to the consistency of the composition. When this is thinned by the addition of spirit, and allowed to stand, it lets fall a curdy-looking sediment, and to this circumstance may be ascribed the frequent breaking of hats. My method of proceeding is, first, to dissolve the gums, by agitation, in twice the due quantity of spirits, whether of wood or wine, and then, after complete solution, draw off one half the spirit in a still, so as to bring the stiffening to a proper consistency. No sediment subsequently appears on diluting this solution, however much it may be done. Both the spirit and alkali stiffenings for hats made by the following recipes, have been tried by some of the first houses in the trade, and have been much approved of:

Spirit Stiffening.—7 pounds of orange shellac; 2 pounds of gum sandarac; 4 oz. of gum mastic; $\frac{1}{2}$ pound of amber resin; 1 pint of solution of copal; 1 gallon of spirit of wine, or wood naphtha.

The shellac, sandarac, mastic, and resin, are dissolved in the spirit, and the solution of copal is added last.

Alkali stiffening.—7 Pounds of common block shellac; 1 pound of amber resin; 4 oz. gum thus; 4 oz. gum mastic; 6 oz. borax; $\frac{1}{2}$ pint of solution of copal.

The borax is first dissolved in a little warm water (say 1 gallon); this alkaline liquor is now put into a copper pan (heated by steam), together with the shellac, resin, thus, and mastic, and allowed to boil for some time, more warm water being added occasionally until it is of a proper consistence; this may be known by pouring a little on a cold slab, somewhat inclined, and if the liquor runs off at the lower end, it is sufficiently fluid. If, on the contrary, it sets before it reaches the bottom, it requires more water. When the whole of the gums seem dissolved, half a pint of wood naphtha must be introduced, with the solution of copal; then the liquor must be passed through a fine sieve, and it will be perfectly clear and ready for use. This stiffening is used hot. The hat bodies, before they are stiffened, should be steeped in a weak solution of soda in water, to destroy any acid that may have been left in them (as sulphuric acid is used in the making of the bodies). If this is not attended to, should the hat body contain any acid when it is dipped into the stiffening, the alkali is neutralised, and the gums consequently precipitated. After the body has been steeped in the alkaline solution, it must be perfectly dried in the stove before the stiffening is applied; when stiffened and stoved, it must be steeped all night in water to which a small quantity of the sulphuric acid has been added; this *sets* the stiffening in the hat body, and finishes the process. A good workman will stiffen 15 or 16 hats a day. If the proof is required cheaper, more shellac and resin must be introduced.

TABLE

Of Pressures at which certain Gases are Liquified.

Gas is the name given to those elastic fluids which are permanent under a considerable pressure, and at the temperature zero.

Name of Gas.	Becomes liquid.		Calculated boiling point, barometer = 30 inches.
	At	Under a pressure of	
Sulphurous Acid, . .	59 F.	3 atmospheres	4° Fahr.
Chlorine, ...	60	4 "	22
Ammonia, ...	50	6·5 "	64
Sulphuretted Hydrog.	50	17 "	142
Carbonic Acid, . .	32	36 "	229
Hydrochloric Acid, .	50	50 "	249
Deutoxide of Azote, .	45	50 "	254

TABLE

Showing the Proportionate Strength of Wheels in Horse Power, with a Velocity of 2·27 Feet per Second.

Pitch in inches.	Thickness in inches.	Breadth in inches.	Length in inches.	H.P. at 2·27 feet per second.	H. P. at 5 feet per second.	H. P at 10 feet per second	H. P. at 15 feet per second.	H. P. at 20 feet per second.	H. P. at 25 feet per second.	H. P. at 30 feet per second.
6·725	3·25	43·5	3 9	117·60	259·9	518·0[illegible]	776·2	1031·27	1295·4	1558·6
6·56	3·125	42·6	3·75	110·95	244·38	488·7[illegible]	732·7	977·00	1221·9	1466·7
6·30	3·00	40·9	3·60	102·25	225·34	459·38	675·66	900·00	1126·0[illegible]	1352·2
6·00	2·875	39·00	3·45	96·30	212·00	424·2[illegible]	636·34	848·4	1016·5	1272 68
5·77	2·750	37·34	3·30	85·87	189·11	378·32	572·26	756·5	945·79	1134·84
5·51	2·625	35 8	3·15	77·14	165·55	339·64	511·00	679·64	848·20	975·42
5·25	2·500	34·28	3 00	71·41	157·41	324·58	476·5	629·26	786·95	934·74
4·98	2·375	32·28	2·85	63·88	140·70	276·78	422·15	562·00	703·00	
4·72	2·250	30·68	2·70	57·50	126·70	253·39	371·14	506·60	633·19	
4·46	2·125	30·58	2·55	54 00	118·94	242·24	357·26	474·77	594·71	
4·2	2·00	27·20	2·40	45·33	99·84	199·25	299·50	399·40		
3·83	1·875	25·19	2·25	39·35	86·60	173·34	259 53	346·70		
3·68	1·750	23·92	2·10	34·60	76·16	152·42	288·00	304·84		
3·54	1·625	22·96	1·95	30·56	67·32	134·62	201·94			
3 15	1·500	20·47	1 80	25·58	56·30	112·30	169·00			
2·88	1·375	18·72	1·65	21·33	46·70	93·50	135·90			
2·625	1·250	17·00	1·50	17·70	38·10	77·00				
2·46	1·125	15·99	1·35	14·98	33·00	65·99				
2·10	1·00	13·65	1·20	11·37	25·00	50·00				

Formula $\frac{3^2{\cdot}25 \times 43{\cdot}5}{3{\cdot}9} = 117{\cdot}60$ strength, at 2·27 feet per second

Ft. per sec.		Strength in H. P.		Ft. per sec.		
Then as 2·27	:	117·60	::	5	:	259·9 h. p.

The thickness of cog multiplied by 2·1 equals the pitch, and the thickness of cog multiplied by 1·2 equals the length.

KNOT TABLE

The minutes and seconds of time in which a vessel passes over the measured knot being known, look for the corresponding number in this table, which will be the rate of the vessel in knots per hour.

Sec.	3m.	4m.	5m.	6m	7m.	8m.	9m.	10m.	11m.	12m.	13m.	14m.
0	20·000	15·000	12·000	10·000	8·571	7·500	6·666	6·000	5·454	5·000	4·615	4·285
1	19·890	14·938	11·960	9·972	8·551	7·484	6·654	5·990	5·446	4·993	4·609	4·280
2	19·780	14·876	11·920	9·944	8·530	7·468	6·642	5·980	5·438	4·986	4·603	4·275
3	19·672	14·815	11·880	9·917	8·510	7·453	6·629	5·970	5·429	4·979	4·597	4·270
4	19·564	14·754	11·841	9·890	8·490	7·438	6·617	5·960	5·421	4·972	4·591	4·265
5	19·460	14·694	11·803	9·863	8·470	7·422	6·605	5·950	5·413	4·965	4·585	4·260
6	19·355	14·634	11·764	9·830	8·450	7·407	6·593	5·940	5·405	4·958	4·580	4·255
7	19·251	14·575	11·726	9·809	8·430	7·392	6·581	5·930	5·397	4·951	4·574	4·250
8	19·150	14·516	11·688	9·783	8·413	7·377	6·569	5·921	5·389	4·945	4·568	4·245
9	19·047	14·457	11·650	9·756	8·391	7·362	6·557	5·911	5·381	4·938	4·562	4·240
10	18·947	14·400	11·613	9·729	8·372	7·346	6·545	5·901	5·373	4·931	4·556	4·235
11	18·848	14·342	11·575	9·703	8·352	7·331	6·533	5·891	5·365	4·924	4·551	4·230
12	18·750	14·285	11·538	9·677	8·333	7·317	6·521	5·882	5·357	4·918	4·545	4·225
13	18·652	14·220	11·501	9·651	8·314	7·302	6·509	5·872	5·349	4·911	4·539	4·220
14	18·556	14·173	11·465	9·625	8·295	7·287	6·498	5·863	5·341	4·904	4·534	4·215
15	18·451	14·118	11·428	9·600	8·275	7·272	6·486	5·853	5·333	4·897	4·528	4·210
16	18·367	14·063	11·392	9·574	8·256	7·258	6·474	5·844	5·325	4·891	4·522	4·206
17	18·274	14·008	11·356	9·549	8·238	7·243	6·463	5·834	5·317	4·884	4·516	4·201
18	18·181	13·953	11·323	9·524	8·219	7·229	6·451	5·825	5·309	4·878	4·511	4·196
19	18·090	13·900	11·285	9·490	8·200	7·214	6·440	5·815	5·301	4·871	4·505	4·191
20	18·000	13·846	11·250	9·473	8·181	7·200	6·428	5·806	5·294	4·864	4·500	4·186

KNOT TABLE—(Continued).

Sec.	3m.	4m.	5m.	6m.	7m.	8m.	9m.	10m.	11m.	12m.	13m.	14m.
21	17·910	13·793	11·214	9·448	8·163	7·185	6·417	5·797	5·286	4·858	4·494	4·181
22	17·823	13·740	11·180	9·424	8·144	7·171	6·405	5·787	5·278	4·851	4·488	4·176
23	17·734	13·688	11·145	9·399	8·127	7·157	6·394	5·778	5·270	4·845	4·483	4·171
24	17·647	13·636	11·111	9·375	8·108	7·142	6·383	5·769	5·263	4·838	4·477	4·166
25	17·560	13·584	11·077	9·350	8·090	7·128	6·371	5·760	5·255	4·832	4·472	4·161
26	17·475	13·533	11·043	9·326	8·071	7·114	6·360	5·750	5·247	4·825	4·466	4·157
27	17·391	13·483	11·009	9·302	8·053	7·100	6·349	5·741	5·240	4·819	4·460	4·152
28	17·307	13·432	10·975	9·278	8·035	7·086	6·338	5·732	5·232	4·812	4·455	4·147
29	17·225	13·383	10·942	9·254	8·017	7·072	6·327	5·723	5·224	4·806	4·449	4·142
30	17·143	13·333	10·909	9·230	8·000	7·059	6·315	5·714	5·217	4·800	4·444	4·137
31	17·061	13·284	10·876	9·207	7·982	7·045	6·304	5·705	5·210	4·793	4·438	4·133
32	16·981	13·235	10·843	9·183	7·964	7·031	6·293	5·696	5·202	4·787	4·433	4·128
33	16·901	13·186	10·810	9·160	7·947	7·017	6·282	5·687	5·195	4·780	4·428	4·123
34	16·822	13·138	10·778	9·137	7·929	7·004	6·271	5·678	5·187	4·774	4·422	4·118
35	16·744	13·091	10·764	9·113	7·912	6·990	6·260	5·669	5·179	4·768	4·417	4·114
36	16·667	13·043	10·714	9·090	7·895	6·977	6·250	5·660	5·172	4·761	4·411	4·110
37	16·590	12·996	10·682	9·068	7·877	6·963	6·239	5·651	5·164	4·755	4·406	4·105
38	16·514	12·950	10·651	9·045	7·860	6·950	6·228	5·642	5·157	4·749	4·400	4·100
39	16·438	12·903	10·619	9·022	7·843	6·936	6·217	5·633	5·150	4·743	4·395	4·095
40	16·363	12·857	10·588	9·000	7·826	6·923	6·207	5·625	5·142	4·737	4·390	4·090
41	16·289	12·811	10·557	8·977	7·809	6·909	6·196	5·616	5·135	4·730	4·384	4·086
42	16·216	12·766	10·526	8·955	7·792	6·896	6·185	5·607	5·128	4·724	4·379	4·081

KNOT TABLE—(Concluded).

Sec.	3m.	4m.	5m.	6m.	7m.	8m.	9m.	10m.	11m.	12m.	13m.	14m.
43	16·143	12·711	10·495	8·933	7·775	6·883	6·174	5·598	5·121	4·718	4·374	4·077
44	16·071	12·676	10·465	8·911	7·758	6·870	6·164	5·590	5·114	4·712	4·368	4·072
45	16·000	12·631	10·434	8·889	7·741	6·857	6·153	5·581	5·106	4·706	4·363	4·067
46	15·929	12·587	10·404	8·867	7·725	6·844	6·143	5·572	5·099	4·700	4·358	4·063
47	15·859	12·543	10·375	8·845	7·708	6·831	6·132	5·564	5·091	4·693	4·353	4·058
48	15·789	12·500	10·345	8·823	7·692	6·818	6·122	5·555	5·084	4·687	4·347	4·054
49	15·721	12·456	10·315	8·801	7·675	6·805	6·112	5·547	5·077	4·681	4·342	4·049
50	15·652	12·413	10·286	8·780	7·659	6·792	6·101	5·538	5·070	4·675	4·337	4·044
51	15·584	12·371	10·256	8·759	7·643	6·779	6·091	5·530	5·063	4·669	4·332	4·040
52	15·517	12·329	10·227	8·737	7·627	6·766	6·081	5·521	5·056	4·663	4·326	4·035
53	15·450	12·287	10·198	8·716	7·611	6·754	6·071	5·513	5·049	4·657	4·321	4·031
54	15·384	12·245	10·169	8·695	7·595	6·741	6·060	5·504	5·042	4·651	4·316	4·026
55	15·319	12·203	10·140	8·675	7·579	6·729	6·050	5·496	5·035	4·645	4·311	4·022
56	15·254	12·162	10·112	8·654	7·563	6·716	6·040	5·487	5·028	4·639	4·306	4·017
57	15·190	12·121	10·084	8·633	7·547	6·704	6·030	5·479	5·020	4·633	4·301	4·013
58	15·125	12·080	10·055	8·612	7·531	6·691	6·020	5·471	5·013	4·627	4·295	4·008
59	15·062	12·040	10·027	8·591	7·515	6·679	6·010	5·463	5·006	4·621	4·290	4·004

CEMENTS.

Shell-lac Cement, or Liquid Glue.—Fine orange shell-lac, bruised, 4 oz.; highly rectified spirit, 3 oz. Digest in a warm place, frequently shaking, till the shell-lac is dissolved. Rectified wood naphtha may be substituted for spirit of wine, where the smell is not objectionable. This is a most useful cement for joining almost any material.

Shell-lac Cement, without Spirit.—Boil 1 oz. of borax in 16 oz. water; add 2 oz. powdered shell-lac, and boil in a covered vessel till the lac is dissolved. This is cheaper than the above, and for many purposes, answers very well. Both are useful in fixing paper labels to tin, and to glass when exposed to damp.

Keller's Armenian Cement, for Glass, China, &c.—Soak 2 dr. of cut isinglass in 2 oz. of water for 24 hours; boil to 1 oz.; add 1 oz. of spirit of wine, and strain through linen. Mix this, while hot, with a solution of 1 dr. of mastic in 1 oz. of rectified spirit, and triturate with ½ dr. powdered gum ammoniac, till perfectly homogeneous.

Dr. Ure's Diamond Cement.—Isinglass, 1 oz.; distilled water, 6 oz.; boil to 3 oz., and add 1½ oz. of rectified spirit. Boil for a minute or two, strain, and add, while hot, first, ½ oz. of a milky emulsion of ammoniac, and then 5 dr. of tincture of mastic.

Hoenle's Cement, for Glass or Earthenware.—Shell-lac, 2 parts; Venice turpentine, 1 part. Fuse together, and form into sticks.

Cheese Cement, for Earthenware, &c.—Mix together white of egg, beaten to a froth, quick-lime, and grated cheese. Beat them to a paste, which forms an excellent cement.

Curd Cement.—Add ½ pint of vinegar to ½ pint of skimmed milk. Mix the curd with the whites of 5 eggs well beaten, and sufficient powdered quick-lime to form a paste. It resists water, and a moderate degree of heat.

Cement for joining Spar and Marble Ornaments, &c.—Melt together 8 parts of resin, 1 of wax, and stir in 4 parts, or as much as may be required, of Paris plaster. The pieces to be made hot.

Hensler's Cement.—Grind 3 parts of litharge, 2 of recently burnt lime, and 1 of white bole, with linseed oil varnish. This is a very tenacious cement, but it takes considerable time to dry.

Singer's Cement, for Electrical Machines and Galvanic Troughs.—Melt together 5 lbs. of resin, and 1 lb. of beeswax; and stir in 1 lb. of red ochre (highly dried, and still warm), and 4 oz. of Paris plaster, continuing the heat a little above 212°, and stirring constantly till all frothing ceases. Or (for troughs), resin, 6 lbs.; dried red ochre, 1 lb.; calcined plaster of Paris, ½ lb.; linseed oil, ½ lb.

Composition for welding Cast Steel.—Take of borax, 10 parts, sal ammoniac, 1 part; grind or pound them roughly together: then fuse them in a metal pot over a clear fire, taking care to continue the heat until all spume has disappeared from the surface. When the liquid appears clear, the composition is ready to be poured out to cool and concrete; afterwards, being ground to a fine powder, it is ready for use. * * * To use this composition: The steel to be welded is first raised to a "bright yellow" heat, it is then dipped among the welding powder, and again placed in the fire, until it attains the same degree of heat as before; it is then ready to be placed under the hammer.

Cast-Iron Cement.—Take of clean iron borings, or turnings, 1 cwt.; of sal-ammoniac 8 oz.; and 1 oz. of flour of sulphur. Mix them thoroughly, and add sufficient water. If the cement is not to be immediately used, care should be taken to keep the mixture soaked in water; if left dry, the cement will heat, and be spoiled.

Cement for Steam Pipe Joints, &c., with Faced Flanges.—To 2 parts of white lead mixed, add 1 part of red lead dry, grind, or otherwise mix them, to a consistence of thin putty, apply interposed layers, with one or two thicknesses of canvas or gauze wire, as the necessity of the case may require.

Glues.—1. A very strong glue is formed by throwing a small quantity of powdered chalk into melted common glue.

2. To make a glue which will resist the action of water—boil one pound of common glue in two quarts of skimmed milk.

Botany Bay Cement.—Take 1 part of Botany Bay gum, and melt and mix it with 1 part of brickdust.

Cap Cement.—As Singer's; but 1 pound of dried Venetian red may be substituted for the red ochre and Paris plaster.

Bottle Cement.—Resin 15 parts; tallow 4 (or wax 3) parts; highly dried red ochre 5 parts. The common kinds of sealing-wax are also used.

Turner's Cement.—Beeswax 1 oz.; resin ½ oz.; pitch ½ oz. Melt, and stir in fine brickdust.

Coppersmith's Cement.—Powdered quick-lime, mixed with bullock's blood, and applied immediately.

Engineers' Cement.—Equal weights of red and white lead, with drying oil, spread on tow or canvas. This is an admirable composition for uniting large stones in cisterns, &c.

Iron Cement for Closing the Joints of Iron Pipes.—Take of iron borings, coarsely powdered, 5 lbs.; of powdered sal-ammoniac 2 oz.; of sulphur 1 oz.; and water sufficient to moisten it. This composition hardens rapidly; but if time can be allowed it sets more firmly without the sulphur. It must be used as soon as mixed, and rammed tightly into the joints.

Cement for Steam Pipes.—Good linseed oil varnish ground, with equal weights of white lead, oxide of manganese, and pipeclay.

Gad's Hydraulic Cement.—Powdered clay 3 lbs.; oxide of iron 1 lb.; and boiled oil to form a stiff paste.

Cements for Masonry of Chambers of Chlorine, &c.—Equal parts of pitch, rosin, and plaster of Paris.

Roman Cement.—1 bushel of slacked lime; 3¼ lbs. of green copperas; and ½ bushel of fine gravel sand. The copperas should be dissolved in hot water. It must be stirred with a stick, and kept stirred continually while in use. Care should be taken to mix at once as much as may be requisite for one entire front, as it is very difficult to obtain the same shade or color a second time. It ought to be mixed the same day it is used. This is the English Roman cement.

The genuine Roman cement consists of the pulvis puteolanus, or puzzolene, a ferruginous clay from Puteoli, calcined by the fires of Vesuvius, lime, and sand, mixed with soft water. The only preparation which the puzzolene undergoes is that of pounding and sifting; but the ingredients are occasionally mixed with bullock's blood and suet, to give the composition greater tenacity.

Seal Engravers' Cement.—Resin 1 part, brickdust 1 part. Mix, with heat.

Marine Cement, commonly called Marine Glue.—Cut caoutchouc into small pieces, and dissolve it, by heat and agitation, in coal naphtha. Add to this solution powdered shell-lac, and heat the whole, with constant stirring, until combination takes place; then pour it, while hot, on metal plates, to form sheets. When used, it must be heated to 280° Fah., and applied with a brush.

Liquid Glue.—Dissolve bruised orange shell-lac in ⅞ of its weight of rectified spirit, or of rectified wood naphtha, by a gentle heat. It is very useful as a general cement and substitute for glue. Another kind may be made by dissolving 1 oz. of borax in 12 oz. of soft water, adding 2 oz. of bruised shell-lac, and boiling till dissolved, stirring it constantly.

Bank Note Glue.—Dissolve 1 lb. of fine glue, or gelatine, in water; evaporate it till most of the water is expelled; add ¼ lb. of brown sugar, and pour it into moulds. Some add a little lemon juice. It is also made with 2 parts of dextrine, 2 of water, and 1 of spirit.

Maissiat's Cement, as an Air-Tight Covering for Bottles, &c.—Melt india-rubber (to which 15 per cent. of wax or tallow may be added), and gradually add finely powdered quick-lime, till a change of odor shows that combination has taken place, and a proper consistence is obtained.

Cement for Attaching Metal Letters on Plate Glass.—Copal varnish 15 parts; drying oil 5 parts; turpentine 3 parts; oil of turpentine

2 parts; liquified glue 5 parts. Melt in a water bath, and add 10 parts of slacked lime.

Japanese Cement.—Mix rice flour intimately with cold water, and boil gently.

French Cement.—Mix thick mucilage of gum arabic with powdered starch.

Stone Cement.—River sand 20 parts; litharge 2 parts; quick-lime 1 part. Mix, with linseed oil.

Plumbers' Cement.—Resin 1 part; brick-dust 2 parts. Mix, with heat.

Parisian Cement.—Gum arabic 1 oz.; water 2 oz.; sufficient starch to thicken.

Cement for Floors.—The following style of floor is well adapted for plain country dwellings: Take two thirds of lime, and one of coal ashes, well sifted, with a small quantity of loam clay; mix the whole together, temper it well with water, and make it up into a heap; let it lie six or seven days, and then temper it again. After this, heap it up for three or four days, and repeat the tempering very high, till it becomes smooth, yielding, tough, and gluey. The ground being then levelled, lay the floor therewith about 2½ or 3 inches thick, making it smooth with a trowel. The hotter the season is the better; when thoroughly dried it makes a capital floor. Should a *better looking* floor be desired, take lime of rag stones, well tempered with white of eggs, and cover the floor half an inch thick with it before the under flooring is too dry. If this be well done, and the floor thoroughly dried, it will look, when rubbed with a little oil, as transparent as metal, or glass.

Common Paste.—To a table-spoonful of flour add gradually half a pint of cold water, and mix till quite smooth; add a pinch of powdered alum (some add a small pinch of powdered rosin), and boil for a few moments, stirring constantly. The addition of a little brown sugar, and a few grains of corrosive sublimate, will, it is said by practical chemists, preserve it for years.

Soft Cement.—Melt yellow wax with half its weight of common turpentine, and stir in a little Venetian red, previously well dried and finely powdered. This cement does very well as temporary stopping for joints and openings in glass and other apparatus, where the heat and pressure are not great.

Lutes, or Cements, for Closing the Joints of Apparatus.—Mix Paris plaster with water to a soft paste, and apply it immediately. It bears nearly a red heat. It may be rendered impervious by rubbing it over with wax and oil.

Another.—Slacked lime, made into a paste with white of egg, or a solution of gelatine

Another. Fat Lute.—Finely powdered clay, moistened with water, and beaten up with boiled linseed oil. Roll it into cylinders,

and press it on the joints of the vessels, which must be perfectly dry. It is rendered more secure by binding it with strips of linen moistened with white of egg.

Another.—Linseed meal beaten to a paste with water.

Another.—Slips of moistened bladder, smeared with white of egg.

Fire and Waterproof Cement.—To half a pint of milk put an equal quantity of vinegar, in order to curdle it; then separate the curd from the whey, and mix the latter with four or five eggs, beating the whole well together. When it is well mixed add a little lime through a sieve, until it has acquired the consistence of a thick paste. With this cement broken vessels may be united. It resists water, and, to a certain extent, fire.

Fire Lutes.—The following composition will enable glass vessels to sustain an incredible degree of heat: Take fragments of porcelain, pulverize, and sift them well, and add an equal quantity of fine clay, previously softened with as much of a saturated solution of muriate of soda as is requisite to give the whole a proper consistence. Apply a thin and uniform coat of this composition to the glass vessels, and allow it to dry slowly before they are put into the fire.

Another.—Equal parts of coarse and refractory clay, mixed with a little hair, form a good lute.

A Cement for Stopping the Fissures of Iron Vessels.—Take two ounces of muriate of ammonia, 1 ounce of flour of sulphur, and 16 ounces of cast-iron filings, or turnings. Mix them well in a mortar, and keep the powder dry. When the cement is wanted take one part of this and twenty parts of clean iron filings, or borings; grind them together in a mortar, mix them with water to a proper consistence, and apply them between the joints. This cement answers for flanges of pipes, &c., about steam-engines.

Genuine Armenian Cement.—"The jewellers of Turkey, who are mostly Armenians," says Mr. Eton, a very intelligent traveller, and at one time a resident and consul in that country, "have a singular method of ornamenting watch cases, &c., with diamonds and other precious stones, by simply glueing or cementing them on. The stone is set in silver or gold, and the lower part of the metal made flat, or to correspond with the part to which it is to be fixed. It is then warmed gently, and the glue applied, which is so very strong that the parts thus cemented never separate. This glue, which will firmly unite bits of glass, and even polished steel, and may of course be applied to a vast variety of useful purposes, is thus made:—Dissolve five or six bits of gum mastic, each the size of a large pea, in as much spirits of wine as will suffice to render it liquid; in another vessel dissolve as much isinglass, previously a little softened in water (though none of the water must be used), in French brandy, or good rum, as will make a two ounce phial of

very strong glue, adding two small bits of gum galbanum, or ammoniacum, which must be rubbed or ground till they are dissolved. Then mix the whole with a sufficient heat, keep the glue in a phial closely stopped, and when it is to be used set the phial in boiling water."

Another.—Thick isinglass glue 1 part; thick mastic varnish 1 part. Melt the glue, mix, and keep it in a closely corked phial. For use, put the phial in hot water.

Elastic Cement for Bells.—Dissolve in good brandy a sufficient quantity of isinglass, so as to be as thick as molasses.

A very strong Carpenters' Glue.—Dissolve an ounce of the best isinglass, with a moderate heat, in a pint of water. Take this solution, and strain it through a piece of cloth, and add to it a proportionate quantity of the best glue, which has been previously soaked for about twenty-four hours, and a gill of vinegar. After the whole of the materials have been brought into a solution, let it once boil up, and strain off the impurities. This glue is well adapted for any work which requires particular strength, and where the joints themselves do not contribute towards the combination of the work; or in small fillets and mouldings, and carved patera, that are held on the surface by the glue.

A Glue for Inlaying Brass or Silver Strings, &c.—Melt your glue as usual, and to every pint add of finely powdered rosin and finely powdered brickdust two spoonfuls each; incorporate the whole together, and it will hold the metal much faster than any common glue.

A strong Glue that will resist Moisture.—Dissolve gum sandarac and mastic, of each ¼ of an ounce, in ¼ of a pint of spirit of wine, to which add ¼ of an ounce of clear turpentine. Now take strong glue, or that in which isinglass has been dissolved; then, putting the gums into a double glue-pot, add by degrees the glue, constantly stirring it over the fire till the whole is well mixed; then strain it through a cloth, and it is ready for use. You may now return it into the glue-pot, and add ½ an ounce of very finely powdered glass; use it quite hot. If you join two pieces of wood together with it you may, when perfectly hard and dry, immerse it in water and the joint will not separate.

A Paste for laying Cloth or Leather on Table Tops.—To a pint of the best wheaten flour add two table spoonfuls of finely powdered rosin, and one spoonful of powdered alum. Mix them well together, put them into a pan, and add by degrees rain water, carefully stirring it till it is of the consistence of thinnish cream; put it into a saucepan over a clear fire, keeping it constantly stirred, that it may not get lumpy. When it is of a stiff consistence, so that the spoon will stand upright in it, it is done enough. Be careful to stir it well from the bottom, for it will burn if not well attended to. Empty it out into a pan, and cover it over till cold, to prevent a

skin forming on the top, which would make it lumpy. This paste is very superior for the purpose, and adhesive. To use it for cloth or baize spread the paste evenly and smoothly on the top of the table, and lay your cloth on it, pressing and smoothing it with a flat piece of wood; let it remain till dry; then trim the edges close to the cross-banding. If you cut it close at first it will, in drying, shrink and look bad where it meets the banding all round. If used for leather, the leather must be first previously dampened, and the paste then spread over it; then lay it on the table, and rub it smooth and level with a linen cloth, and cut the edges close to the banding with a short knife. Some lay their table-cover with glue instead of paste, and for cloth perhaps it is the best method; but for leather it is not proper, as glue is apt to run through. In using it for cloth, great care must be taken that your glue is not too thin, and that you rub the cloth well down with a thick piece of wood made hot at the fire, for the glue soon chills. You may by this method cut off the edges close to the border at once.

Cement Stopping.—Mix equal quantities of sawdust, of the same wood required to be stopped, and clear glue; and with this stop up the holes or defects of the wood. Where the surface is to be japanned or painted, whiting may be used instead of sawdust. Be sure to let the stopping dry before you attempt to finish the surface.

Mahogany-colored Cement.—Melt two ounces of beeswax, and half an ounce of rosin, together; then add half an ounce of Indian red, and a small quantity of yellow ochre to bring the cement to the desired color. Keep it in a pipkin for use.

A Cement to stop Flaws or Cracks in Wood of any Color.—Put any quantity of fine sawdust, of the same wood your work is made with, into an earthen pan, and pour boiling water on it, stir it well, and let it remain for a week or ten days, occasionally stirring it; then boil it for some time, and it will be of the consistence of pulp or paste; put it into a coarse cloth, and squeeze all the moisture from it. Keep for use, and when wanted mix a sufficient quantity of thin glue to make it into a paste; rub it well into the cracks, or fill up the holes in your work with it. When quite hard and dry, clean your work off, and, if carefully done, you will scarcely discover the imperfection.

Fireproof Stucco for Wood, &c.—Take moist gravelly earth (previously washed), and make it into stucco with the following composition: Pearlashes two parts; water five parts; common clay one part. It has been tried on a large scale and found to answer.

Terra Cotta.—Potter's clay, Ryegate sand, and water, each a sufficient quantity. Model and bake.

Pew's Composition for covering Buildings.—Take the hardest and purest limestone (white marble is to be preferred), free from sand, clay, or other matter; calcine it in a reverberatory furnace, pulverize and pass it through a sieve. One part, by weight, is to be mixed

with two parts of clay well baked and similarly pulverized, conducting the whole operation with great care. This forms the first powder. The second is to be made of one part of calcined and pulverized gypsum, to which is added two parts of clay, baked and pulverized. These two powders are to be combined, and intimately incorporated, so as to form a perfect mixture. When it is to be used, mix it with about a fourth part of its weight of water, added gradually, stirring the mass well the whole time, until it forms a thick paste, in which state it is to be spread like mortar upon the desired surface. It becomes in time as hard as stone, allows no moisture to penetrate, and is not cracked by heat. When well prepared it will last any length of time. When in its plastic or soft state, it may be colored of any desired tint.

TABLE

Of Analysis of certain Organic Substances, from the best authorities.

	Carbon.	Hydrogen.	Oxygen.	Nitrogen.	Total.
Sugar, ...	42·225	6·600	51·175	—	100
Starch, . . .	44·250	6·674	49·076	—	100
Gum, ...	42·682	6·374	50·944	—	100
Lignin, . . .	52·53	5·69	41·78	—	100
Tannin, . . .	52·590	3·825	43·585	—	100
Indigo, . . .	73·260	2·500	10·43	13·81	100
Camphor, . .	73·38	10·67	14·61	·34	100
Caoutchouc, . .	87·2	12·8	—	—	100
Albumen, . .	52·883	7·540	23·872	15·705	100
Fibrin, . . .	53·36	7·021	19·685	19·934	100
Casein, . . .	59·781	7·429	11·409	21·381	100
Urea, ...	18·9	9·7	26·2	45·2	100
Gelatine, . . .	47·881	7·914	27·207	16·998	100
Picromel, . .	54·53	1·82	43·65	—	100
Hordein, . . .	44·2	6·4	47·6	1·8	100
Emelin, . . .	64·57	7·77	22·95	4·3	100
Veratrin, . . .	66·75	8·54	19·60	5·04	100
Cinchonin, . .	77·81	7·37	5·93	8·89	100
Quinin, . . .	75·76	7·52	8·61	8·11	100
Brucin, . . .	70·88	6·66	17·39	5·07	100
Strychnin, . .	76·43	6·70	11·06	5·81	100
Narcotin, . .	65·00	5·50	26·99	2·51	100
Morphin, . .	72·340	6·366	16·299	4·995	100

TABLE

To Calculate the Pitch of a Toothed Wheel, when the radius and number of teeth are given; and the RADIUS, *when the pitch and number of teeth are given, from* 10 *to* 159 *teeth.*

No. of Teeth.	Radius.	No. of Teeth.	Radius.	No. of Teeth.	Radius.	No. of Teeth.	Radius.	No. of Teeth.	Radius.
10	1·618	40	6·373	70	11·144	100	15·918	130	20·692
11	1·774	41	6·532	71	11·303	101	16·077	131	20·851
12	1·932	42	6·691	72	11·463	102	16·236	132	21·010
13	2·089	43	6·850	73	11·622	103	16·395	133	21·169
14	2·247	44	7·009	74	11·781	104	16·554	134	21·328
15	2·405	45	7·168	75	11·940	105	16·713	135	21·488
16	2·563	46	7·327	76	12·099	106	16·873	136	21·647
17	2·721	47	7·486	77	12·258	107	17·032	137	21·806
18	2·879	48	7·645	78	12·417	108	17·191	138	21·965
19	3·038	49	7·804	79	12·576	109	17·350	139	22·124
20	3·196	50	7·963	80	12·735	110	17·509	140	22·283
21	3·355	51	8·122	81	12·895	111	17·668	141	22·442
22	3·513	52	8·281	82	13·054	112	17·827	142	22·602
23	3·672	53	8·440	83	13·213	113	17·987	143	22·761
24	3·830	54	8·599	84	13·370	114	18·146	144	22·920
25	3·989	55	8·758	85	13·531	115	18·305	145	23·079
26	4·148	56	8·917	86	13·690	116	18·464	146	23·238
27	4·307	57	9·076	87	13·849	117	18·623	147	23·397
28	4·465	58	9·235	88	14·008	118	18·782	148	23·556
29	4·624	59	9·394	89	14·168	119	18·941	149	23·716
30	4·788	60	9·558	90	14·327	120	19·101	150	23·875
31	4·942	61	9·712	91	14·486	121	19·260	151	24·034
32	5·101	62	9·872	92	14·645	122	19·419	152	24·193
33	5·260	63	10·031	93	14·804	123	19·578	153	24·352
34	5·419	64	10·190	94	14·963	124	19·737	154	24·511
35	5·578	65	10·349	95	15·122	125	19·896	155	24·620
36	5·737	66	10·508	96	15·281	126	20·055	156	24·830
37	5·896	67	10·667	97	15·440	127	20·214	157	24·989
38	6·055	68	10·826	98	15·600	128	20·374	158	25·148
39	6·214	69	10·985	99	15·759	129	20·533	159	25·307

RULE 1.—Divide the required radius by the radius opposite the given number of teeth in the table; the quotient will be the required pitch of the wheel.

Example. To find the pitch of a wheel whose radius is 43 inches, that shall contain 90 teeth:

Required radius $43 \div 14{\cdot}327 = 3$-inch pitch.

RULE 2.—Multiply the radius opposite the given number of

teeth in the table, by the pitch required; the product will be the required radius of the wheel.

Example. To find the radius of a wheel that shall contain 48 teeth of 2½-inch pitch:

In the Table, radius 7·645 × 2·5 = 19$\frac{1}{10}$ inches nearly.

CABLES.

TABLE

For finding the Strain that may safely be applied to a good Hempen Cable.

Circum.	Pounds.	Circumfer.	Pounds.	Circumfer.	Pounds.
6·	4320·	10·25	12607·5	14·50	25230·
6·25	4687·5	10·50	13230·	14·75	26107·5
6·50	5070·	10·75	13867·5	15·	27000·
6·75	5467·5	11·	14520·	15·25	27907·5
7·	5880·	11·25	15187·5	15·50	28830·
7·25	6307·5	11·50	15870·	15·75	29767·5
7·50	6750·	11·75	16567·5	16·	30720·
7·75	7207·5	12·	17280·	16·25	31687·5
8·	7680·	12·25	18007·5	16·50	32670·
8·25	8167·5	12·50	18750·	16·75	33667·5
8·50	8670·	12·75	19507·5	17·	34680·
8·75	9187·5	13·	20280·	17·25	35707·5
9·	9720·	13·25	21067·5	17·50	36750·
9·25	10267·5	13·50	21870·	17·75	37807·5
9·50	10830·	13·75	22687·5	18·	38880·
9·75	11407·5	14·	23520·	18·25	39967·5
10·	12000·	14·25	24367·5		

To ascertain the Strength of Cables.—Multiply the square of the circumference in inches by 120, and the product is the weight the cable will bear in pounds, with safety.

To ascertain the Strength of Ropes.—Multiply the square of the circumference in inches by 200, and it gives the weight the rope will bear in pounds, with safety.

To ascertain the Weight of Manilla Ropes and Hawsers.—Multiply the square of the circumference in inches by ·03, and the product is the weight in pounds of a foot in length.

This is but an approximation, sufficiently correct for many purposes.

TABLE

Showing the Size of Cables and Anchors proportional to the Tonnage of Vessels.

Tonnage of vessels.	Cables. Circumfer. in inches.	Chain Cables. Diam. in inches.	Proof in tons.	Weight of Anchor in pounds.	Weight of a fathom of chain.	Weight of a fathom of Cable.
5	3·	·$\frac{5}{16}$	·$\frac{3}{4}$	56	5·$\frac{1}{2}$	2·1
8	4·	·$\frac{3}{8}$	1·$\frac{3}{4}$	84	8·	4·
10	4·$\frac{1}{2}$	·$\frac{7}{16}$	2·$\frac{1}{2}$	112	11·	4·6
15	5·$\frac{1}{2}$	·$\frac{1}{2}$	4·	168	14·	6·5
25	6·	·$\frac{9}{16}$	5·	224	17·	8·4
40	6·$\frac{1}{2}$	·$\frac{5}{8}$	6·	336	24·	9·8
60	7·	·$\frac{11}{16}$	7·	392	27·	11·4
75	7·$\frac{1}{2}$	·$\frac{3}{4}$	9·	532	30·	13·
100	8·	·$\frac{13}{16}$	10·	616	36·	15·
130	9·	·$\frac{7}{8}$	12·	700	42·	18·9
150	9·$\frac{1}{2}$	·$\frac{15}{16}$	14·	840	50·	21·
180	10·$\frac{1}{2}$	1·	16·	952	56·	25·7
200	11·	1·$\frac{1}{16}$	18·	1176	60·	28·2
240	12·	1·$\frac{1}{8}$	20·	1400	70·	33·6
270	12·$\frac{1}{2}$	1·$\frac{3}{16}$	21·	1456	78·	36·4
320	13·$\frac{1}{2}$	1·$\frac{1}{4}$	22·$\frac{1}{2}$	1680	86·	42·5
360	14·	1·$\frac{5}{16}$	25·	1904	96·	45·7
400	14·$\frac{1}{2}$	1·$\frac{3}{8}$	27·	2072	104·	49·
440	15·$\frac{1}{2}$	1·$\frac{7}{16}$	30·	2240	115·	56·
480	16·	1·$\frac{1}{2}$	33·	2408	125·	59·5
520	16·$\frac{1}{2}$	1·$\frac{9}{16}$	36·	2800	136·	63·4
570	17·	1·$\frac{5}{8}$	39·	3360	144·	67·2
620	17·$\frac{1}{2}$	1·$\frac{11}{16}$	42·	3920	152·	71·1
680	18·	1·$\frac{3}{4}$	45·	4200	161·	75·6
740	19·	1·$\frac{13}{16}$	49·	4480	172·	84·2
820	20·	1·$\frac{7}{8}$	52·	5600	184·	93·3
900	22·	1·$\frac{15}{16}$	56·	6720	196·	112·9
1000	24·	1·	60·	7168	208·	134·6

TABLE

For finding the Strain that may be applied to a Hempen Rope with safety.

Circum.	Pounds.	Circumfer.	Pounds.	Circumfer.	Pounds.
1·	200·	3·50	2450·	6·	7200·
1·25	312·5	3·75	2812·5	6·25	7812·5
1·50	450·	4·	3200·	6·50	8450·
1·75	612·5	4·25	3612·5	6·75	9112·5
2·	800·	4·50	4050·	7·	9800·
2·25	1012·5	4·75	4512·5	7·25	10512·5
2·50	1250·	5·	5000·	7·50	11250·
2·75	1512·5	5·25	5512·5	7·75	12012·5
3·	1800·	5·50	6050·	8·	12800·
3·25	2112·5	5·75	6612·5		

TABLE

Of Weight of Copper Rods or Bolts, from $\frac{1}{4}$ to 4 inches in diameter, and 1 foot in length.

Diam.	Pounds.	Diameter.	Pounds.	Diameter.	Pounds.
$\cdot\frac{1}{4}$	·1892	$1\cdot\frac{1}{8}$	3·8312	$2\cdot\frac{3}{8}$	17·0750
$\cdot\frac{5}{16}$	·2956	$1\cdot\frac{3}{16}$	4·2688	$2\cdot\frac{1}{2}$	18·9161
$\cdot\frac{3}{8}$	·4256	$1\cdot\frac{1}{4}$	4·7298	$2\cdot\frac{5}{8}$	20·8562
$\cdot\frac{7}{16}$	·5794	$1\cdot\frac{5}{16}$	5·2140	$2\cdot\frac{3}{4}$	22·8913
$\cdot\frac{1}{2}$	·7567	$1\cdot\frac{3}{8}$	5·7228	$2\cdot\frac{7}{8}$	25·0188
$\cdot\frac{9}{16}$	·9578	$1\cdot\frac{7}{16}$	6·2547	3·	27·2435
$\cdot\frac{5}{8}$	1·1824	$1\cdot\frac{1}{2}$	6·8109	$3\cdot\frac{1}{8}$	29·5594
$\cdot\frac{11}{16}$	1·4307	$1\cdot\frac{9}{16}$	7·3898	$3\cdot\frac{1}{4}$	31·9722
$\cdot\frac{3}{4}$	1·7027	$1\cdot\frac{5}{8}$	7·9931	$3\cdot\frac{3}{8}$	34·4815
$\cdot\frac{13}{16}$	1·9982	$1\cdot\frac{3}{4}$	9·2702	$3\cdot\frac{1}{2}$	37·0808
$\cdot\frac{7}{8}$	2·3176	$1\cdot\frac{7}{8}$	10·6420	$3\cdot\frac{5}{8}$	39·7774
$\cdot\frac{15}{16}$	2·6605	2·	12·1082	$3\cdot\frac{3}{4}$	42·5680
1·	3·0270	$2\cdot\frac{1}{8}$	13·6677	$3\cdot\frac{7}{8}$	45·4550
$1\cdot\frac{1}{16}$	3·4170	$2\cdot\frac{1}{4}$	15·3251	4·	48·4330

Weight of a copper rod 12 inches long and 1 in. diameter = 3·039 lbs.

Weight of a brass rod 12 inches long and 1 inch diameter = 2·86 lbs.

TABLE

Of the Weight of Riveted Copper Pipes, from 5 to 30 inches in diameter, from 3 to $\frac{5}{16}$ thick, and 1 foot in length.

Diameter in inches.	Thickness in 16ths.	Weight in pounds.	Diameter in inches.	Thickness in 16ths.	Weight in pounds.	Diameter in inches.	Thickness in 16ths.	Weight in pounds.
5·	3	12·497	9·½	4	30·598	19·	4	60·142
5·	4	16·880	10·	4	32·208	19·	5	75·233
5·½	3	13·628	11·	4	35·200	20·	5	78·208
5·½	4	18·395	12·	4	38·456	21·	5	82·984
6·	3	14·765	13·	4	41·456	22·	5	86·771
6·	4	19·908	14·	4	44·640	23·	5	90·571
6·½	3	15·897	15·	4	47·646	24·	5	94·308
6·½	4	21·415	15·	5	59·588	25·	5	98·122
7·	3	17·034	16·	4	50·752	26·	5	101·897
7·	4	22·932	16·	5	63·470	27·	5	105·700
7·½	4	24·447	17·	4	53·856	28·	5	109·446
8·	4	25·961	17·	5	67·844	29·	5	113·221
8·½	4	27·471	18·	4	57·037	30·	5	116·997
9·	4	28·985	18·	5	71·258			

The above weights include the laps on the sheets for riveting and caulking.

The weights of the rivets are not added; the *number* per linear foot of pipe depends upon the distance they are placed apart, and their *size* upon the diameter of the pipe.

TABLE

Showing the Capacity of Cisterns in Gallons.

For each 10 Inches in Depth.

Feet Diam.		Feet Diam.		Feet Diam.		Feet Diam.	
2	19·5	5	122·40	8	313·33	12	705·
2½	30·6	5½	148·10	8½	353·72	13	827·4
3	44·06	6	176·25	9	396·56	14	959·6
3½	59·97	6½	206·85	9½	461·40	15	1101·6
4	78·33	7	239·88	10	489·20	20	1958·4
4½	99·14	7½	275·40	11	592·40	25	3059·9

TABLE

Containing the weight of a Square Foot of Copper and Lead in lbs. avoirdupois, from $\frac{1}{32}$ to $\frac{1}{2}$ an inch in thickness, advancing by $\frac{1}{32}$.

Thickness.	Copper.	Lead.	Thickness.	Copper.	Lead.
$\frac{1}{32}$	1·45	1·85	$\frac{1}{4}$ and $\frac{1}{32}$	13·07	16·62
$\frac{1}{16}$	2·90	3·70	$\frac{1}{4}$ " $\frac{1}{16}$	14·52	18·47
$\frac{3}{32}$	4·35	5·54	$\frac{1}{4}$ " $\frac{3}{32}$	15·97	20·31
$\frac{1}{8}$	5·80	7·39	$\frac{3}{8}$	17·41	22·16
$\frac{1}{8}$ and $\frac{1}{32}$	7·26	9·24	$\frac{3}{8}$ " $\frac{1}{32}$	18·87	24·00
$\frac{1}{8}$ " $\frac{1}{16}$	8·71	11·08	$\frac{3}{8}$ " $\frac{1}{16}$	20·32	25·85
$\frac{1}{8}$ " $\frac{3}{32}$	10·16	12·93	$\frac{3}{8}$ " $\frac{3}{32}$	21·77	27·70
$\frac{1}{4}$	11·61	14·77	$\frac{1}{2}$	23·22	29·55

TABLE

Of the Weight of a Square Foot of Sheet Iron in lbs. avoirdupois, the thickness being the number on the wire gauge.—No. 1 is $\frac{5}{16}$ of an inch; No. 4, $\frac{1}{4}$; No. 11, $\frac{1}{8}$, &c.

No. on wire gauge, .	1	2	3	4	5	6	7	8
Pounds avoirdu.,	12·5	12	11	10	9	8	7·5	7
No. on wire gauge, .	9	10	11	12	13	14	15	16
Pounds avoirdu.,	6	5·68	5	4·62	4·31	4	3·95	3
No. on wire gauge, .	17	18	19	20	21	22		
Pounds avoirdu.,	2·5	2·18	1·93	1·62	1·5	1·37		

TABLE

Of the Weight of a Square Foot of Boiler Plate Iron, from $\frac{1}{8}$ to 1 inch thick, in lbs. avoirdupois.

$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1 in.
5	7·5	10	12·5	15	17·5	20	22·5	25	27·5	30	32·5	35	37·5	40

TABLE

Showing the Quantity of Water per Linear Foot in Pumps, or Vertical Pipes of different Diameters.

Diameter of pump in inches.	Number of gallons per linear foot.	Number of cubic feet per linear foot.	Diameter of pump in inches.	Number of gallons per linear foot.	Number of cubic feet per linear foot.
2	·136	·0218	8	2·176	·3490
2¼	·172	·0276	8¼	2·314	·3712
2½	·212	·0340	8½	2·456	·3940
2¾	·257	·0412	8¾	2·603	·4175
3	·306	·0490	9	2·754	·4417
3¼	·359	·0576	9¼	2·909	·4666
3½	·416	·0668	9½	3·068	·4923
3¾	·478	·0766	9¾	3·232	·5184
4	·544	·0872	10	3·400	·5454
4¼	·614	·0985	10¼	3·572	·5730
4½	·688	·1104	10½	3·748	·6013
4¾	·767	·1230	10¾	3·929	·6302
5	·850	·1363	11	4·114	·6599
5¼	·937	·1503	11¼	4·303	·6902
5½	1·028	·1649	11½	4·496	·7212
5¾	1·124	·1803	11¾	4·694	·7529
6	1·224	·1963	12	4·896	·7853
6¼	1·328	·2130	12½	5·312	·8521
6½	1·436	·2304	13	5·746	·9217
6¾	1·549	·2489	13½	6·196	·9939
7	1·666	·2672	14	6·664	1·0689
7¼	1·787	·2866	15	7·650	1·2271
7½	1·912	·3067	16	8·704	1·3962
7¾	2·042	·3275	18	11·016	1·7670

Examples illustrative of the Utility of the Table.

1. Required the quantity of water lifted by each stroke of the bucket of a 9½-inch pump, the length of the stroke being 2¼ feet.

$3{\cdot}068 \times 2{\cdot}25 = 6{\cdot}903$ gallons, each stroke.

2 What length of stroke with a 6-inch pump will be necessary to discharge 44 gallons of water per minute, the number of strokes being 18 in the given time?

$$\frac{44}{1{\cdot}224 \times 18} = 2 \text{ feet, the length of stroke.}$$

3. What must be the diameter capable of raising 25 cubic feet of water per minute, the length of the stroke being 2½ feet, and making 16 effective strokes per minute?

$$\frac{25}{2{\cdot}5 \times 16} = {\cdot}625, \text{ or } 10\tfrac{3}{4} \text{ inches, nearly.}$$

Properties of Atmospheric Air.—It is by the oxygen of the atmosphere that combustion is supported. The common combustibles of nature are chiefly compounds of carbon and hydrogen, which, during combustion, combine with the oxygen of the atmosphere, and are converted into carbonic acid and watery vapor, different species of fuel requiring different quantities of oxygen. The quantity required for the combustion of a pound of coal varies from two to three lbs. Sixty cubic feet of atmospheric air will produce 1 lb. of oxygen.

The pressure or fluid properties of the atmosphere oppose bodies in passing through it, the opposing resistance increasing as the square of the velocity of the body, and the resistance per square foot in lbs. as its velocity in feet per second, multiplied into ·002288. Thus, suppose a locomotive engine in a still atmosphere, at a velocity of 25 miles per hour, presents a resisting frontage of 20 feet; required the amount of opposing resistance at that velocity.

25 miles per hour equal 36·67 feet per second.

Then $36{\cdot}67^2 \times {\cdot}002288 \times 20 = 61{\cdot}5$ lbs., constant opposing force.

TABLE

Showing the Number of Threads to an Inch in V-thread Screws.

Diam. in inches,	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
No. of threads,	20	18	16	14	12	11	10	9	8	7	7	6
Diam. in inches,	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	
No. of threads,	6	5	5	$4\frac{1}{2}$	$4\frac{1}{2}$	4	4	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{4}$	
Diam. in inches,	$3\frac{3}{4}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{3}{4}$	5	$5\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	6		
No. of threads,	3	3	$2\frac{7}{8}$	$2\frac{7}{8}$	$2\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{5}{8}$	$2\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{1}{2}$		

The depth of the threads should be half their pitch. The diameter of a screw, to work in the teeth of a wheel, should be such that the angle of the threads does not exceed 10°.

TABLE

Of the component parts of one English pound avoirdupois of 7000 grains of the following varieties of Wood. [MUSHET.]

Description of Wood	Water, Hyd. gas, Carb. acid.	Carbon.	Ashes.	Color and degree of saturation of the alkaline principle.
Oak, ...	5382·6	1587·8	29·6	grey, sharply alkaline.
Ash, ...	5688·2	1258·0	53·8	whitish blue, shrp. alk.
Birch, ...	5650·2	1224·4	125·4	brownish red, shrp. alk.
Norway Pine, .	5630·9	1344·3	24·8	brown, not at all alk.
Mahogany, . .	5147·0	1784·4	68·6	grey, sharply alkaline.
Sycamore, . .	5544·0	1381·4	74·6	pure white, weakly alk.
Holly, ...	5524·4	1394·3	81·3	pure white, sharply alk.
Scotch pine, .	5816·7	1151·9	31·4	brown, perceptibly alk.
Beech, . . .	5737·3	1395·9	66·8	greyish white, shrp. alk.
Elm, ...	5576·6	1370·2	53·2	grey, partially alkaline.
Walnut, . . .	5496·5	1446·4	57·1	pure white, light as down, weakly alk.
American Maple	5553·2	1393·1	53·7	dark grey, sharply alk.
Do. Black Beech, .	5425·9	1301·8	72·3	brown, sharply alkaline.
Laburnum, . .	5196·4	1721·0	82·6	white & grey, partly alk.
Lignum Vitæ, .	5083·0	1880·0	35·0	grey, sharply alkaline.
Sallow, . . .	5626·0	1294·8	79·2	light grey, sharply alk.
Chestnut, . .	5341·3	1629·6	29·1	grey, sharply alkaline.

TABLE

Of Properties of Gases.

Atmospheric air being the standard of comparison, or 1000.

Names.	Specific gravity.	Names.	Specific gravity.
Hydriodic acid gas, .	4340	Carbonic oxide gas, .	972
Chlorine " " .	2500	Carburetted hydrogen " .	972
Carbonic " " .	1527	Prussic acid " .	937
Nitrous oxide " .	1527	Ammoniacal " .	590
Cyanogen " .	1805	Steam of water " .	623
Oxygen " .	1111	Hydrogen " .	69

TABLE

Of Change Wheels for Screw-cutting; the leading Screw being of ½ inch pitch, or containing 2 threads in an inch.

Number of threads in inch of screw.	Number of teeth in: Lathe spindle-wheel.	Number of teeth in: Leading screw wheel.	Number of threads in inch of screw.	Number of teeth in: Lathe spindle-wheel.	Number of teeth in: Wheel in contact with spindle-wh.	Number of teeth in: Pinion in contact with spindle-wh.	Number of teeth in: Leading screw-wheel.	Number of threads in inch of screw.	Number of teeth in: Lathe spindle-wheel.	Number of teeth in: Wheel in contact with spindle-wh.	Number of teeth in: Pinion in contact with screw-wheel	Number of teeth in: Leading screw-wheel.
1	80	40	8¼	40	55	20	60	19	50	95	20	100
1¼	80	50	8½	90	85	20	90	19½	80	120	20	130
1½	80	60	8¾	60	70	20	75	20	60	100	20	120
1¾	80	70	9½	90	90	20	95	20¼	40	90	20	90
2	80	90	9¾	40	60	20	65	21	80	120	20	140
2¼	80	90	10	60	75	20	80	22	60	110	20	120
2½	80	100	10½	50	70	20	75	22½	80	120	20	150
2¾	80	110	11	60	55	20	120	22¾	80	130	20	140
3	80	120	12	90	90	20	120	23¾	40	95	20	100
3¼	80	130	12¾	60	85	20	90	24	65	120	20	130
3½	80	140	13	90	90	20	130	25	60	100	20	150
3¾	80	150	13½	60	90	20	90	25½	30	85	20	90
4	40	80	13¾	80	100	20	110	26	70	130	20	140
4¼	40	85	14	90	90	20	140	27	40	90	20	120
4½	40	90	14¼	60	90	20	95	27½	40	100	20	110
4¾	40	95	15	90	90	20	150	28	75	140	20	150
5	40	100	16	60	80	20	120	28½	30	90	20	95
5½	40	110	16¼	80	100	20	130	30	70	140	20	150
6	40	120	16½	80	110	20	120	32	30	80	20	120
6½	40	130	17	45	85	20	90	33	40	110	20	120
7	40	140	17½	80	100	20	140	34	30	85	20	120
7½	40	150	18	40	60	20	120	35	60	140	20	150
8	30	120	18¾	80	100	20	150	36	30	90	20	120

Temperature and Weight of the Atmosphere at various heights.

Height.	Temperature.	Water heavier than the air.
Level of the sea, . . .	60°	860 times.
One mile above, . . .	43	1,083 "
Two miles above, . . .	26	1,363 "
Three miles above, . .	9	1,716 "
Four miles above, . . .	−8	2,160 "
Five miles above, . . .	−25	2,719 "

TABLE

Showing how to discover the Quantity and Weight of Water in Pipes of any given size.

Diameter in inches.	Quantity in cubic inches.	Quantity in imperial gallons.	Weight in lbs. avoirdupois.
½	14·14	0·051	0·51
1	56·55	0·205	2·05
1½	127·23	0·460	4·60
2	226·19	0·818	8·18
2½	353·43	1·278	12·78
3	508 94	1·841	18·41
3½	692·72	2·506	25·06
4	904·78	3·272	32·72
4½	1145·11	4·142	41·42
5	1413·72	5·113	51·13
5½	1710·60	6·187	61·87
6	2035·75	7·363	73·63
6½	2389·18	8·641	86·41
7	2770·88	10·022	100·22
7½	3180·86	11·505	115·05
8	3619·11	13·090	130·90
8½	4085·64	14·777	147·77
9	4580·44	16·567	165·67
9½	5103·52	18·459	184·59
10	5654·87	20·453	204·53
10½	6234·49	22·550	225·50
11	6842·39	24 748	247·48
11½	7478·56	27·049	270·49
12	8143·01	29·452	294·52

This table shows the quantity and weight of water contained in one fathom of length of pipes of different bores from 1 inch to 12 inches in diameter, advancing by half inch. The weight of a cubic foot of water is taken at 1000 ounces avoirdupois, and the imperial gallon at 10 lbs.

Multipliers used for ascertaining the quantity of Tallow, Oakum, and Oil that can be contained in Tanks for use of Steam-vessels.

Tallow, ...	59 lbs. in a cubic foot.
Oakum, ...	11 lbs. in a cubic foot.
Oil, ...	6·23 galls. in a cubic foot.
Coal, ...	45 cubic feet to a ton.

PROPERTIES OF BODIES.

TABLES, *combining the Specific Gravities and other Properties of Bodies. Water the standard of Comparison, or* 1000.

PROPERTIES OF METALS.

Names.	Specific gravity.	Melting points in degrees of Fah.	Contraction in parts of an inch per linear foot from the fluid to the average temp. in solid state.	Ultimate cohesive strength of an in. sq. prism in tons	Scale of wire-drawing ductility.	Scale of laminable ductility	Ratio of hardness.	Scale as conductors of electricity.	Ratio of power in the conduction of heat.
Platinum, ...	19500	3280	—	—	3	5	—	—	3·8
Pure gold, ...	19258	2016	—	—	1	1	1·8	3	10·0
Mercury, ...	13500	—	—	—	—	—	—	—	—
Lead, ...	11352	612	·319	·81	8	7	1·0	6	1·8
Pure silver, . . .	10474	1873	—	—	2	2	2·4	2	9·7
Bismuth, ...	9823	476	·156	1·45	—	—	2·0	—	—
Copper, cast, . .	8788	1996	·193	8·51	—	—	—	—	—
" wrought, .	8910	—	—	15·08	5	3	2·8	1	8·9
Brass, cast, . . .	7824	1900	·210	8·01	—	—	to any degree	—	—
" sheet, . . .	8396	—	—	12·23	6	6	—	—	8·6
Iron, cast, . . .	7264	2786	·125	7·87	—	—	to any degree	—	—
" bar, . . .	7700	—	·137	25·00	4	8	4·7	4	3·7
Steel, soft, . . .	7833	—	·133	58·91	—	—	—	—	—
" hard, . . .	7816	—	—	—	—	—	to any degree	—	—
Tin, cast, ...	7291	442	·278	2·11	8	4	1·2	5	3·0
Zinc, cast, . . .	7190	778	·329	5·06	7	8	1·6	7	3·6

PROPERTIES OF STONES, EARTHS, &c.

Names.	Specific gravity.	Weight of a cubic foot in lbs.	Cubic feet in a ton.	Tons required to crush 1½ in. cubes
Marble, average	2720	170·00	13	9·25
Granite, do.	2651	165·68	12½	6·2
Purbeck stone,	2601	162·56	13¾	9·0
Portland do., .	2570	160·62	14	4·5
Bristol do., .	2554	159·62	14	—
Millstone, . .	2484	155·25	14½	—
Paving stone, .	2415	150·93	14¾	5·7
Craigleith do.,	2362	147·62	15	5·0
Grindstone, .	2143	133·93	16¾	6·6
Chalk, British,	2781	173·81	12¾	0·5
Brick, ...	2000	125·00	17	0·8
Coal, Scotch, .	1800	81·15	27½	—
" Newcastle,	1270	79·37	27¼	—
" Staffordsh.	1240	77·50	29	—
" Cannel, .	1238	77·37	29	—

Properties of Woods.

Names.	Specific gravity, water, 1000.	Average wt. of a cubic foot in lbs.	Cubic feet in a ton.	Ultimate cohesive strength of an inch square prism in lbs.	Comparative Stiffness.	Comparative Strength.	Comparative Resilience.
English oak,	934	58	38½	11880	100	100	100
Riga do,	872	54	41½	12888	93	108	125
Dantzic do.,	756	47	48	12780	117	107	99
American do.,	672	42	53	10253	114	86	64
Beech,	852	48	45	12225	77	103	138
Alder,	800	46	48½	9540	63	80	101
Plane,	640	40	55	10935	78	92	108
Sycamore,	604	38	59	9630	59	81	111
Chestnut,	610	38	59	10656	67	89	118
Ash,	845	52	43	14130	89	119	160
Elm,	673	42	53	9720	78	82	86
Mahogany, Spanish,	800	50	45	7560	73	67	61
" Honduras,	637	40	55	11475	93	96	99
Walnut,	671	42	53	8800	49	74	111
Teak,	750	46	48½	12915	126	109	94
Poona,	640	40	55	12350	99	104	82
African oak,	944	59	38	17200	101	144	138
Poplar,	383	34	66	5928	44	50	57
Cedar,	561	33	68	7420	28	62	106
Riga fir,	753	47	48	9540	98	80	64
Memel do.,	546	34	66	9540	114	80	56
Scotch do.,	528	33	68	7110	55	60	65
Christ. white deal,	590	37	60	12346	104	104	104
American white spruce,	551	34	66	10296	72	86	102
Yellow pine,	461	28	80	11853	95	99	103
Pitch pine,	660	41	54½	9796	73	82	92
Larch,	530	31	72	12240	79	103	134
Cork,	240	15	149				

Fusing Point of various Metals.

The fusing points of the more refractory substances are only to be ascertained approximately, on account of the doubtful accuracy of the indications given by the *pyrometers* at very high temperatures.

The pyrometer constructed of platinum is the most delicate, although the rate of its expansion must be uncertain as it approaches its own fusing point. The following are considered to be the fusing points of metals:

	Fahr.		Fahr.
Platinum, ...	3080°	Silver, ...	1830°
Wrought iron, . . .	2910	Zinc, ...	700
Steel, ...	2500	Lead, ...	590
Gold, ...	2190	Bismuth, ...	500
Cast iron, ...	2100	Tin, ...	450
Copper, ...	1920		

A dull red heat is estimated as 1480°; a bright red heat as 1830°; and a white heat as 2370° to 2910°, Fah.

Table *of Properties of Liquids.*

Names.	Specific grav. water, 1000.	Weight of an imp. gallon in lbs.	Names.	Specific grav. water, 1000.	Weight of an imp. gallon in lbs.
Acid, sulphuric, .	1850	18·5	Oils, expressed:		
" nitric, . .	1271	12·7	linseed, ...	940	9·4
" muriatic, .	1200	12·0	sweet almond, .	932	9·3
" fluoric, . .	1060	10·6	whale, ...	923	9·2
" citric, . .	1034	10·3	hempseed, . .	926	9·3
" acetic, . .	1062	10·6	olive, ...	915	9·2
Water from the Baltic, . . .	1015	10·2	Oils, essential: cinnamon, . .	1043	10·4
Water from the Dead Sea, . .	1240	12·4	lavender, . .	894	8·9
			turpentine, . .	870	8·7
Water from the Mediterranean, .	1029	10·3	amber, ...	868	8·7
			Alcohol, ...	825	8·2
Water, distilled, .	1000	10·0	Ether, nitric, . .	908	9·1
			Proof spirit, . .	922	9·2
			Vinegar, . . .	1009	10·1

Axle Grease.

1. The popular axle grease of the celebrated Mr. Booth is made as follows:

Dissolve ½ lb. common soda in 1 gallon of water, add 3 lbs. of tallow and 6 lbs. of palm oil (or 10 lbs. of palm oil only). Heat them together to 200° or 210° Fah.; mix, and keep the mixture constantly stirred till the composition is cooled down to 60° or 70°.

2. Another and thinner composition is made with ½ lb. of soda, 1 gallon of water, 1 gallon of rape oil, and ½ lb. of tallow, or palm oil.

3. The French compound, called Liard, is thus made:—Into 50 parts of finest rape oil put 1 part of caoutchouc, cut small. Apply heat, until it is nearly all dissolved.

4. Mankettrick's lubricating compound consists of 4 lbs. of caoutchouc (dissolved in spirits of turpentine), 10 lbs. of common

soda, 1 lb. of glue, 10 gallons of oil, and 10 gallons of water. Dissolve the soda and glue in the water by heat, then add the oil, and lastly the caoutchouc, stirring them until perfectly incorporated.

5. The following is the ordinary kind of axle-grease in common use:—1 part of fine black lead, ground perfectly smooth, with 4 parts of lard. Some recipes add a little camphor.

TABLE
Of Fusibility of Metals.
As given by M. Thenard.

1.—*Fusible below a red heat.*

CENTIGRADE.

Mercury, ...	—39°	
Potassium, ...	+58	Gay Lussac and Thenard.
Sodium, ...	90	Do. do.
Tin, ...	210	Newton.
Bismuth, ...	256	Do.
Lead, ...	260	Biot.
Tellurium, ...	a little less fus. than lead	Klaproth.
Arsenic, ...	undetermined	
Zinc, ...	370	Brongniart.
Antimony, ...	a little below a red heat	

2.—*Infusible below a red heat.*

PYROMETER OF WEDGWOOD.

Silver, ...	20°	Kennedy.
Copper, ...	27	Wedgwood.
Gold, ...	32	Do.
Cobalt, ...	a little less difficult to melt than iron	
Iron, ...	130	Wedgwood.
"	158	Sir G. McKenzie.
Manganese, . . .	160	Guyon.
Nickel, ...	160	Richter.
Palladium, . . . Molybdenum, . . . Uranium, ... Tungsten, ... Chromium, . . .	Nearly infusible, and to be obtained at a forge heat only in small buttons.	
Titanium, ... Cerium, ... Osmium, ... Iridium, ... Rhodium, ... Platinum, ... Columbium, . . .	Infusible at the forge furnace. Fusible at the oxy-hydrogen blowpipe.	

TABLE

Containing the Quantities of Water, in cubic feet, that will be discharged over a Weir per minute, for every inch in its breadth, when the depths of the Water from the surface to the top edge of the wasteboard do not exceed eighteen inches.

Depth of the Water in inches.	Cubic feet per minute, according to Du Buat's formula.	Cubic feet per minute, according to experiments made in Scotland.	Depth of the Water in inches.	Cubic feet per minute, according to Du Buat's formula.	Cubic feet per minute, according to experiments made in Scotland.
1	0·403	0·428	10	12·748	13·535
2	1·140	1·211	11	14·707	15·632
3	2·095	2·226	12	16·758	17·805
4	3·225	3·427	13	18·895	20·076
5	4·507	4·789	14	21·117	22·437
6	5·925	6·295	15	23·419	24·888
7	7·466	7·933	16	25·800	27·413
8	9·122	9·692	17	28·258	30·024
9	11·884	10·564	18	30·786	32·710

TABLE

Of the Composition of different Gunpowders.

KINDS.	Nitre.	Charcoal.	Sulphur.
Royal Mills at Waltham Abbey, England, ...	75	15	10
France, national establishm't.	75	12·5	12·5
French, for sportsmen, . . .	78	12	10
French, for mining, ...	65	15	20
United States of America, . .	75	12·5	12·5
Prussia, ...	75	13·5	11·5
Russia, ...	73·78	13·59	12·63
Austria (musket), ...	72	17	16
Spain, ...	76·47	10·78	12·75
Sweden, ...	76	15	9
Switzerland (a round powder)	76	14	10
Chinese, ...	75	14·4	9·9
Theoretical propor. (as above)	75	13·23	11·77

Alloys.

Alloys having a Density greater than the Mean of their Constituents.	Alloys having a Density less than the Mean of their Constituents.
Gold and zinc.	Gold and silver.
Gold and tin.	Gold and iron.
Gold and bismuth.	Gold and lead.
Gold and antimony.	Gold and copper.
Gold and cobalt.	Gold and iridium.
Silver and zinc.	Gold and nickel.
Silver and lead.	Silver and copper.
Silver and tin.	Silver and iron.
Silver and bismuth.	Iron and bismuth.
Silver and antimony.	Iron and antimony.
Copper and zinc.	Iron and lead.
Copper and tin.	Tin and lead.
Copper and palladium.	Tin and palladium.
Copper and bismuth.	Tin and antimony.
Lead and antimony.	Nickel and arsenic.
Platinum and molybdenum.	Zinc and antimony.
Palladium and bismuth.	

TABLE

Showing the estimated Power of Man or Horse as applied to Machinery.

Application of the Power.	Lbs. avr. at the rate of 220 feet per minute.		Lbs. avr. at the rate of one foot per minute.
A man is supposed to be capable of lifting or carrying	27·273		6000
A man is supposed to be capable of turning the winch of a crane with a force equal to	28·637		6300
When the united efforts of two men are applied to the winch of a crane, the handles being at right angles, each man exerts a force equal to	33·499	or	7350
A man is supposed to exert a power in pumping equal to	17·335		3814
In ringing, a man exerts a force equal to .	38·955		8570
And in rowing, ...	40·955		9010
The power of a horse is equal to	150		33000

TABLE

Of the Speed and Force of Wind, at different velocities.

Velocity of the Wind in Miles per hour.	Velocity of the Wind in Feet per second.	Force in lbs. avoirdupois per square foot.	Common Appellations given to the Wind.
1	1·47	·005	Hardly perceptible.
2	2·93	·020	Just perceptible.
3	4·40	·044	
4	5·87	·079	Gentle, pleasant wind.
5	7·33	·123	
10	14·67	·492	Pleasant, brisk gale.
15	22·00	1·107	
20	29·34	1·968	Very brisk.
25	36·67	3·075	
30	44·01	4·429	High winds.
35	51·34	6·027	
40	58·68	7·873	Very high
45	66·01	9·963	
50	73·35	12·300	A storm or tempest.
60	88·02	17 715	A great storm.
80	117·36	31·490	A hurricane.
100	146·70	49·200	A violent hurricane, which wrenches and tears up trees, forces dwellings and minor buildings from their foundations, and drives them before it

Note.—The following rule is used to find the force of wind acting perpendicularly upon a surface:—Multiply the surface in feet by the square of the velocity in feet, and the product by ·002288. The result is the force in pounds avoirdupois.

TABLE *showing the Height of the Boiling Point, Fah., at different Heights of the Barometer.*

Barometer.	Boiling Point.	Barometer.	Boiling Point.
Inches.	Degrees.	Inches.	Degrees.
31	213·57	28½	209·55
30½	212·79	28	208·69
30	212·00	27½	207 84
29½	211·20	27	206·96
20	210·38		

In a vacuum water boils at 98° to 100°, according as the vacuum is more or less perfect.

TABLE

Of the sizes of Nuts, equal in strength to their Bolts.

Diam. of bolt in in.	Short diameter of nut in in.	Diam. of bolt in in.	Short diameter of nut in inches.	Diam. of bolt in in.	Short diameter of nut in in.
$\frac{1}{4}$	$\frac{3}{8}$	$1\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$4\frac{7}{16}$
$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{2}$	$2\frac{11}{16}$	$2\frac{5}{8}$	$4\frac{5}{8}$
$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{5}{8}$	$2\frac{7}{8}$	$2\frac{3}{4}$	$4\frac{15}{16}$
$\frac{5}{8}$	$1\frac{1}{16}$	$1\frac{3}{4}$	$3\frac{1}{8}$	$2\frac{7}{8}$	$5\frac{1}{8}$
$\frac{3}{4}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$3\frac{3}{8}$	3	$5\frac{3}{8}$
$\frac{7}{8}$	$1\frac{9}{16}$	2	$3\frac{9}{16}$	$3\frac{1}{4}$	$5\frac{7}{8}$
1	$1\frac{3}{4}$	$2\frac{1}{8}$	$3\frac{3}{4}$	$3\frac{1}{2}$	$6\frac{5}{16}$
$1\frac{1}{8}$	2	$2\frac{1}{4}$	4	$3\frac{3}{4}$	$6\frac{3}{4}$
$1\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$4\frac{1}{4}$	4	$7\frac{1}{8}$

Note.—The depth of the head should equal the diameter of the bolt; the depth of the nut should exceed it, in the proportion of 9 or 10 to 8.

TABLE

Showing the Power of various Species of Fuel.

Species of Fuel.	Effect in lbs. of water heated 1° by one lb. of fuel.	Effect in lbs. of water converted into steam of 220°.	Quantity to convert a cubic foot of water into low pressure steam.	Quantity to convert a cubic foot of water into steam, allowing 10 per cent. for loss.
	lbs.	lbs.	lbs.	lbs.
Caking coal, . . .	9800	8·4	7·45	8·22
Coke, ...	9000	7·7	8·1	9·00
Splint coal, ...	7900	6·75	9·25	10·28
Oak wood, dry, . .	6000	5·13	12·2	13·6
Ordinary oak, . . .	3600	3·07	20·31	22·6
Peat compact, of ordinary dryness, .	3250	2·8	22·5	25·0

TABLE

Of the Ratios of the Successive Hardnesses of Bodies.

Substances.	Hardness.	Specific Gravity.	Substances.	Hardness.	Specific Gravity.
Diamond from Ormus,	20	3·7	Sardonyx, ...	12	2·6
Pink Diamond, . . .	19	3·4	Occidental amethyst,	11	2·7
Bluish Diamond, . . .	19	3·3	Crystal, ...	11	2·6
Yellowish Diamond, .	19	3·3	Cornelian, ...	11	2·7
Cubic Diamond, . .	18	3·2	Green Jasper, ...	11	2·7
Ruby, ...	17	4·2	Reddish yellow do. .	9	2·6
Pale ruby, from Brazil,	16	3·5	Schoerl, ...	10	3·6
Deep blue sapphire, .	16	3·8	Tourmaline, ...	10	3·0
Do., paler, ...	17	3·8	Quartz, ...	10	2·7
Topaz, ...	15	4·2	Opal, ...	10	2·6
Whitish topaz, . . .	14	3·5	Chrysolite, ...	10	3·7
Ruby spinell, ...	13	3·4	Zeolite, ...	8	2·1
Bohemian topaz, . .	11	2·8	Fluor, ...	7	3·5
Emerald, ...	12	2·8	Calcareous spar, . .	6	2·7
Garnet, ...	12	4·4	Gypsum, ...	5	2·3
Agate, ...	12	2·6	Chalk, ...	3	2·7
Onyx, ...	12	2·6			

Ductility and Malleability of Metals.

Ductility is the property of being drawn out in length without breaking. This property is possessed in a pre-eminent degree by gold and silver, as also by many other metals, by glass in the liquid state, and by many semi-fluid resinous and gummy substances. The spider and the silkworm exhibit the finest natural exercise of ductility, upon the peculiar viscid secretions from which they spin their threads. When a body can be readily extended in all directions under the hammer it is said to be malleable; and when into fillets, under the rolling press, it is said to be laminable.

There appears, therefore, to be a real difference between ductility and malleability; for the metals which draw into the finest wire are not those which afford the thinnest leaves under the hammer, or in the rolling press. Of this fact iron affords a good illustration. Among the metals permanent in the air seventeen are ductile and sixteen are brittle. But the most ductile cannot be wire-drawn or laminated to any considerable extent without being annealed from time to time during the progress of the extension, or rather the sliding of the particles alongside of each other, so as to loosen their lateral cohesion.

TABLE
Of the Ratio of the Ductility and Malleability of Metals.

Metals ductile and malleable, in alphabetical order.	Brittle metals in alphabetical order.	Metals in the order of their wire-drawing ductility.	Metals in the order of their laminable ductility
Cadmium.	Antimony.	Gold.	Gold.
Copper.	Arsenic.	Silver.	Silver.
Gold.	Bismuth.	Platinum.	Copper.
Iron.	Cerium?	Iron.	Tin.
Iridium.	Chromium.	Copper.	Platinum.
Lead.	Cobalt.	Zinc.	Lead.
Magnesium.	Columbium.	Tin.	Zinc.
Mercury.	Iridium.	Lead.	Iron.
Nickel.	Manganese.	Nickel.	Nickel.
Osmium.	Molybdenum.	Palladium?	Palladium?
Palladium.	Osmium.	Cadmium?	Cadmium?
Platinum.	Rhodium.		
Potassium.	Tellurium.		
Silver.	Titanium.		
Sodium.	Tungsten.		
Tin.	Uranium.		
Zinc.			

Conducting Powers of Various Substances.

The conducting power of wood is very low; the softer woods being lower in this respect than those which are harder. Of metals, and some other substances, the following is the order, according to Despretz:

Gold,	1000	Tin,	304
Silver,	973	Lead,	180
Copper,	898	Marble,	24
Platinum,	381	Porcelain,	12
Iron,	374	Tile,	11
Zinc,	363		

Radiating Power of Various Substances.

Bodies that have polished surfaces radiate heat less than those that are roughened, and metallic surfaces less than those of more imperfect conductors. The following are the proportions of some of each, according to Leslie:

Lamp-black,	100	Rough lead,	45
Water,	100	Mercury,	20
Writing-paper,	98	Polished lead,	19
Glass,	90	Polished iron,	15
Tissue-paper,	88	Tin, silver, copper, and gold,	12
Ice,	85		

Reflecting Powers of Various Substances.

Heat is reflected from the surface on which its rays fall, in the same manner as light; the angle of reflection being opposite and equal to that of incidence. The metals are the strongest reflectors of heat, in the following order, according to Leslie:

Brass, ...	100	Lead, ...	60
Silver, ...	90	Tinfoil rubbed with mer.,	10
Tinfoil, ...	85	Glass, ...	10
Block-tin, ...	85	Glass, waxed or oiled, .	5
Steel, ...	70		

Power of Various Substances to Transmit Heat.

All bodies capable of transmitting heat are, more or less, transparent, though their powers of transmitting heat and light are not in the same relative proportions; as the following list of the relative powers of equal masses, determined by Melloni, will show:

Air, ...	100	Rape-seed Oil, ...	2
Rock salt, transparent, .	92	Tourmaline, green, . .	7
Flint-glass, ...	67	Sulphuric Ether, . . .	21
Bisulphuret of Carbon, .	63	Gypsum, ...	20
Calcareous spar, transparent,	62	Sulphuric Acid, ...	17
Rock-crystal, ...	62	Nitric Acid, ...	15
Topaz, brown, ...	57	Alcohol, ...	15
Crown-glass, ...	49	Alum, in crystals, . . .	12
Oil of turpentine, . . .	31	Water, ...	11

TABLE

Showing the Scale of Proofs for Chain Rigging close-linked, &c.; the extreme Length of Links not to exceed five diameters of their size in Iron.

Diam. of Links.	Testing Weight.	Max. Strain.	Minimum Strain.		Diam. of Links.	Test. Wght.	Maximum Strain.	Minimum Strain.	
Inches.	Tons.	Tons.	Tons.	Cwt.	Inches.	Tons	Tons.	Tons.	Cwt.
$1\frac{5}{8}$	$31\frac{5}{8}$	75	68	0	$\frac{11}{16}$	$5\frac{5}{8}$	14	13	10
$1\frac{1}{2}$	27	64	58	0	$\frac{5}{8}$	$4\frac{5}{8}$	12	10	15
$1\frac{3}{8}$	$22\frac{5}{8}$	54	49	0	$\frac{9}{16}$	$3\frac{3}{4}$	10	$8\frac{3}{4}$ nearly.	
$1\frac{1}{4}$	$18\frac{3}{4}$	45	41	0	$\frac{1}{2}$	3	7	6	18
$1\frac{1}{8}$	$15\frac{1}{4}$	37	34	0	$\frac{7}{16}$	$2\frac{1}{4}$	6	5	2
1	12	30	28	0	$\frac{3}{8}$	$1\frac{5}{8}$	4	3	0
$\frac{15}{16}$	$10\frac{1}{2}$	26	25	0	$\frac{5}{16}$	$1\frac{1}{8}$	3	2	14
$\frac{7}{8}$	$9\frac{1}{8}$	23	22	0	$\frac{9}{32}$	$\frac{7}{8}$	none broken.	none broken.	
$\frac{13}{16}$	$7\frac{7}{8}$	20	20	0	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{3}{4}$	1	14
$\frac{3}{4}$	$6\frac{3}{4}$	17	16	0	$\frac{3}{16}$	$\frac{13}{34}$	$1\frac{1}{16}$	0	19

MASONRY.

Of the different kinds of Masonry.

Masonry, in the general acceptation of the term, is the art of cutting or squaring stones, to be applied to the purposes of build ing; or, in a more limited sense, it is the art of joining stones toge ther with mortar, or otherwise.

The ancients enumerate seven different methods in which they arranged the stones of their buildings. Vitruvius thus classes them: three of hewn or squared stones, threw of unhewn, and one a mixture of both methods.

1. *Net masonry.* This is represented in fig. 33, within the area D E F G, where the stones are squared and placed upon one of the angles, their joints thus forming a net-like appearance. This method, though very neat, is wanting in firmness and strength; for the oblique position of the stones, in regard to each other, gives them a tendency to separate rather than to form a compact assemblage of parts that unite in supporting each other. Whenever this form of masonry is employed, it is consequently necessary to keep the work together by a border of stones, having some other arrangement, one that is not only capable of supporting itself, but of overcoming the resistance of the net-like form. This is shown in the same figure at A B C; and where the network is merely a casing of stone to the brickwork of a wall, it will be found to answer tolerably well, and looks very neat.

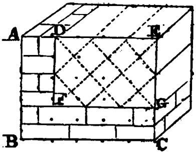


Fig. 1.

2. *Bound masonry* is that represented in fig. 2, and is remarkably strong. The perpendicular joints in each course fall directly in the middle of the stones composing the course below and above it; and while it has every requisite of solidity, the joints have, at the same time, a regular and pleasing appearance.

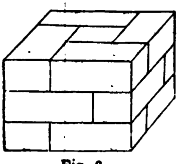


Fig. 2.

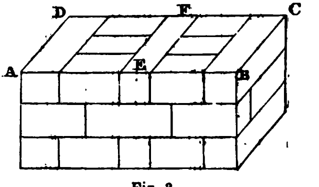


Fig. 3.

3. *Greek masonry* is that represented in fig. 3, where every alternate stone, as shown at A D, E F, and B C, is made of the whole thickness of the wall, and serves to bind together the stones

which compose the external and internal faces of the building; and this may be called double binding, as from the perpendicular joints being somewhat similarly situated to that in bound masonry, it has also an additional binding, by extending to the courses above and below it, thus forming a compact and durable wall, which resists every effort to separate in any direction.

4. *Masonry by equal courses.* This method of uniting stones is shown in fig. 4, and only differs from the bound masonry in its being composed of unhewn stones, or rather in being formed of stones that are not so accurately cut, nor the edges so perfectly squared; it being only necessary that the external face should be level, and the horizontal joints at equal distances from each other, care being taken at the same time that the perpendiculars are so situated as to bind the courses above and below them.

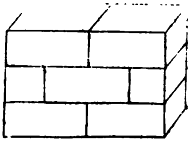


Fig. 4.

5. *Masonry by unequal courses* This is represented in fig. 5, and is, like the last, formed of unhewn stones, without any regularity as to their size, it being sufficient that each course is made to bind with the preceding, and the only regularity observed is in the joining which separates each course, the courses themselves being of unequal thickness, as shown at A B C D.

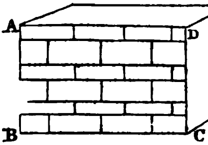


Fig. 5.

6. *Masonry filled up in the middle,* as shown in fig. 6, is formed of unhewn stones of unequal courses, and the middle, as at D, is filled up with stones thrown in at random among the mortar.

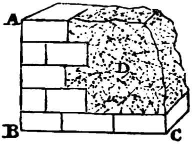


Fig. 6.

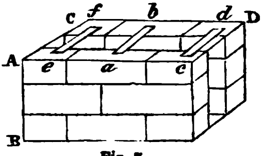


Fig. 7.

7. *Compound masonry* is, as its name imports, a mixture of the other kinds. It is represented in fig. 7, where the external course A B is formed of bound masonry, and the corresponding internal course is at some distance from it, but held to the former by means of iron cramps, as shown at *a*, *b*, *c*, *d*, *e*, *f*, the space between being filled in with small stones or flints thrown into the mortar.

The Methods of Joining Stone.

As the strength and durability of masonry depend as much on the method employed, and the care taken in making all the joints to correspond accurately with each other, as in the quality of the material employed, some remarks will be required in explanation of the methods of joining stone. We shall, therefore, enumerate the several means adopted by workmen, and, where necessary, notice the purposes to which each method is best adapted, giving some cautions to secure success in practice, and to save the workman unnecessary labor and trouble.

The joints in masonry are either secured by the means of mortar, cement, or plaster of Paris, or the courses are held together by cramps, joggles, mortice, and tenoning, or dovetailing.

1. Joining by mortar, or cements. It is absolutely necessary that the joints should be perfectly smooth, and touch in every part; and the stones must be so square as to bed well on each other, that is to say, they must not have such irregular faces as to roll, or, in technical terms, be winding to each other. The greatest care must be taken by the workman to have his mortar of a proper consistence—not too thin, as in drying it would shrink from the work, nor too thick, for that would prevent the stones from bedding properly. The best way in irregular masonry, or in that composed of small stones thrown, as it were, between the regular work, as in compound masonry, is to saturate fresh lime with water, and, while hot, to pour it on the work, which hardens and consolidates the whole into one solid mass. This method is much used in joining soft stones and brickwork, and is calculated to promote the strength and solidity of the work.

2. Joining by cramps. Cramping is performed by inserting into the two pieces of stone, which are to be bound together, a piece of iron or some other metal, the ends of which, bent at right angles, are inserted in a cavity cut in each stone, the cavities being so large as to admit the iron easily; melted lead is then poured in to fill the vacant space, and, when cold, a chisel is driven into it, so that it may press close to the work; for all metals expand by fusion, and obstacles may prevent them from contracting in cooling. Cramps composed of copper are, in many cases, very preferable to those made of iron, for they are less likely to oxidize, or rust, or to be affected by the lime or mortar. It would be of advantage to coat the cramps, if made of iron, with some substance that would defend them from the effects of damp. We may here remark, that the channel made to receive the cramp should be dovetailed, to prevent the lead from coming out, which it is otherwise apt to do, in the course of time. The only objection to the use of copper cramps, in preference to iron, is their expense, which in large public works is not of any importance, and, for common purposes, iron answers very well; but the more malleable or tough the iron

the better it is, as it is more calculated to resist the different temperatures to which the work may be exposed.

3. Joining by joggles. The method of securing the joints of masonry by means of joggles is chiefly adopted for securing the joints of columns or pillars; and consists in sinking a cavity in the two pieces in such a manner as to make them correspond with each other, and inserting in that cavity a piece of metal, stone, or even wood, so that any lateral thrust may not be able to separate them. This method may, with very great advantage, be applied in the construction of domes, and works of the same nature, where it is necessary to avoid the lateral thrust as much as possible.

We may here take the opportunity of mentioning a plan proposed by Dr. Hutton, in his edition of Oznamare's Mathematical Recreations, for taking away the lateral thrust of domes and cupolas. The following is the problem proposed, and the solution given:

"*How to construct a hemispherical arch, or what the architects call an arc en cul-de-four, which shall have no thrust on its piers.*

"Let A B, fig. 8, be two contiguous voussoirs, which we will suppose to be three feet in length, and eighteen inches in breadth. Cut out on the contiguous sides two cavities, in the form of a dovetail, four inches in depth, with an aperture of the same extent, *a*, *b*, five or six inches in length, and as much in breadth. This cavity will serve to receive a double key of cast-iron, as shown in fig. 9, or of common forged-iron, which is still more secure, as it is not so brittle. These two voussoirs will thus be connected together in such a manner that they cannot be separated without breaking the dovetail at the re-entering angle; but, as each of its dimensions in this place will be four inches, it will be easily seen that an immense force would be required to produce that effect; for we are taught, by well-known experiments on the strength of iron, that it requires a force of four thousand five hundred pounds to break a bar of forged iron an inch square, by the arm of a lever of six inches; consequently, two hundred and eighty-eight thousand pounds would be necessary to break a bar of sixteen square inches, like that in question. Hence there is reason to conclude, that these voussoirs will be connected together by a force of two hundred and eighty-eight thousand pounds; and as they will never experience an effort to disjoin them nearly so great, as might easily be proved by calculation, it follows that they may be considered as one piece."

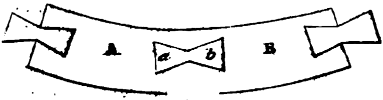


Fig. 8.


Fig. 9.

They might be still further strengthened in a very considerable degree, for the height of these dovetails might be made double, and

a cavity might be cut in the middle of the bed of the upper voussoir, fit to receive it entirely: the dovetail could not then be broken without breaking the upper voussoir also; but it may be easily seen that, to produce this effect, an immense force would be required.

The second method proposed by Dr. Hutton is more properly by the aid of joggles. Let A and B, fig. 10, be two contiguous voussoirs, and C, fig. 11, the inverted voussoir of the next course, which ought to cover the joint between A and B. Each of the voussoirs A and B being divided into two parts, as *a b* and *c d*; then if at *a b* and *c d* we sink an hemispherical cavity, in which to introduce a globe of very hard marble; and in the upper voussoir, fig. 10, we sink similar cavities, *b c*; this, when laid on *b c*, fig. 11, will form a secure joint without any lateral thrust; and the two courses cannot be separated without a force adequate to either break the solid stone, or disunite the marble globe; a force almost inconceivable, or at least one far superior to that produced by the arch; the whole dome, or cupola, is, in fact, one solid mass, and can exert no lateral thrust upon the walls on which it is raised. Marble globes are recommended, because iron is liable to rust; but, if the joggles were made of iron, and covered with pitch before they were placed in the cavities, there would be little to fear from rust; and particularly as the iron is inclosed in the substance of the stone, and quite excluded from the action of atmospheric causes.

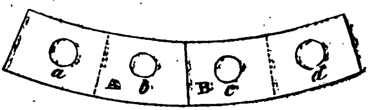


Fig. 10.

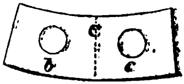


Fig. 11.

Little need be said in this place as to morticing and tenoning, or dovetailing, except that they differ slightly from the same operations in joiners' work; for, as cement is used in the joining, they need not be so accurately cut, and are made shorter and thicker than those formed by the joiner, it being sufficient that the parts of each piece to be joined enter into each other at most five or six inches, even in large masses of stone. In small pieces, an inch or an inch and a half is sufficient; for, if the tenon or dovetail be too long, it will decrease the solidity of the joint. For greater security, a small channel is frequently cut in the shoulder of the joint, and melted lead is poured into it, which, filling up the space round the tenon or dovetail, makes the joint more secure, and the work firm and solid.

In laying some sorts of stones, it is desirable, as far as possible, to place them in the same direction as they had when in the quarry, or, as it is termed by workmen, bedways of the stone; for, if laid in other directions, they are liable to peel and split by the action of the atmosphere.

BRICKLAYING.

Foundations.

The best soils for building upon are gravel, chalk, and stone rock.

Those most to be guarded against are sands, bog earth, clays, and made earth (no matter how hard). Where these occur, avoid piling (except in water works); plank the foundations through the centre of the walls, place long tassels in the piers, lay in chain bond, let the plates be stout, and in one piece, the whole length of each wall; all that is required is to so bind the building that it may settle altogether, and not partially.

In doubtful foundations, it is advisable to have a trench dug out to the depth of 2 feet to 3 feet below the footings of the brickwork, and about twice the width of the footings, which is to be filled up with *concrete*, composed of stone lime ground and ballast, or coarse gravel, to be mixed with water, in the proportion of one of lime to five or six of gravel; immediately that it is made up it must be shot into the trench from a stage, 6 feet above, which will cause it to fall in a solid mass; and in a few hours afterwards it will be as firm as a rock.

It is strongly recommended to have good plates; whatever may be slighted in other parts these should not be neglected—they are the soul and support of a building, and cannot, if put in too small, be taken out and replaced, as other timbers may; the difference in large houses will rarely amount to twenty-five dollars.

Bond the work—English bond—using all whole bricks, a course of stretchers and headers alternately.

Particular care must be taken that all the internal joints of brickwork are well flushed up with mortar; too frequently the workmen are apt to neglect doing it; the consequence is, that all the interior joints are hollow, and allow the damp to penetrate to the inside, no matter how thick the wall may be. Another serious defect in brickwork is in not properly bonding the facing to the backing, particularly if the facing be malms or bricks, which cost an extra price; generally the headers are only bats or half bricks, instead of being a stretcher or a whole brick to bond in with the brickwork at the back; there ought to be at least one stretcher in every 3 feet to each course, if there be not the wall will split or divide into two thicknesses.

In building arches of a large span, it is advisable to build them in half brick rims, with vertical or radiating bond every 3 or 4 feet in the girt; if this latter precaution be not adopted, the consequence will be, that when the centre is struck, the rims will divide and weaken the arch, and perhaps cause a total failure.

In selecting bricks, clap them together—if they ring well, and,

vhen broken, show that they are burnt through, they will answer he purpose. A hard clamp burnt gray stock is all that is wanted or strength; for water-works and foundations use clinker burnt nark stocks. Avoid samnel or place bricks, and chuffy stocks, and generally prefer hand tempering to pugging the clay.

In mixing of mortar, it is advisable to see that the laborer properly turns up the mortar, and that the lime is thoroughly incorporated with the sand throughout; *avoid using too much water*, as t drowns the lime and weakens it; in large works it is best to mix the lime and sand in a mill—cement must be mixed in small quantities.

TABLE

Showing the Quantity of Earth to be removed, the Number of Bricks and Gallons in one foot in depth or length.

Diam. in the clear.		½ Brick rim.					1 Brick rim.				Contents in gallons.
		Feet cube of digging.		Number of bricks.		Feet cube of digging.		Number of bricks.		
ft.	in.			laid dry.	in mortar			laid dry.	in mortar	
0	9	1	8	23	19	4	0	60	50	2½
1	0	2	4	28	23	4	9	70	58	5
1	3	3	1	33	27	5	9	80	66	7½
1	6	4	0	38	31	7	1	90	74	11
1	9	4	8	43	35	8	3	102	82	15
2	0	5	9	48	41	9	6	112	92	19½
2	3	7	1	53	44	11	0	122	100	25
2	6	8	3	58	48	12	6	132	108	30½
3	0	11	0	68	57	15	9	154	126	44
3	6	14	2	79	65	19	6	174	142	60
4	0	17	7	89	73	23	8	194	159	78
4	6	21	6	100	82	28	3	214	176	100
5	0	26	0	110	90	33	2	234	192	122
5	6	30	7	120	98	38	5	254	209	149
6	0	35	8	130	107	44	2	276	226	176
6	6	41	3	140	115	50	3	296	242	206
7	0	47	2	150	123	56	7	316	260	239
7	6	53	5	160	131	63	6	336	276	275
8	0	60	1	170	140	70	9	358	292	313
8	6	67	2	180	148	78	5	378	308	354
9	0	74	7	191	156	86	6	398	326	396
10	0	90	8	212	174	103	9	438	360	489

In the measurement of brickwork no allowance is to be made in quantity for small or difficult works.

Flues to be measured solid.

Timbers inserted in walls not to be deducted.

Two inches to be allowed for bedding plates, where no brickwork is over them.

All cuttings to be measured superficially, excepting to bird's mouths and squint quoins, which are to be run.

The net quantity of brickwork being found, it is to be reduced to the standard thickness of a brick and a half, and brought into statute rods of 5½ yards square, or 272 superficial.

Ovens, coppers, and solid walls, of irregular thickness, to be cubed and brought into the standard thickness, by multiplying by 8 (the number of 1½ inches in a foot), and dividing by 9 (the number of 1½ inches in a brick and a half, or 13½ inches, the standard thickness).

Facings of all descriptions to be measured and charged extra, per foot superficial.

272 feet superficial is a rod of brickwork, 1½ brick, or 13½ inches thick, the standard thickness, to which all brickwork, of whatever thickness, is reduced.

306 cubic feet, or 11⅓ cubic yards, equal to 1 rod of reduced brickwork.

4352 stock bricks to 1 rod reduced, 4 courses 1 foot high.

4533 ditto, if the 4 courses measure 11½ inches high.

These calculations are without allowing any waste, which is more than amply compensated in dwelling-houses, by not deducting flues and bond timber; in such work, 4300 stocks, or 4500 place, are sufficient.

5371 bricks laid dry to 1 rod.

4900 ditto in wells and circular cesspools.

A rod of brickwork contains 285 feet cube of bricks, and 71 feet of mortar (4 courses to a foot); which will weigh, upon an average calculation, 15 tons.

A rod of brickwork requires 1½ cubic yard of chalk lime, and 3 single loads or yards of drift; or 1 cubic yard of stone lime, and 3½ single loads or yards of sand; or 36 bushels of cement, and 36 of sharp sand.

16 bricks to a foot of reduced brickwork.

7 ditto to a foot super of facing.

10 ditto to a foot super of gauged arches.

30 bricks on edge, and 45 bricks flat, to 1 yard of brick-nogging.

36 stocks laid flat, and 52 ditto on edge, to 1 yard of paving.

36 paving bricks laid flat, and 82 ditto on edge, ditto.

A load of mortar, 27 feet cube, requires 9 bushels of lime and 1 yard of sand. A hod contains 20 bricks.

Lime and sand loses one third of its bulk when made into mortar —likewise cement and sand.

The proportion of mortar, or cement, when made up, to the lime, or cement and sand before made up, is as 2 to 3.

Lime, or cement and sand, to make mortar, require as much water as is equal to one third of their bulk, or about $5\frac{1}{2}$ barrels for a rod of brickwork built with mortar.

PLASTERING.

Thickness of Compo.	Inch yards.	$\frac{3}{4}$ inch yards.	$\frac{1}{2}$ inch yds.
1 bushel of cement will cover	$1\frac{1}{8}$	$1\frac{1}{2}$	$2\frac{1}{4}$
1 do. and 1 of sand do.	$2\frac{1}{4}$	3	$4\frac{1}{2}$
1 do. and 2 do. do.	$3\frac{3}{8}$	$4\frac{1}{2}$	$6\frac{3}{4}$
1 do. and 3 do. do.	$4\frac{1}{2}$	6	9
($\frac{5}{8}$ inch is the usual thickness.)			
1 cwt. of mastic and 1 gallon of oil		$1\frac{1}{2}$	$2\frac{1}{4}$

1 cubic yard of chalk lime, 2 yards of road drift or sand, and 3 bushels of hair, will cover 75 yards of *render and set* on brick, and 70 yards on lath, or 65 yards *plaster and render* 2 *coats and set* on brick, and 60 yards on lath; floated work will require about the same as 2 coats and set.

1 bundle of laths and 500 nails will cover about $4\frac{1}{2}$ yards.

Mortar.

1 hundred of lime contains 25 striked bushels, or 100 pecks. It is a measure 3 feet square, and 3 feet 1 inch deep. 1 chaldron of lime is equivalent to 57·765 cubic feet, or rather more than 2 hundred.

18 heaped bushels, 22 striked bushels, or 1 yard cube, a single load of sand, mortar, &c.

1 double load is equal to 36 heaped bushels.

1 hod of mortar is equal to 1134 cubic inches, or 8 duodecimal inches, or 9 × 9, and 14 inches long.

2 hods of mortar make a bushel nearly.

Cement.

1 barrel of cement is 5 bushels, and weighs 3 cwt. 1 rod of brickwork, in cement, requires 36 bushels of cement and 36 bushels of sand.

1 yard, or 9 feet superficial of 14 inches, or $1\frac{1}{2}$ brickwork, in cement, requires about $2\frac{1}{2}$ bushels.

1 yard superficial of pointing to brickwork, in cement, requires about one eighth of a bushel.

1 yard square of plastering, in cement, requires three fourths of a bushel.

Carpentry and Plastering are measured by the square foot or yard; or, in moulded and ornamental work, by the linear foot. In extensive work the square of 100 feet is also used.

Paving is measured by the square yard.

Digging, &c.

23½ cubic feet of sand, 17½ ditto clay, 18 ditto earth, 13 ditto chalk, equal to a ton.

A cubic yard of earth, before digging, will occupy about 1½ cubic yard when dug.

27 cubic feet, or 1 cubic yard, contains 21 striked bushels, which is considered a *single load*, and double these quantities a *double load*.

18 cubic feet of night soil, 1 ton.

2½ tons of ditto is the quantity a cart contains; 6 feet long, 3 feet 3 inches wide, by 2 feet 4 inches deep, or 45 feet cube.

Coarse Stuff.

Coarse stuff, or lime and hair, as it is sometimes called, is prepared in the same way as common mortar, with the addition of hair procured from the tanner, which must be well mixed with the mortar by means of a three-pronged rake, until the hair is equally distributed throughout the composition. The mortar should be first formed, and when the lime and sand have been thoroughly mixed, the hair should be added by degrees, and the whole so thoroughly united that the hair shall appear to be equally distributed throughout.

Fine Stuff.

This is made by slaking lime with a small portion of water, after which so much water is added as to give it the consistence of cream. It is then allowed to settle for some time, and the superfluous water is poured off, and the sediment is suffered to remain till evaporation reduces it to a proper thickness for use. For some kinds of work it is necessary to add a small portion of hair.

Stucco for Inside of Walls.

This stucco consists of fine stuff already described, and a portion of fine washed sand, in the proportion of one of sand to three of fine stuff. Those parts of interior walls are finished with this stucco which are intended to be painted. In using this material, great care must be taken that the surface be perfectly level, and to secure this it must be well worked with a floating tool or wooden trowel. This is done by sprinkling a little water occasionally on the stucco, and rubbing it in a circular direction with the float, till the surface has attained a high gloss. The durability of the work very much depends upon the care with which this process is done, for if it be not thoroughly worked it is apt to crack.

Gauge Stuff.

This is chiefly used for mouldings and cornices which are run or formed with a wooden mould. It consists of about one fifth of plaster of Paris, mixed gradually with four fifths of fine stuff. When the work is required to set very expeditiously, the proportion of plaster of Paris is increased. It is often necessary that the plaster to be used should have the property of setting immediately it is laid on, and in all such cases gauge stuff is used, and consequently it is extensively employed for cementing ornaments to walls or ceilings, as well as for casting the ornaments themselves.

Higgins' Stucco.

To fifteen pounds of the best stone lime add fourteen pounds of bone ashes, finely powdered, and about ninety-five pounds of clean, washed sand, quite dry, either coarse or fine, according to the nature of the work in hand. These ingredients must be intimately mixed, and kept from the air till wanted. When required for use, it must be mixed up into a proper consistence for working with lime water, and used as speedily as possible.

Parker's Cement.

This cement, which is perhaps the best of all others for stucco, as it is not subject to crack or flake off, is now very commonly used, and is formed by burning argillaceous clay in the same manner that lime is made; it is then reduced to powder, by the process described in a previous part of this work. The cement, as used by the plasterer, is sometimes employed alone, and sometimes it is mixed with sharp sand; and it has then the appearance, and almost the strength, of stone. As it is impervious to water, it is very proper for lining tanks and cisterns.

Hamelein's Cement.

This cement consists of earthy and other substances insoluble in water, or nearly so; and these may be either those which are in their natural state, or have been manufactured, such as earthenware and china; those being always preferred which are least soluble in water, and have the least color. When these are pulverized, some oxide of lead is added, such as litharge, gray oxide, or minium, reduced to a fine powder; and to the compound is added a quantity of pulverized glass or flint-stones, the whole being thoroughly mixed and made into a proper consistence with some vegetable oil, as that of linseed. This makes a durable stucco or plaster, that is impervious to wet, and has the appearance of stone.

The proportion of the several ingredients is as follows:—to every five hundred and sixty pounds of earth, or earths, such as pit sand, river sand, rock sand, pulverized earthenware or porcelain, add forty pounds of litharge, two pounds of pulverized glass or flint, one pound of minium, and two pounds of gray oxide of lead. Mix

the whole together, and sift it through sieves of different degree of fineness, according to the purposes to which the cement is to b applied.

The following is the method of using it:—To every thirty pound weight of the cement in powder add about one quart of oil, eithe linseed, walnut, or some other vegetable oil, and mix it in the sam manner as any other mortar, pressing it gently together, either b treading on it, or with the trowel; it has then the appearance o moistened sand. Care must also be taken that no more is mixed a one time than is required for use, as it soon hardens into a soli mass. Before the cement is applied, the face of the wall to be plas tered should be brushed over with oil, particularly if it be applie to brick, or any other substance that quickly imbibes the oil; if t wood, lead, or any substance of a similar nature, less oil may b used.

Maltha, or Greek Mastic.

This is made by mixing lime and sand in the manner of mortar and making it into a proper consistency with milk or size, instea of water.

Plaster in imitation of Marble.

This species of work is exquisitely beautiful when done wit taste and judgment, and is so like marble to the touch, as well a appearance, that it is scarcely possible to distinguish the one from the other. We shall endeavor to explain its composition, and th manner in which it is applied; but so much depends upon th workman's execution, that it is impossible for any one to succeed in an attempt to work with it without some practical experience.

Procure some of the purest gypsum, and calcine it until the large masses have lost the brilliant sparkling appearance by which they are characterized, and the whole mass appears uniformly opaque. This calcined gypsum is reduced to powder, and passed through a very fine sieve, and mixed up, as it is wanted for use, with Flanders glue, isinglass, or some other material of the same kind. This solution is colored with the tint required for the scagliola, but when a marble of various colors is to be imitated, the several colored compositions required by the artist must be placed in separate vessels, and they are then mingled together in nearly the same manner that the painter mixes his color on the pallet. Having the wall or column prepared with rough plaster, it is covered with the composition, and the colors intended to imitate the marble, of whatever kind it may be, are applied when the floating is going on.

It now only remains to polish the work, which, as soon as the composition is hard enough, is done by rubbing it with pumice-stone, the work being kept wet with water applied by a sponge. It is then polished with Tripoli and charcoal, with a piece of fine linen, and finished with a piece of felt, dipped in a mixture of oil and Tripoli, and afterwards with pure oil.

Composition.

This is frequently used, instead of plaster of Paris, for the ornamental parts of buildings, as it is more durable, and becomes in time as hard as stone itself. It is of great use in the execution of the decorative parts of architecture, and also in the finishings of picture frames, being a cheaper method than carving, by nearly eighty per cent.

It is made as follows: Two pounds of the best whitening, one pound of glue, and half a pound of linseed oil are heated together, the composition being continually stirred until the different substances are thoroughly incorporated. Let the compound cool, and then lay it on a stone covered with powdered whitening, and heat it well until it becomes of a tough and firm consistence. It may then be put by for use, covered with wet cloths to keep it fresh. When wanted for use it must be cut into pieces, adapted to the size of the mould, into which it is forced by a screw press. The ornament, or cornice, is fixed to the frame or wall with glue, or with white lead.

To make Glass Paper.

Take any quantity of broken glass (that with a greenish hue is the best), and pound it in an iron mortar. Then take several sheets of paper, and cover them evenly with a thin coat of glue, and, holding them to the fire, or placing them upon a hot piece of wood or plate of iron, sift the pounded glass over them. Let the several sheets remain till the glue is set, and shake off the superfluous powder, which will do again. Then hang up the papers to dry and harden. Paper made in this manner is much superior to that generally purchased at the shops, which chiefly consists of fine sand. To obtain different degrees of fineness, sieves of different degrees of fineness must be used.

To make Stone Paper.

As, in cleaning wood-work, particularly deal and other soft woods, one process is sometimes found to answer better than another, we may describe the manner of manufacturing a stone paper, which, in some cases, will be preferred to sand paper, as it produces a good face, and is less liable to scratch the work. Having prepared the paper as already described, take any quantity of powdered pumice-stone, and sift it over the paper through a sieve of moderate fineness. When the surface has hardened, repeat the process till a tolerably thick coat has been formed upon the paper, which, when dry, will be fit for use.

WOODWORK, CARPENTRY, &c.

Decay of Wood.

Some woods decay much more rapidly than others; but they will *all*, in some situations, lose their fibrous texture, and with it their properties. To ascertain the causes which act upon woods, and effect their destruction, is an important object both to the builder and to the public.

Cause of the Decay of Timber.

All vegetable as well as animal substances, when deprived of life, are subject to decay.

If the trunk or branch of a tree be cut horizontally it will be seen that it consists of a series of concentric layers, differing from each other in color and tenacity. In distinct species of trees these layers present very different appearances, but in all cases the outer rings are more porous and softer than the interior. Wood is essentially made up of vessels and cells, and the only solid parts are those coats which form them. These vessels carry the sap which circulates through the tree, gives life and energy to its existence, and is the cause of the formation of leaves, flowers, and fruit. But when the tree is dead, and the sap is still in the wood, *it becomes the cause of vegetable decomposition by the process of* FERMENTATION. There are five distinct species of vegetable fermentation—the saccharine, the coloring, the vinous, the acetous, and the putrefactive. We are indebted to Mr. Kyan for the discovery that albumen is the cause of putrefactive fermentation, and the subsequent decomposition of vegetable matter.

Circumstances favorable to Vegetable Decomposition.

Wood is not equally liable to decay under all circumstances. When thoroughly dried it is not so quickly decomposed as when in its green state, for in the latter condition it has in itself all the elements of destruction, and it is scarcely possible to prevent the effect if it be then used in building. But supposing the timber to be perfectly seasoned, it is more liable to decay under some circumstances than in others. Timber is most durable when used in very dry places.

When timber is constantly exposed to the action of water, the decomposition effected will depend upon the nature and chemical composition of the substance. A portion of wood may be soluble in water, but other parts are not; so that after a definite period, the continued action of water upon a piece of timber ceases, and if it can sustain the influence of this cause until that period there is no termination to its endurance, except from those casualties which it might have been able to bear in its original state, but cannot after the removal of that portion of its substance soluble in water.

Should a piece of timber that has been for a long time exposed to water be brought into the air and dried, IT WILL BECOME BRITTLE AND USELESS.

When wood is alternately exposed to the influence of dryness and moisture it decays rapidly. It appears, from experiments, that after all the matter usually soluble in water has been removed, a fresh maceration and contact of the air produces a state of matter in that which is left which renders it capable of solution. A piece of timber may then in this manner be more and more decomposed, until at last the whole mass is destroyed. The builder is sometimes compelled to use wood in places where it will be exposed to alternate dryness and moisture; fencing, weather boarding, and other works, are thus exposed. In all these cases he may anticipate the destructive process, and provide against it. The wood used in such situations should be thoroughly seasoned, and then painted or tarred; but, *if it be painted when not thoroughly seasoned,* THE DESTRUCTION WILL BE HASTENED, for the evaporation of the contained vegetable juices is prevented.

There is one other circumstance to be considered—the influence of moisture associated with heat. Within certain limits the decomposition resulting from moisture increases with the temperature. The access of the air is not absolutely necessary to the carrying on of this process, but water is; and as it goes on, carbonic acid gas and hydrogen gas are given off. The woody fibre itself is not free from this decomposition, for, as the carbonaceous matter is abstracted by fermentation, it becomes more susceptible of this change. This statement is proved by the circumstance, that when quicklime is added to the moisture the decomposition is accelerated, for it abstracts carbon; but the carbonate of lime produces no such effect: a practical lesson may be learnt from this fact; if timbers be bedded in mortar, decomposition must follow, for it is a long time before it can absorb sufficient carbonic acid to neutralize the effect, and the dampness which is collected by contact with the wet mortar increases the effect. When the wood and the lime are both in a dry state no injury results, and it is well known that lime protects wood from worms.

When the destructive process first becomes visible it is by the swelling of the timber, and the formation of a mould or fungus upon its surface. This fungus or cryptogamic plant rapidly increases, and soon covers over the whole surface of a piece of timber, having a white, grayish-white, or brownish hue. When the seeds of destruction are thus once sown they cannot be readily eradicated. Heat and moisture may be considered the prominent causes of the rapid decomposition of vegetable substances. When wood is completely and constantly covered with water this effect is not produced; and we have an example in the fact, that, although those parts of a vessel which are subject to an occasional moisture are liable to dry rot, yet those parts which are constantly beneath the water are not ever thus affected; and although the head of a pile, which may be now and then wetted by the casual rise of the tide,

and is then dried again by the sun, may be decomposed, yet *those parts which are always covered with water have been found in a solid state after* CENTURIES *of immersion.*

Means of Preventing Decay.

Something may be done towards the prevention of decay by felling the timber at a proper season. A tree may be felled too soon or too late, in relation to its age and to the period of the year. A tree may be so young that no part of it shall have the proper degree of hardness, and even its heart-wood may be no better than sap-wood; or a tree may be felled when it is so old that the wood, if not decayed, may have become brittle, losing all the elasticity of maturity. The time required to bring the several kinds of trees to maturity varies according to the nature of the tree and the situation in which it may be growing. Authors differ a century as to the age at which oak should be felled, some say one hundred, and others two hundred years; it must, then, be regulated according to circumstances.

But it is also necessary that the timber trees should be felled at a proper season of the year; that is to say, when their vessels are least loaded with those juices which are ready for the production of sap-wood and foliage. *The timber of a tree felled in spring or in autumn would be especially liable to decay;* for it would contain the element of decomposition. Midsummer and midwinter are the proper times for cutting, as the vegetative powers are then expended.

There are some trees, the bark of which is valuable, as well as the timber; and as the best time for felling is not the best for stripping the bark, it is customary to perform these labors at different periods. The oak-bark, for instance, is generally taken off in early spring, and *the timber is felled* AS SOON AS THE FOLIAGE IS DEAD; and *this method is found to be highly advantageous to the durability of the timber.* The sap-wood is hardened, and all the available vegetable juices are expended in the production of foliage. Could this plan be adopted with other trees, it would be desirable; but the barks are not sufficiently valuable to pay the expense of stripping.

Seasoning Timber.

Supposing all these precautions to be taken in felling timber, it is still necessary to season it; that is, to adopt some means by which it may be dried, so as to throw off all the juices which are still associated with the fibres of the wood. As soon as the timber is felled, it should be removed to some dry place; and, being piled in such a manner as to admit a circulation of air, remain in log for some time, as it has a tendency to prevent warping. The next process is to cut the timber into scantlings, and to place these upright in some dry situation, where there is a good current of air, avoiding the direct rays of the sun. The more gradually the

process of seasoning is carried on, the better will be the wood for all the purposes of building. Mr. Tredgold says, "It is well known to chemists, that slow drying will render many bodies less easy to dissolve; while rapid drying, on the contrary, renders the same bodies more soluble. Besides, all wood, in drying, loses a portion of its carbon, and the more in proportion as the temperature is higher. There is in wood that has been properly seasoned a toughness and elasticity which is not to be found in rapidly dried wood. This is an evident proof that firm cohesion does not take place when the moisture is dissipated in a high heat. Also, seasoning by heat alone, produces a hard crust on the surface, which will scarcely permit the moisture to evaporate from the internal part, and is very injurious to the wood.

"For the general purposes of carpentry, *timber should not be used in less than two years after it is felled;* and this is the least time that ought to be allowed for seasoning. For joiners' work it requires four years, unless other methods be used; but, for carpentry, natural seasoning should have the preference, unless the pressure of the air be removed."

Many artificial methods of seasoning timber have been proposed; and a brief notice of some of those which have been found most useful will be required.

Seasoning by a Vacuum.

All the vegetable and animal juices are kept in their particular vessels by the pressure of the atmosphere: remove that pressure, and the animal fluids could no longer be retained by the veins and arteries; and the vegetable fluids would exude and appear on the surface of the plant. Place a small piece of wood beneath the receiver of an air-pump, and exhaust the air, and in a short time the wood will be covered with drops of the liquid which can no longer be retained, as the atmospheric pressure is removed. Mr. Langton thought that this might be applied to the extraction of those vegetable juices in timber, known to be the cause of its decay. An arrangement was therefore adopted, by which large masses of timber might be inclosed in a vessel having such machinery as would be necessary to exhaust the air, heat being at the same time employed so as to vaporize the exuded juices. The vapor is conveyed away by pipes surrounded by cold water, and is condensed into liquid having a sweet taste. This process is deserving of more attention than has hitherto been given to it.

Water Seasoning.

It has been stated, by various writers, that wood immersed in water for about a fortnight, and then dried, is better suited for all the purposes of the joiner. There can be no doubt that immersion in water tends to neutralize the effect of the saccharine matter, by dilution or an almost absolute removal. This process has also the effect of rendering the wood less liable to crack and warp; but, if

we judge by Duhamel's experiments, it injures the strength of the material, and should not, therefore, be adopted in any instance where the timber is to be employed by the carpenter. Evelyn recommends boards that are to be used for flooring to be seasoned in this way: "Lay your boards," he says, "a fortnight in water, (if running the better, as at a mill-pond head;) and then setting them upright in the sun and wind, so as it may pass freely through them, turn them daily; and thus treated, even newly-sawn boards will floor far better than those of a many years' dry seasoning, as they call it." Timber intended for *ship building* may be immersed in *sea water;* but that which is to be used for *houses* ought to be placed in *fresh* water; for if timber, or any other building material, be impregnated with salt, it will ever be wet, for salt attracts moisture so readily that it may be used approximately as a hygrometer. Plaster or mortar made with *salt* water *will always sweat* with a moist atmosphere; and timber intended for the house carpenter, if impregnated with salt, will always be *damp*, or covered with a crystallized efflorescence. Much injury, however, is sometimes done by not *thoroughly* immersing the timber; the carpenter should therefore be careful when he employs this method of seasoning, that the timber is *entirely* covered with water, and that it be not exposed to its action for too long a time.

Seasoning by Smoking and Charring.

Authors who have written upon the seasoning of timber have spoken of the effects of smoke, and the carbonization of the surface. We have adopted the same arrangement, but it will be necessary to caution the reader against a misconception of a very inaccurate expression. Timber cannot be seasoned by either smoking or charring. but seasoned timbers may be made more capable of resisting the effects of certain situations by these processes. Should a piece of timber, containing the vegetable juices, be smoked or charred, it would be a means of accelerating decomposition; for preventing all means of evaporation, the common sources of protection would become sources of destruction. But when timber is to be used in situations where it is liable to be attacked by worms, or to produce fungi, it may be desirable to smoke or to char it.

Seasoning by Boiling or Steaming.

Timber is sometimes seasoned by steaming or boiling, both of which means are frequently adopted by ship-builders. The strength of timber appears to be somewhat impaired by these processes, but it is generally less liable to shrink or crack. Duhamel states that he boiled a piece of wood, and then dried it upon a stove, but in drying it, it lost part of its substance, as well as the water contained, and, upon a repetition, he found that it had lost still more of its weight. Four hours' exposure to steam or boiling water is sufficient for timbers of ordinary dimensions, and the drying afterwards goes

on very rapidly, but it should be done as gradually as possible. The joiner frequently finds it necessary to steam or boil wood, to bend it into a particular curve, and also the ship-builder. It has been stated by writers on ship-building, that boiling increases the durability of timber; and, in proof of this, they inform us that the planks in the bow of a ship, which are bent in this way, are never affected by the dry rot.

It may now be inquired whether, after the most perfect seasoning, timber is secured against the process of decay? To this question a negative answer must be given. However well the timber may be seasoned, it will certainly rot if placed in a damp situation, the rapidity of the decomposition depending upon the nature and state of the wood, and the activity of the destroying agent. As the builder seldom attempts any other seasoning than that which depends upon drying his timbers, it is absolutely necessary that he should carefully avoid the rise of damp, and adopt every means in his power to prevent this evil. Timbers are usually placed in contact with walls, but it must not be supposed that this is sufficient to keep them from the access of damp, for they are frequently the conducting media. Brickwork very readily absorbs moisture, and also throws it upwards, so that the ends of timbers are in contact with the very source of mischief. To prevent the rise of damp upwards, it is common to use, for a few feet above the foundations, cement, a substance impervious to water, instead of mortar, or to place between the courses zinc or slate. But that these plans may be effective, the basement walls should be surrounded with an open area, for, if in contact with the earth on their sides, they can be of no value. To prevent dampness from entering in front, the brickwork should be covered with compo, or some substance impermeable to water.

Another thing to be considered, for the security of timbers, is to arrange, in every plan of a building, for a perfect circulation of air. Ventilation is a most important requisite in the construction of a building, although it is generally a matter of very little importance in the consideration of those who have to plan or construct buildings. The ventilation of roofs is by no means difficult, but there are often so many obstacles to the ventilation of flooring that the designer will not give sufficient attention to his subject to provide against them. These things, however, are not matters of speculation, to be attended to by those who have no higher employment, but are absolutely necessary for the construction of a work that is intended to survive the builder.

The attention of scientific men has been recently directed to the experiments made by Mr. Kyan. Having made a great number of experiments with a view to ascertain the primary cause of vegetable decomposition, he was at last convinced that albumen was that cause, and that to neutralize its effects would be to prevent decomposition. Some plan was required similar to that adopted in tanning. The gelatin in animal bodies is quite as liable to decom-

position as the albumen of vegetables; but when tannin, the infusion of oak bark, is combined with it, the destructive properties are lost, and the animal matter becomes durable, and almost incapable of decay. Reasoning upon this effect, Mr. Kyan imagined that it might be possible to prevent vegetable decomposition by causing the albumen to form a combination with some other substance; and, knowing the affinity of corrosive sublimate for the albumen, he entered upon a series of experiments, which led him to propose the use of that substance as a protection for timber.

Mr. Kyan inferred that, as wood consists of various successive layers, in which the albumen, or juices containing albumen, circulated freely, it is quite certain, as these juices within the wood, with the watery parts, fly off by the leaves, that the albumen remains behind, and it is probable that this albumen, which from its nature is peculiarly prone to enter into new combinations, is the thing in wood which begins the tendency to decomposition, and produces ultimate decay, whether that decomposition is attended with the formation of cryptogamic substances, or whether in the less organized form, the change occurs with the simple production of what has been called the dry rot. Mr. K. conceived, therefore, if albumen made a part of wood, the latter would be protected by converting that albumen into a compound of protochloride of mercury and albumen; and he proceeded to immerse pieces of wood in this solution, and obtained the same result as that which he had ascertained with regard to the vegetable decoctions. Having done so, it became necessary to employ various modes of experiment, as well as comparative experiments. Now it is not clear in what part of the wood the vegetable albumen may be found, though it exists more especially in that part of the tree which is denominated the alburnum or sap, and is found between the heart-wood and the innermost layer of bark. The experience of all practical men has confirmed the opinion, that this portion of wood is the first to decay.

It is probable that, as the alburnum becomes successive layers of wood, it loses a quantity of albumen; or that, in consequence of the pressure which takes place by the addition of each successive layer, it becomes so situated as to lose a part of its exposure to the vessels where a change may occur, and therefore becomes, in some measure, protected; for that which is one year alburnum or sap, may be, and indeed generally is, proper wood the next.

The mode in which the application of the solution takes place is in tanks, which may be constructed of different dimensions, from twenty to eighty feet in length, six to ten in breadth, and three to eight in depth. The timber to be prepared is placed in the tank, and secured by a cross-beam to prevent its rising to the surface. The wood being thus secured, the solution is then admitted from the cistern above, and for a time all remains perfectly still. In the course of ten or twelve hours, the water is thrown into great agitation by the effervescence occasioned by the expulsion of the air

fixed in the wood, by the force with which the fluid is drawn in by chemical affinity, and by the escape of that portion of the chlorine, or muriatic acid gas, which is disengaged during the process. In the course of twelve hours this commotion ceases, and in the space of seven to fourteen days, varying according to the diameter of the wood, the change is complete, so that as the corrosive sublimate is not an expensive article, the albumen may be converted into an indecomposable substance at a very moderate rate, and the seasoning will take place in the course of two or three weeks."

Mr. Kyan's method of seasoning has been already tested, under circumstances so severe, that they may be said to have proved its efficiency. A piece of oak was five years in the fungus pit in Woolwich yard, London, a place notorious for the rapid and almost instantaneous destruction of vegetable matter, and it was as sound when taken out as when put in. This was the most severe test to which the method could be subjected, and its having sustained the trial is a proof of the value of the discovery. It has, however, been objected to the process, that the impregnation of timber with corrosive sublimate must unfit it for use in ship-building; but Mr. Kyan has furnished evidence to the contrary, and proves that salubrity is one advantage. We strongly recommend the builder to make experiments himself upon wood prepared by Mr. Kyan, by using it in places where decay is rapid.

Framing of Timbers.

When timbers are framed together, it is with the intention of supporting some weight, or resisting the strains to which the materials may be exposed in the situations where they are to be placed. Horizontal or vertical timbers are not always of themselves sufficiently strong to sustain the pressure to which they may be subject, but they need assistance, and it then becomes a question, how can the materials intended to assist be best applied, and what are the smallest scantlings that can be adapted? Two things must be studied—stability and economy. It has been often stated that these two results cannot be accomplished by the same arrangement, but as the forces which are to be opposed have usually a direct application, so the system by which they are to be resisted may, usually, be of a simple construction.

Composition and Resolution of Forces.

Two great mechanical principles lie at the base of all proper attempts to estimate the nature of the forces which may be exerted upon substances in particular situations; these principles are called the composition and the resolution of forces.

The resolution of forces is the means of finding any two or more forces which may resist or control the pressure of any one force. The composition of forces consists in finding the direction and amount of one force that is capable of producing the same effect as

two or more forces acting in different directions. This is, in fact only the reverse of the resolution of forces, and the two are strictly speaking, but one principle; and if the one process be understood, the other must be almost so of necessity. Nor may the student pass over this part of the work, under a fear that it is too mathematical for him to understand, for he can never be certain that the roofs or other framing which he may design will support the weights they are intended to carry, if he does not know how to calculate the action of the weights or forces by which they may be pressed.

Let B D, fig. 1, be the king-post of a roof, and let B A, B C, be

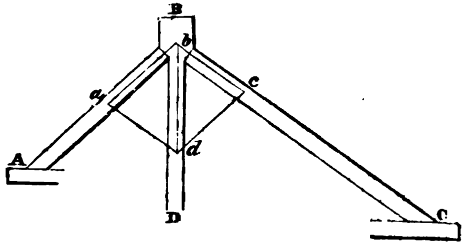


Fig. 1.

the rafters: they are framed together for the purpose of carrying some weight; and the question is this—are they sufficiently strong to carry the weight which is to be placed upon them? To determine this we must refer to the resolution of forces. Let us suppose some determined weight to rest upon the point B. Then, by some scale of equal parts, draw a line B *d*, equal to the number of pounds, hundred weights, or tons, resting upon the point B, and draw *d a* parallel to B C, and *dc* parallel to B A. Now measure the line *a* B by the same scale, and it will give the number of pounds, hundred weights, or tons, by which A B is strained, and *c* B will give the strain upon B C. But, in the drawing affixed, the rafter B C is longer than the rafter B A; but this does not at all affect the weight, for it remains the same, whatever may be the length of the beam which carries it; but it is necessary to remember that, by increasing the length of the beam, it is rendered less capable of supporting the weight, and a proportionate increase of dimensions must be allowed. But should the direction of the beam be changed, a very different result will be obtained, for in every case the pressure will be increased or decreased. The strain upon the beam B A, fig. 2, will now be measured by the line *a b*, and that upon B *c* by *b c*. In fact, a very slight alteration of position may, under certain circumstances, enormously increase or decrease a strain. It will be scarcely necessary to explain how two or more

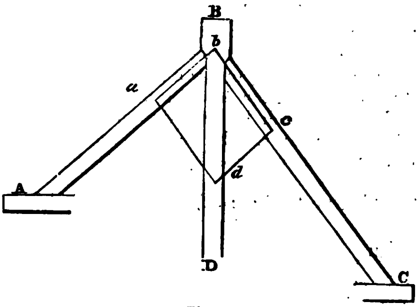


Fig. 2.

forces may be composed, and the single force, acting in a certain direction, be calculated.

Leaving the subject of the composition and resolution of forces, after a statement of the principle, we may proceed to explain the construction and arrangement of those parts of a building which belong to the carpenter. And, first of all, we may speak of roofs.

The Construction of Roofs.

The simplest method of constructing a roof is to place horizontal timbers from wall to wall, but this method is only suited to very short bearings, and does not readily throw off the water which may fall upon its covering. The Egyptians constructed flat roofs. To prevent this inconvenience, a roof may be made as an inclined plane; and such a construction has advantages, though its want of uniformity and beauty, and also its want of strength, proportioned to the amount of timber employed, are objections to its use; but still it is stronger than the flat roof, and readily carries off the water that may fall upon it. The best form for a roof is that in which there are two sides, equally inclined to the horizon, and resting in a line called the ridge of the roof. The angle which the inclined side forms with the horizon is called the pitch. In countries where there is a cold climate, and snow is apt to fall in large quantities, the roof is high; in warm countries the roof is low. In Gothic architecture the roof is generally high pitched, and it is so consonant with the style that it often forms a prominent feature in these buildings. There are not so many advantages in high pitched roofs as most persons suppose, and there are many disadvantages. The additional force of the wind upon a high roof is a serious objection, and when parapets are employed it is so far from preventing the effects of a heavy fall of rain or snow that the gutters are so filled that the pipes cannot carry off the water fast enough, or, being stopped by the dirt carried down by the velocity of the water, an overflow is occasioned. The height of roofs is now generally between one third and one sixth of the span.

It is the carpenter's business to frame the timbers of roofs, and sometimes he is required to design them, and he should therefore know how to obtain the strength and other qualities required, with the smallest possible amount of timber.

A piece of timber, in whatever way it may be placed, except when vertical, will bend or sag, that is to say, its upper side will form itself into a concave surface. The more horizontal the timber is placed the more it will always sag, and as the distance between the points on which it rests is increased, so it has greater liabilities of bending. To prevent this effect as much as possible, arrangements must be made for the support of the beam in some intermediate points. Now, it may be supported from either above or below. If there should be any walls between those on which the ends of the timber rest, these will be sufficient for all the purposes required; if not, the same result must be produced by a system of framing.

The timbers which compose a roof are known by different names, according to the uses for which they are employed, and the situations in which they are placed. The principal timbers of a roof are the following, but they are not all used in every roof: the tie-beams, wall-plates, collar-beams, king-posts, queen-posts, struts, principal rafters, common rafters, ridge-piece, collar-beams, purlins, and pole-plates.

The TIE-BEAM (A), fig. 3, is a horizontal piece of timber, which

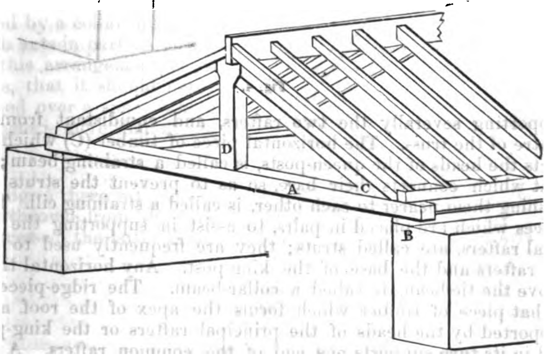


Fig. 3.

extends from wall to wall, and rests upon the WALL-PLATES (B) at each end. It is employed for the purpose of connecting the feet of the principal rafters (C), which would otherwise have a tendency to push out the walls by their own weight, and the weight of the materials placed upon them. In roofs of large span, it is necessary that the tie-beam should be well supported in some point or points, between the ends on which it is supported, for if this be not done it will sag and draw either one or both of the principal rafters towards its centre, and thus destroy the stability of the framing. The KING-POST (D) is sometimes used for this purpose. It

is a piece of timber placed in a vertical position, connecting the point where the two principal rafters meet, and the centre of the tie-beam.

When the king-post is not thought to be sufficient to support the pressure which may be on the framing, QUEEN-POSTS (B), fig. 4, may be used, which are pieces of timber placed in an upright position,

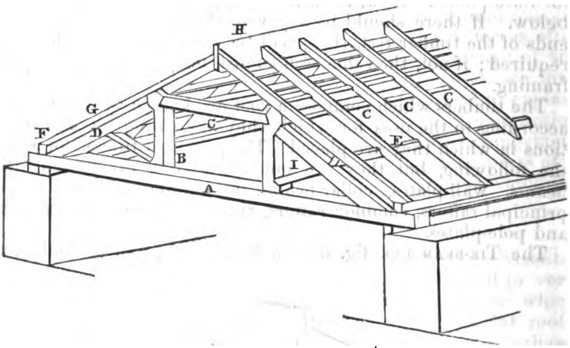


Fig. 4.

supporting severally the two rafters, and equidistant from the centre of the truss. The horizontal piece of timber (C) which connects the heads of the queen-posts, is called a straining-beam; and that which connects their base, so as to prevent the struts from pushing them nearer to each other, is called a straining cill. Those pieces which are placed in pairs, to assist in supporting the principal rafters, are called struts; they are frequently used to unite the rafters and the base of the king-post. Any horizontal timber above the tie-beam is called a collar-beam. The ridge-piece (H) is that piece of timber which forms the apex of the roof and is supported by the heads of the principal rafters or the king-posts, and in its turn supports one end of the common rafters. A pole-plate is a beam over the walls, supported by the principal rafters or the tie-beam, and is intended to carry the lower ends of the common rafters. Purlins (E) are horizontal timbers, between the pole-plates and ridge-piece. The small spars (*c c*), which are parallel to the principal rafters, and are supported by the ridge-plate, purlins, and pole-plates, are called common rafters.

The Dimensions of Timbers used in a Roof.

However accurately a roof may be designed, it is unfit for its purpose if the dimensions of the parts be not accurately proportioned. To accomplish this, some experience is required, and a

knowledge of the strength of timbers, under particular circumstances.

There are two things to be secured—a sufficient strength to support the weights to be carried without sagging, and to do that without burdening the walls or other parts of the building over which the roof is thrown. This is not always an easy task, for roofs are sometimes to be made in such forms as prevent the adoption of those means which would otherwise immediately accomplish the object. Sometimes a very large roof must be made flat, at other times a lantern-light must be provided in its centre; and, in a third case, it may be necessary to erect a dome. In designing for these and other roofs, attention should be paid to the character and success of similar works already executed, and the artist should study the points of similarity and difference between these and his own work, so as to provide against dangers, which may peculiarly affect his building.

Examples of Roofs.

Fig. 5 is a roof, the rafters of which are only supported by a collar-beam (C), which acts in part as a tie; but this arrangement is so feeble, that it should never be used over a space where the span is more than fifteen feet.

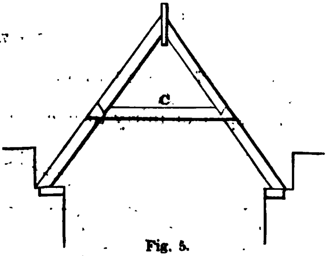


Fig. 5.

In fig. 6 there is the addition of a tie-beam (A), and the strain is here thrown from the collar to the tie-beam; the former being compressed, the latter in a state of tension. As there is no arrangement in this truss to support the tie-beam, and to prevent it from sagging, it is unfit for a span of more than twenty-five feet.

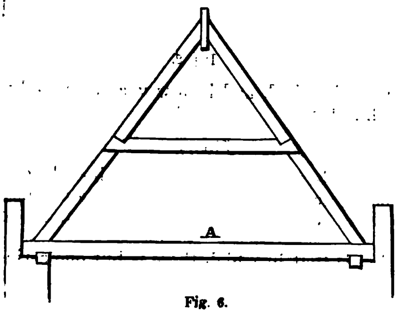


Fig. 6.

To prevent the inconveniences resulting from the sagging of the tie-beam, a king-post (P) and struts (SS) may be introduced, as

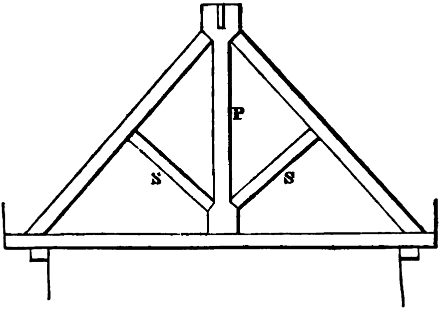


Fig. 7.

shown in fig. 7. This form of roof is very well adapted for a span of twenty-five feet.

For a span of thirty to five-and-forty feet, the truss represented

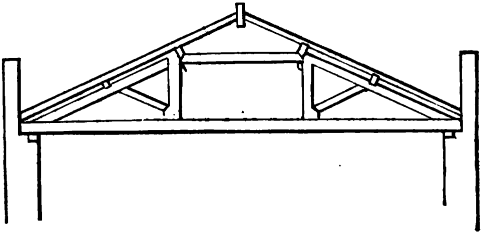


Fig. 8.

in fig. 8 is very well suited, and is now very commonly adopted by architects and builders.

Floors.

The timbers which support the flooring boards, and the ceiling of a room beneath, are called, in carpentry, the naked flooring.

There are three kinds of naked flooring—single, double, and framed.

Single flooring is that in which there is but one series of joists, as shown in fig. 9, where A A A are joists, and B the flooring-boards. To make a single floor as strong as possible, the joists should be thin but deep, sufficient thickness being always allowed for the nailing of the flooring boards. Two inches by six is the smallest

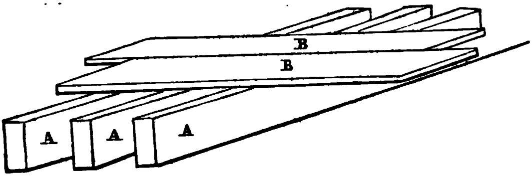


Fig. 9.

dimension for joists; for a length of twenty feet they should be about three inches thick, and twelve inches deep.

Sometimes the joists cannot have in a particular place a bearing upon the walls, and then a piece of timber is framed between the nearest joists. This is done where flues, fire-places, and stairs interfere. The timber thus used is called a trimmer, and the two joists on which it is supported are called trimming-joists, and should be made a little stronger than the common joists. Thus, in fig. 10,

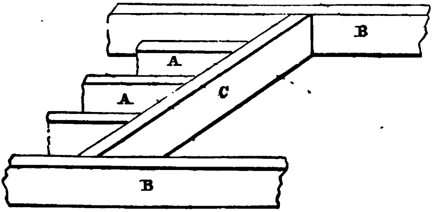


Fig. 10.

A A are common joists, B B trimming joists, and C a trimmer. When the bearing is more than seven or eight feet, the joists should be strutted; that is to say, short pieces of board should be fitted between the joists, so as to form a continued line from wall to wall. These struts greatly strengthen the floor, and prevent the joists from sinking; but it is not desirable to mortice them into the joists, as that process has the effect of weakening the joists themselves.

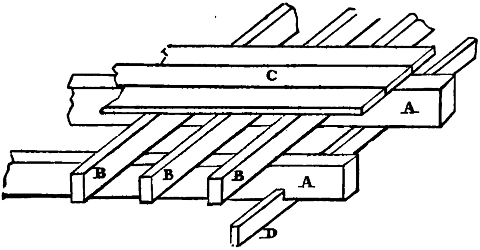


Fig. 11.

Double flooring is that in which there are two tiers of joists, the binding joists, as A A, in fig. 11, which in fact support the floor, and the bridging joists B B. In this kind of flooring, the binders extend from wall to wall, and the bridging joists are notched down upon them. Beneath the binders we have a third tier of timbers (D), which are pulley-morticed into the binders, and are called ceiling joists.

When the binding joists are framed into a large piece of timber, called a girder, the floor is said to be a double framed floor. Thus in fig. 12 A is the girder, B a binding joist, C a bridging joist, D D

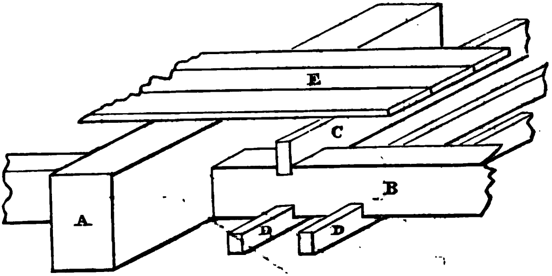


Fig. 12.

ceiling joists, and E flooring boards. This kind of floor is decidedly the best when it is necessary to provide for a good and even ceiling, for although single floors may be made very strong for a great bearing, yet the ceilings are always liable to crack.

It is not easy to obtain timber for girders of much more than twenty feet scantling, and they are therefore trussed. Trusses are used in both floors and roofs, but we have not thought it desirable to interrupt the course of explanation we have given, by a reference to any particulars concerning this branch of carpenter's work; yet it is necessary that we should now make a few remarks upon it.

Trusses.

When timbers are so framed together as to support weights, they are called trusses. It frequently happens that a piece of timber, in itself incapable of supporting a weight, may, when cut into scantlings of different dimensions, and framed together, not only carry that weight, but also support a much greater. The bow and string roof, invented by Mr. Smart, is an example in point.

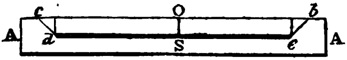


Fig. 13.

Let A A, in fig. 13, be a piece of timber, which we will suppose to be insufficient of itself to carry a particular weight; from this cut the pieces *o*, *s*, *e*, *b*, and *o*, *s*, *d*, *c*. Then let these pieces be raised as in fig. 14, and a key be placed between them at the apex; and it will form a very strong truss, which may be made still more capable of resisting a strain, by the application of struts.

Fig. 14.

The principal rafters of a roof are so called because they are trussed. It is not necessary to truss all the rafters in a roof, and it would be very expensive to do so; and therefore trusses are placed at particular distances from each other, according to the weight to be carried; and they are formed in different ways, according to the span over which they are thrown.

It has been already stated, that girders are sometimes trussed, and should always be when their bearing is much more than twenty feet. We have often seen trusses which, so far from strengthening the girders, have decidedly weakened them. Large girders are sometimes sawn down the middle, and when reversed, are bolted together with slips of wood between them. It has been supposed that this strengthens, and is adopted for this purpose; but the supposition is erroneous, though the plan is certainly a good one, for it allows a free circulation of air between the pieces, and facilitates the emission of any dampness that may be in the timber.

A strong girder may be made as strong, in fact, as any truss of the same depth, by bolting two pieces of timber together, or by confining them with iron hoops, the ends of the girder being smaller than the centre, so as to allow the hoops to be driven tighter, and confine the beams.

In fig. 15 we have given a representation of a strong truss

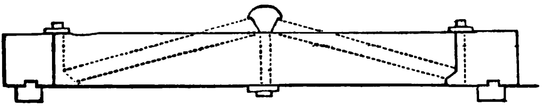


Fig. 15.

girder, the truss post and the abutment pieces being made of wrought iron.

Of Connecting Timbers.

It is sometimes impossible to obtain timbers of the length required for the several parts of a building, and it is then necessary to join two or more pieces together, so as to form them into one piece, and to injure the stability as little as possible. This process

is called scarfing, and the parts of the joints which come in contact are called scarfs, and are usually connected by iron bolts.

There are many ways of scarfing, every builder adopting that one which appears to him the best under the circumstances in which the timber is to be employed. Two or three different methods may be mentioned, leaving the workman to examine those which he may happen to meet with in practice, and the various designs which have been given by writers on the art of building.

Fig. 16 shows the means of scarfing without diminishing the

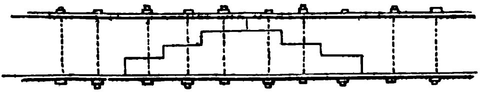


Fig. 16.

length of the pieces. This is done by the introduction of a third piece, having the form of steps, and all the pieces being united together by bolts and plates.

Fig. 17 is a representation of a scarfing, which is very simple, and frequently used, though there is a considerable loss of timber.

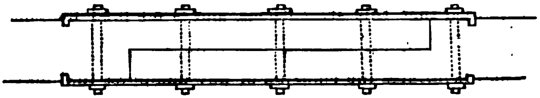


Fig. 17.

The pieces to be united are connected by iron bolts, an iron plate being placed on both sides.

Fig. 18 represents a form of scarfing, adapted to a beam, which

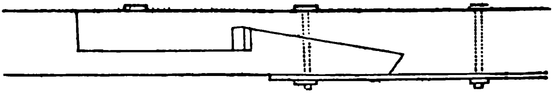


Fig. 18.

has to support a cross strain. In many arrangements, the whole strain is supported by the straps and bolts, but in this they do not, in consequence of the indentation.

Timber Partitions.

Rooms and passages are often separated by timber partitions, which are so formed as to be covered with lath and plaster. In fig. 19 we have given a design for the framing of a partition, with a door through it; A A are the door-posts, B the head, C the sill, D D are braces which support the quartering, and are assisted by

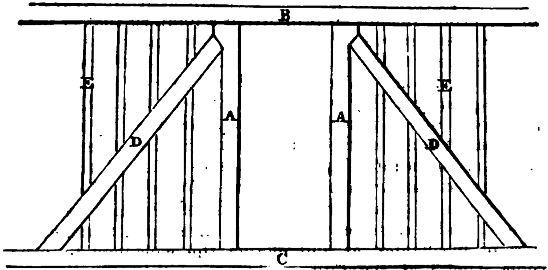


Fig. 19.

the struts, E E. It will be quite evident from a glance at the drawing, that the door-posts help to sustain the braces and struts; while they in return prevent the fall of the door-posts. Braces may be introduced in various ways, but strength is the object for which they ought to be introduced, a circumstance which is very frequently entirely forgotten by carpenters. In some instances, it may be found desirable to introduce a simple truss into a design for partitions.

The carpenter usually connects his timbers either by notching, or by mortice and tenon. Dovetail joints are sometimes used in carpentry, but they ought never to be adopted, for they will always draw when the timber shrinks, and the oblique surface of the dovetail tends to force the timbers apart, acting as though it were a wedge.

Gluing Joints.

In general, nothing more is necessary to glue a joint, after the joint is made perfectly straight, or, in technical terms, out of winding, than to glue both edges while the glue is quite hot, and rub them lengthwise until it has nearly set. When the wood is spongy, or sucks up the glue, another method must be adopted, one which strengthens the joints, while it does away with the necessity of using the glue too thick, which should always be avoided; for the less glue there is in contact with the joints, provided they touch, the better; and when the glue is thick, it chills quickly, and cannot be well rubbed out from between the joints. The method to which we refer is, to rub the joints on the edge with a piece of soft chalk, and, wiping it so as to take off any lumps, glue it in the usual manner; and it will be found, when the wood is porous, to hold much faster than if used without chalking.

Of the different Methods of joining Woodwork.

Many workmen are not aware of the proportion which a piece made to fit into another should have towards that into which it is

fitted, so as to produce the greatest strength with the least possible waste of material; or how to proportion a joint, so that it shall not fail or give way before another. In too many instances, the method of joining woodwork is regulated by no other rule than the fancy of the workman. It is not difficult to explain why joiners' work so frequently fails; why the parts separate with a trifling strain; or, from being bound too tightly together, fly and split in all directions. It is not so frequently from the bad execution of the work, as from the want of an adequate estimate of the strength required to resist the stress on the joint. We shall, then, describe the several kinds of joints, or the methods of framing and joining timber; and, under each head, give such directions, founded on the principles of mechanics, as will enable the workman to proceed with some degree of *certainty;* and not, as is too frequently the case with artisans, observe no other rules than those which custom has authorized, and practice made familiar.

Dovetailing.

We have given, in the cuts, several examples of dovetailing. The parts which fit into each other are known by different names; the projecting piece, represented in fig. 20, is called the pin of the dovetail; and the aperture into which it is fitted, as shown in fig.

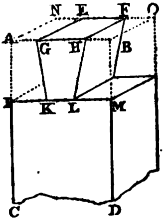


Fig. 20.

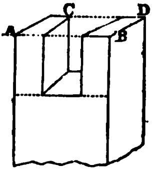


Fig. 21.

21, is called the socket. Now the strength of a dovetail depends upon so proportioning the pin and the socket as to enable them to support, rather than destroy, each other. Let A B C D, fig. 20, be a scantling, which is required to be joined to another, by means of a single dovetail. The strength of the joint depends on the form of the dovetail, as well as on the proportion it bears to the parts cut away. We shall endeavor to lay down the principle on which the greatest strength may be secured. Having squared the end of the scantling, and gauged it to the required thickness, A I K L M, divide I M into three equal parts, at K and L. Let K L be the small end of the dovetail, and make the angles I K G and M L H equal to about 75 and 80 degrees respectively; and make G E and H F parallel to A N and B O. Then cut away the parts A I K G E N, and B M L H F O, and having formed the socket to correspond, by

marking the form of the dovetail on the top of the piece A B C D, fig. 21, and cutting away accordingly, the pieces may be fitted together, as shown in fig. 22. It may be here observed, that the bevel of the dovetail, that is, the angle I K G, fig. 20, may be either more or less than has been mentioned, according to the texture of the wood. Hard, close-grained woods, not apt to rive or split, will admit of a greater bevel than those which are soft, or subject to split; thus the bevel of a dovetail in deal must be less than in hard oak, or in mahogany. It is a great fault to make a dovetail too beveling, for instead of adding to the strength of the joint, as some persons suppose, it weakens it; for provided the bevel is sufficient to prevent the possibility of pulling the pieces apart, the less the bevel that is given the better. It must have been observed, that there is a great difference between the dovetail made by the cabinet-maker and by the joiner; the former has very little bevel, the latter very much; the former looks neat, and is at the same time strong; while the latter, appearing to aim at strength, looks clumsy, and is at the same time much the weaker of the two.

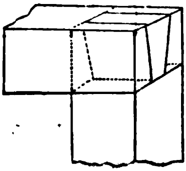


Fig. 22.

Fig. 23 represents the dovetail in common use for drawer-fronts. When it is required to hide the appearance of the joint in front, the board A B C D is cut with the pin, and A E F B with the socket. The pins in this sort of dovetail are in general from about three quarters of an inch to an inch apart, according to the size of the pieces to be joined.

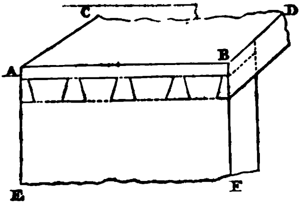


Fig. 23.

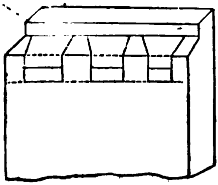


Fig. 24.

Fig. 24 represents the pin part of a lap dovetail, which, when put together, shows only a joint, as if the pieces were rebated together, as shown in fig. 25. A B C D represents the pin, E F G H the socket, and when put together the line H G is only seen as a joint; and if the corner A B is rounded to the joint G H, it will appear as if only mitred together. This kind of dovetail is very useful for many purposes where neatness is required, such as in making boxes.

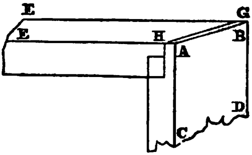


Fig. 25.

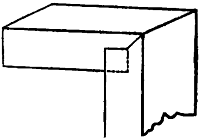


Fig. 26.

Fig. 26 represents a still neater dovetail; and, as the edges are mitred together, it is termed a mitred dovetail; and is the same as that shown in fig. 6, except that instead of the square shoulder, or rebat, in A B, it is cut into a mitre, and the other piece is made to correspond.

Another very neat as well as expeditious method of joining pieces of wood, and it is somewhat analogous to dovetailing, is shown in fig. 27. The joint is first formed into a mitre, and the pieces are then keyed together, either by making a saw kerf in a slanting direction, as at A B, or by cutting out a piece, as at C D, in the form of a dovetail. The first method, A B, is called, amongst workmen, keying together; the second, C D, key-dovetailing.

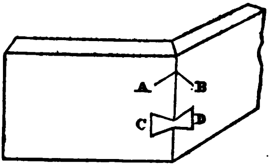


Fig. 27.

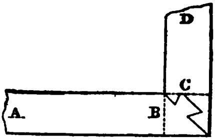


Fig. 28.

The last method to be mentioned is that shown in fig. 28, and may be termed mitre dovetail grooving; the part A B being formed with shoulders cut to the required bevel, and a piece left for the pin dovetail, which is inserted into the socket dovetail, made to correspond to it in the piece C D, which has been previously formed into a mitre. This method, though not much employed, may be used with great advantage in many instances, particularly when it is required to join pieces together the lengthway of the grain.

Mortice and Tenon.

Under this head, we shall endeavor to give some rules necessary to be observed in attempting to proportion the parts of the mortice and tenon, so that they may be equally strong, or that the tenon may not be more likely to give way than the checks of the mortice; for this is the principal thing to be avoided. The workman

frequently allows too little substance for the tenon, lest he should weaken the mortice; and sometimes he falls into the opposite error; facts which clearly prove that he is not acquainted with a means of obtaining a maximum of strength with a given quantity of material.

Figs. 29 and 30 represent a simple mortice and tenon. The dotted

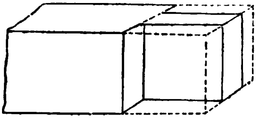


Fig. 29.

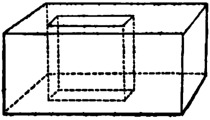


Fig. 30.

lines show the parts to be cut away. To show the thickness of the tenon, and consequently the width of the mortice, we have here one tenon and two shoulders, that is, three parts; one of which is to be allowed for the tenon, and two for the shoulders; and this will in general be found the best proportion, for if the tenon be more than that, it will weaken the shoulders of the mortice. Now if we have, as is frequently the case, two tenons in one piece, as represented in fig. 31, there will be five parts, two tenons, and three shoulders; so that each tenon will be one fifth of the thickness of the stuff, for the shoulders are all equal to the tenons. This rule may be generally observed, unless the tenon is at a considerable distance from the end of the stuff, and then something more may be allowed for its thickness, as the mortice is then not so liable to split; but it should in no case, however sound the timber, or tough the material, be more than two out of four parts; that is to say, it can never be safe to make the tenon more than half the thickness of the stuff, and that only under particular circumstances, when the mortice is near the middle of the scantling, for the piece in which the mortice is cut would, in other cases, be considerably weakened.

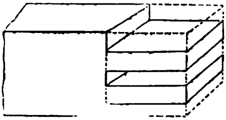


Fig. 31.

There is frequently in joiners' work a shoulder at the bottom of the tenon that fits into the piece in which the mortice is cut, as represented in fig 32; and the tenon is divided into two parts, as there shown, which, when the stuff is wide, is a good method, as it strengthens the piece in which the mortice is cut, without weakening, in the same proportion, the mortice itself; and we would suggest, in this case, that the piece B C, cut out from between the tenons A B

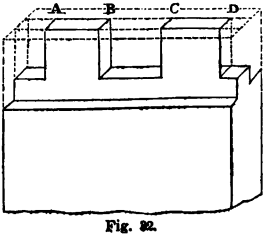


Fig. 32.

and D C, be nearly, if not quite, one third of the distance A D; for if much less, the piece left between the mortice will add but very little to the strength of the piece in which the mortice is made; and the tenon would be stronger in proportion to the mortice-piece than necessary. It may be here observed, that if the width of the tenon be much more than four times its thickness, additional strength will be gained by dividing the tenons into two or more parts, as shown in the figure, particularly if we allow a small piece at the bottom of the tenon, as represented in the drawing.

Grooving and Lapping.

This method of joining wood-work is analogous to that of morticing and tenoning. When it is required to join two boards together by means of a tongue and groove, the groove should never exceed one third of the thickness; and often, if the piece for the tongue be formed of hard wood and liable to split, one quarter of the thickness will be sufficient. When a panel is let into a groove in the style, the joiner is often guided by the thickness of the panel itself, which should never be less than one third the thickness of the style.

In making a groove across the grain, as for partitions, it will be best, in most cases, to make it about a fifth or sixth of the substance of the stuff. But, if the groove be formed into a dovetail, one quarter the thickness will be better, and the dovetail should be made a little tapering, but not too much. It should, in fact, be so made as to go almost home without requiring a blow from a hammer or mallet to drive it into its place until it has nearly attained it; and all joints should be easily separated with a gentle blow before they are glued. In a lap-joint, that is, in lapping two pieces together, supposing them of equal thickness, half the substance of each should be cut away; and, if of unequal thickness, the lap should be made in the thinner piece, of about two thirds or three quarters of its thickness, according to the substance of the thicker piece; thus endeavoring in this, as in all other cases, to avoid weakening one piece more than another.

Bending and Gluing-up.

In bending and gluing-up stuff for sweep-work, much judgment is necessary, and, as the methods are various, we shall mention a few which the workman may apply, as occasion may require, one method being preferable to another, according to the nature of the work in hand.

The first and most simple method is that of sawing kerfs or notches on one side of the board, thereby giving it liberty to bend in that direction; but this method, though very ready and useful for many purposes, weakens the work, and may cause it to break when strains are thrown on the piece. But a tolerably strong sweep may be made in this manner, if, after sawing the kerfs

(particular care being taken to make them regular and even, and to saw them at regular depths), some strong glue be rubbed into each kerf. When bent into the required sweep, a piece of strong canvas should be glued over the kerfs themselves, and the glue be left to harden in the position to which the stuff is bent.

Another method is to glue up the stuff in thin thicknesses, in a cawl or mould, made with two pieces of thick wood cut into the required sweep. This method, if done with care, that is, making the several pieces of equal thickness throughout, of wood free from knots, is perhaps the best that can be devised for strength and accuracy. It is also a practice sometimes to glue up a sweep in three thicknesses, making the middle piece the contrary way of the grain to the outside and inside pieces, which run lengthwise. This method, though frequently used for expedition, is much inferior to the above, as the different pieces cannot shrink together, and consequently the joint between them is apt to give way.

A solid piece, if not too thick, may be sometimes bent into the form required. If a piece of timber be well soaked upon the intended outside of the curve, it may be bent into position, and if kept in that position till cold will retain the curvature that is given to it.

The only other method of forming a curve, necessary for us to mention, is that of cutting out solid pieces to the required sweep, and gluing them upon one another till they have the thickness required, taking care that the joints are alternately in the centre of each piece below it, something in the manner of courses of bricks one above the other. In this case, it will be necessary, if the work be not painted, to veneer the whole with a thin piece, after it has been thoroughly dried and planed level, and then made somewhat rough with either a rasp or toothing-plane. But the joiner must adopt one plan or another, according to circumstances.

Scribing.

Scribing is the operation by which a piece of wood-work is made to fit against an irregular surface. Thus, for instance, the plinth of a room is made to meet or correspond with the unevenness of the floor. To determine the portion which is to be cut off from a partition, or any wood-work where a floor or ceiling is irregular, it is only necessary to open the compasses to a width equal to the greatest distance between the plinth and the floor; and, passing one leg over the uneven surface, the other leg will leave a mark on the plinth. If the wood be cut away on that line, a surface will be obtained which will make a good joint with the floor or ceiling. But the chief use of the art of scribing is to enable the joiner so to connect the moulding of panels or cornices, that when placed together, they shall seem to form a regular mitre-joint. This method has certainly one advantage over the common method of mitring, for, if the stuff should shrink, little or no alteration will be made in the appearance, but, under the same circumstances, a mitre

would open, and the joint would be shown. The method adopted is this: To cut one piece of the moulding to the required mitre, and then, instead of cutting the other to correspond with it, cut away the parts of the first piece to the edge of the first moulding which will then fit to the other moulding, and appear as a regular mitre.

Finishing of Joiners' Work.

Joiners' work is generally intended to increase the beauty of a building. When a joiner works in wainscot, oak, or mahogany, his chief object must be to obtain a surface perfectly smooth and even. When the framing is glued together, the glue which oozes out, and may be spilt upon the work, must be allowed to remain a few minutes and chill, and may then be carefully scraped off with a chisel; and the parts which cannot be thus cleaned may be washed with a sponge dipped in hot water and squeezed nearly dry. This not only saves trouble in operations which follow, but prevents staining, always produced when glue is suffered to remain till quite hard, particularly on wainscot, which turns black in every joint or place where the glue is suffered to remain. After this operation, which, though it may appear tedious to some workmen, will be found a saving of time, the work should remain till perfectly dry; and, when the joints and other parts have been levelled with a smoothing plane, the whole surface may be passed under a smooth scraper, and finished with fine glass-paper. It will be sometimes necessary, when the grain is particularly cross, to damp the entire surface with a sponge "to raise the grain," and then again to apply the glass-paper. The work will then be ready for polishing with wax, or varnishing, and the good appearance of the work will be in proportion to the time and trouble expended in the process.

In cleaning pine, the same precautions must be taken for the removal of glue left upon the joints, or spilt upon the work, as already described. This being done, the work may be cleaned off with a piece of glass-paper that has been rubbed with chalk, or, in some cases, with a piece of hearthstone. The work is then ready for the painter; but as there are knots and other places where the turpentine contained in the wood is apt to ooze out, either with or without the increase of heat, and thus spoil the appearance of the finishing, those parts are done over with a composition, and the process is called priming. This is properly the painter's business; but it must sometimes be done by the joiner, for the sake of saving his work. The composition used for this purpose is made with red lead, size, and turpentine, to which is sometimes added a small quantity of linseed oil. Priming has also the advantage of preventing the knots from being seen through the paint. Some workmen omit in this composition the oil and the turpentine, but the size of itself is apt to peel off, and does not thoroughly unite itself with the wood.

Another method of cleaning-off pine is sometimes adopted. When the surface has been made quite smooth with the plane, it is rubbed with a piece of chalk, and the whole is cleaned with a piece of fine pumice-stone, as in the former process it was done with glass paper; but if the grain should be still rough, the work may be damped with a sponge, and the operation repeated when dry.

As, in finishing interior work, it is now customary to imitate the graining of different kinds of wood, it is necessary that the joiners work should be well finished; for if a good even surface be not provided, it will be impossible for the painter to produce the effect he desires. Every defect in the ground will, in fact, be more visible under a delicate graining than when the surface is covered with successive coats of color; but, even in the latter case, work well prepared will not only look better, but the color will not be so apt to chip and peel off as when the surface is not properly levelled.

TERMS USED IN BUILDING.

Abacus.—The upper member of the capital of a column, that on which the architrave rests. It has different forms in the several orders: In the Tuscan or Doric, it is a square tablet; in the Ionic, its edges are moulded; in the Corinthian, its sides are concave, and frequently enriched with carving.

Abutment.—That part of a pier from which the arch springs.

Acanthus.—A plant whose leaves are carved on the Corinthian and Composite capital. They are differently disposed, according to circumstances; and the leaves of the laurel and parsley are sometimes employed in their place.

Acroterium.—A pedestal on the angle or apex of a pediment, intended as a base for sculpture.

Altitude.—The perpendicular height of anything in the direction of the plumb line. The length of a body is measured on the body itself, and remains constant, its altitude varies according to its inclination to or from the perpendicular.

Alto Relievo.—A sculpture, the figures of which project from the surface on which they are carved.

Amphiprostylos.—An order of Grecian temples, having columns in the back as well as the front.

Amphitheatre.—A double theatre, employed by the ancients for public amusements. The colosseum at Rome, built by Vespasian, is one of these.

Annulet.—A small square moulding, used to separate others; the fillet which separates the flutings of a column is sometimes known by this term.

Antæ.—Pilasters attached to a wall, receiving an entablature, and having bases and capitals differing according to the order employed, but always unlike those of the columns.

Antepagmenta.—A term in ancient architecture, the architraves round doors.

Apophyge.—That part of a column which connects the upper fillet of the base and the under one of the capital with the cylindrical part of the shaft.

Aræostylos.—That style of building in which the columns are distant from one another from four to five diameters. Strictly speaking, the term should be limited to an intercolumniation of four diameters, which is only suited to the Tuscan order.

Arch.—Such an arrangement, in a concave form, of building materials, as enables them, supported by piers or abutments, to carry weights and resist strains.

Arch-buttress.—Sometimes called a flying buttress; an arch springing from a buttress or pier against a wall.

Architrave.—That part of the entablature which rests upon the capital of a column, and is beneath the frieze. It is supposed to represent the principal beam of a timber building.

Area.—This term is applied to superficies, whether of timber, stone, or other material, and is the superficial measurement; that is, the length multiplied into the breadth. The word area sometimes signifies an open space.

Arris.—The line in which two surfaces meet each other.

Ashler.—Common freestone, as it comes from the quarry, generally about nine inches thick, but of different superficial dimensions.

Ashlering.—Quartering, to which laths are nailed.

Astragal.—A small moulding with a semicircular profile, sometimes plain and sometimes ornamented.

Attic Order.—A term used to denote the low pilasters which are placed over orders of columns or pilasters, and frequently employed in the decorations of an attic.

B.

Baluster.—A small pillar or pilaster, supporting a rail.

Balustrade.—A series of balusters connected by a rail.

Band.—A square member. To distinguish the situation in which it is placed, or the order in which it is used, an adjective is frequently prefixed; thus, a dentil or a modillion band.

Base.—The lower division of a column. The Grecian Doric has no base, and the Tuscan has only a single torus on a plinth.

Bead.—A circular moulding, which lies level with the surface of the material in which it is formed. When the moulding projects, or several are joined, it is called reeding.

Beak.—A small fillet in the under edge of a projecting cornice,

intended to prevent the rain from passing between the cornice and fascia.

Beam.—A piece of timber in a building laid horizontally, and intended to support a weight, or to resist a strain.

Beam-filling.—The masonry, or brickwork, between beams or joists.

Bearer.—A vertical support.

Bearing.—The length between bearers, or walls; thus, if a beam rests on walls twenty feet apart, the bearing is said to be twenty feet.

Bed Mouldings.—Those mouldings between the corona and the frieze.

Bevil.—An instrument used by workmen for taking angles. In form it resembles a square, but the blade is moveable about a centre. When the two sides of any solid body have such an inclination to each other as to form an angle greater or less than a right angle, the body is said to be beviled.

Bond.—A term used to signify the connection between the parts of a piece of workmanship. In bricklaying and masonry, it is that connection between bricks, or pieces of stone, which prevents one part of the building from separating itself from another.

Bond Timber.—Timber laid in walls to tie or bind them together.

Brace.—A piece of timber placed in an inclined position, and used in partitions or roofs, to strengthen the framing. When a brace is employed to support a rafter, it is called a strut.

Bressummer.—A beam, or iron tie, intended to carry an external wall, and itself supported by piers or posts.

Bricknoggin.—Brickwork between quartering.

Buttress.—A mass of stone or brick-work intended to support a wall, or to assist it in sustaining the strain that may be upon it. Buttresses in Gothic architecture are used for ornament as well as strength.

C.

Cabling.—Cylindrical pieces filling up the lower part of the flutes of a column.

Camber.—To give a convexity to the upper surface of a beam.

Cantalivers.—Pieces of wood or stone beneath the eaves to support them, or mouldings above them.

Capital.—That part of a column or pilaster beneath the entablature; or, in other words, the uppermost member of a column or pilaster. The capital is variously formed, according to the order: Thus, we have the Tuscan, Doric, Ionic, Corinthian, and Composite capitals, and many others, that have been invented since the times of the Greeks and Romans.

Caryatides.—Figures of women, introduced to support an entablature, instead of columns.

Casement.—Applied to a window which is hung upon hinges in place of lines and weights.

Casting.—The warping or shrinking of timber or wood-work, occasioned by an insufficient strength, or by an unequal exposure to the weather, and want of proper seasoning.

Cavetto.—A concave moulding, the quadrant of a circle.

Centering.—The framing upon which an arch is turned.

Clamping.—When one piece of wood is so fixed into the end of another as to prevent it from splitting or casting, it is said to be clamped. The pieces may be united with a mortice and tenon, or with a groove and tongue.

Collar Beam.—A beam framed between two principal rafters.

Console.—An ornament cut on the key-stone of an arch, sometimes in the form of a scroll, at other times to represent a human face.

Content.—The amount of any substance in rods, yards, feet, or inches whether solid or superficial.

Coping.—The stone which covers the top of a wall or parapet.

Corbel.—A bracket, or piece of timber projecting from a wall: in Gothic architecture, usually carved with some grotesque figure.

Cornice.—The combination of mouldings which finishes or crowns an entablature.—The term is also applied to the mouldings which finish the walls and ceiling of a room, hall, or passage, filling up the angle which they make.

Crown.—A term applied to the uppermost or highest part of an arch, that in which the key-stone is fixed.

Cyma.—A moulding with a waved or crooked profile, partly convex, partly concave. It is called by workmen an ogee. When the hollow part of the moulding is uppermost, it is called a cyma-recta; when the convex part is above, a cyma-reversa.

D.

Dado.—That flat part of the base of a column between the plinth and the cornice. It is of a cubical form, and from thence takes its name

Dentils.—Square blocks introduced as ornaments into cornices of the Doric, Ionic, and Corinthian orders. A small circular piece is sometimes cut out, and at other times they are fluted.

Die.—A square cube.

Door Frame.—The case in which a door opens and shuts, consisting of two uprights and one horizontal piece, connected together by mortices and tenons.

Dormer.—A window made in the sloping part of a roof, or above the entablature.

Dovetailed.—When two pieces of wood are fastened together, by letting the pieces of one into apertures formed in the other, of a

shape somewhat resembling a fan or dovetail, they are said to be dovetailed.

Drops—Ornaments in the Doric entablature resembling bells placed immediately under the triglyphs.

Dwarf Wall.—A wall that has a less height than that of the story in which it is used.

E.

Eaves.—The edge of a roof or slating which overhangs a wall, and is designed to carry off the water, without flowing down the wall.

Echinus.—A moulding, the profile of which is the quadrant of a circle turned outwards, or in some instances a conic section. It is said to resemble the shell of the chestnut.

Ellipse.—That curve called by workmen an oval.

Entablature.—That assemblage of mouldings, &c., which are supported by the column. It consists of three parts—the architrave, frieze, and cornice.

Entasis.—The swelling of a column.

Eustylos—That intercolumniation in which the columns are placed two diameters and a quarter from each other.

Eye.—A term sometimes used in architecture to denote a small window in a pediment. The middle of the Ionic volute, that is, the circle within which the different centres for drawing it are found, is known by the same name.

F.

Façade.—The face or front of a building; strictly speaking, the principal front.

Fascia.—A flat broad member, in architecture, but of small projection. It is used to denote the flat members into which the architrave is divided, and these are called fasciæ.

Feather-edged.—Boards or planks thicker at one edge than the other.

Fillet.—A small square moulding, of slight projection. In carpentry, it means a piece of wood to which boards are nailed.

Flashings.—Pieces of lead so let into the wall as to lap over a gutter.

Flatting.—Painting, which has no gloss on its surface, being worked with turpentine. It is used for finishing.

Flutes.—Vertical channels cut in the shafts of columns and pilasters, sometimes meeting one another at a sharp edge, and at other times having a fillet between them.

Flyers.—Stairs which rise without winding.

Flue.—The aperture of a chimney.

Footings.—The courses of brick or stone at the foundation of a wall.

Frieze.—The flat member in an entablature, separating the architrave from the cornice.

Furring.—A means of restoring an irregular framing by the addition of small pieces of wood nailed to the framing itself.

Fust.—The shaft of a column.

G.

Gable.—The upright triangular end of a building at the ends of a roof.

Girder.—The largest piece of timber in a floor, that into which the joists are framed.

Groin.—The intersection of two arches.

Groove.—A rectangular channel cut in stone or timber; such as that which is cut in the stiles to receive the panel of a door.

Grounds.—Those pieces of wood imbedded in the plastering of walls to which skirting and other joiners' finishings are attached.

Guttæ.—See "Drops."

Gutter.—A valley between the parts of a roof, or between the roof and parapet, intended to carry off the rain.

H.

Half Round.—A moulding in a semicircular form, projecting from the surface.

Headers.—Bricks laid with their short face in front.

Hips.—Those pieces of timber placed in an inclined position at the corners or angles of a roof.

I.

Impost.—The combination of mouldings which form the capital of a pier.

Insulated.—A term applied to a column which is unconnected with a wall, or to a building, that stands detached from others.

Intercolumniation.—The space between two columns.

Intertie.—Small pieces of timber placed horizontally between, and framed into, vertical pieces to tie them together.

J.

Jambs.—The side pieces of an opening in a wall, such as door-posts, and the uprights at the side of window frames.

Joggle-piece.—A post to receive struts.

Joists.—Those pieces of timber which are framed into a girder, bressummer, or otherwise, to support a ceiling or a floor.

K.

Key-stone.—That stone in the top or crown of an arch which is in a perpendicular line with the centre.

King-post.—The centre post of a trussed framing, intended to support the tie-beam and struts.

Knee.—A piece of timber bent to receive some weight, or to relieve a strain.

L.

Lantern.—A frame in the dome or cupola of a building to give light. The term is applied to some kinds of fanlights, that is, the frame over a door to light a passage or corridor.

Lining.—That joiners' work which covers an interior surface.

Lintels.—The pieces of timber which lie horizontally over the jambs of windows and doors.

M.

Mantel.—The cross-piece which rests on the jamb of a chimney.

Metopa.—The interval between the triglyphs in the Doric order.

Minute.—The sixtieth part of the diameter of a column.

Modillion.—An ornament in the Ionic, Corinthian, and Composite orders. It is a sort of bracket, and should be placed under the corona.

Module.—The semi-diameter of a column, and is divided into thirty minutes. It is the measure by which the architect determines the proportions between the parts of an order.

Mortise.—A method of joining two pieces of wood; a hole being made in one of such a size as to receive the tenon or projecting piece formed on the other.

Mosaic.—A term applied to pavements, and other work, when formed of various materials of different shapes and colors, laid in a kind of stucco, so as to present some pattern or device. The ancients were very successful in the execution of Mosaic, and many fine specimens remain to this day.

Mullion.—Upright posts or bars which divide the lights in a Gothic window.

N.

Naked.—This term is applied, in architecture, to a plain surface, or that which is unfinished; as the naked walls, the naked flooring —that is, uncovered. The word is sometimes applied to flat surfaces before the mouldings and other ornaments have been fixed.

Newel.—The centre round which the stairs wind in a circular staircase.

Nosings.—The rounded and projecting edges of the treads of stairs.

O.

Obelisk.—A slender pyramid.

Ogee.—A moulding, consisting of a portion of two circles turned in contrary directions, so that it is partly concave and partly convex, and somewhat resembles the letter S.

Order.—An assemblage of parts having certain proportions to one another. There are five orders of architecture—Tuscan, Doric, Ionic, Corinthian, and Composite—all of which were invented by the ancients, and are now employed by the moderns.

Oval.—A curve line, the two diameters of which are of unequal length, and is allied in form to the ellipse. An ellipse is that figure which is produced by cutting a cone or cylinder in a direction oblique to its axis, and passing through its sides. An oval may be formed by joining different segments of circles, so that their meeting shall not be perceived, but form a continuous curve line. All ellipses are ovals, but all ovals are not ellipses; for the term oval may be applied to all egg-shaped figures, those which are broader at one end than the other, as well as to those whose ends are equally curved.

Ovolo.—A convex projecting moulding whose profile is the quadrant of a circle.

P.

Panel.—A compartment inclosed in a frame, into which it is framed or grooved.

Parapet.—A low wall generally about breast-high, on the top of bridges or buildings.

Pargetting.—Rough plastering, commonly adopted for the interior surface of chimneys.

Pedestal.—That arrangement on which columns are sometimes placed: it is divided into three parts—the cornice, the die, and the base.

Pediment.—A low triangular crowning ornament in the front of a building, and over doors and windows. Pediments are sometimes made in the form of a segment of a circle.

Pier.—A square, or other formed mass, used to strengthen or support a building; it sometimes signifies that mass of stone or brickwork between the arches of a bridge, and from which they spring, or against which they abut. But the term is usually employed to designate the solid part between the doors or windows of a building.

Pilaster.—A square pillar insulated, or engaged to the wall, and is usually enriched with a capital and base.

Piles.—Large timbers, usually shod with pointed iron caps,

driven into the ground for the purpose of making a secure foundation.

Pillar.—An irregular, insulated column. It differs from a column in having no architectural proportion, being either too massive or too slender.

Pinnacle.—A small spire used to ornament Gothic buildings.

Pitch of a Roof.—The proportion obtained by dividing the span by the height; thus we speak of its being one half, one third, one fourth.

Plinth.—The solid support of a column or pedestal.

Plumb-line.—An instrument to determine perpendiculars; it consists of a piece of lead attached to a string.

Porch.—The vestibule or entrance to a building.

Portico.—A kind of gallery or piazza, frequently erected in front of large buildings.

Posts.—Square timbers set on end; the term is especially applied to those which support the corners of a building, and are then framed into the bressummer or cross-beam, under the walls.

Pricking-up.—The first coat of plaster worked upon laths.

Profile.—The outline; the contour of a part, or the parts compassing an order.

Pugging.—The stuff laid upon sound boarding to prevent the passage of sound from one story to another.

Punchtons.—Short pieces of timber employed to support a weight when the bearing is too distant.

Purlines.—Those pieces of timber which lie across the rafters to prevent them from sinking.

Putlogs.—Pieces of timber used in building a scaffold; they are those which lie at right angles to the line of wall, and rest on the scaffold poles or ledgers.

Pyramid.—A solid massive edifice which rises from a square or rectangular base, and terminates in a point called the vertex.

Q.

Quarter Round.—See "Ovolo."

Quarters.—Pieces of timber used in an upright position for partitions. Quarters may be either single or double; the single are generally two inches thick, and four inches broad; the double are four inches square. The quarters are never placed at a greater distance than fourteen inches from each other.

Quirk.—A piece of ground taken out of a plot. The term is also applied to a particular form of moulding, one which has a sudden convexity.

Quoins.—The corners of a building; they are called rustic quoins when they project from the wall, and have their edges chamfered off.

R.

Rabbet or Rebate.—A groove or channel in the edge of a board.

Rafters.—Those timbers which form the inclined sides of a roof.

Raking.—Means literally inclining, and is applied to those mouldings which, instead of maintaining the horizontal line, are suddenly bent out of their course.

Rails.—Those pieces in framing which lie in a horizontal position are called rails; those which are perpendicular are called stiles; hence two rails and two stiles inclose a panel. The term is also applied to those pieces in fences or paling which go from post to post.

Relief.—The projection which a figure has from the ground on which it is carved.

Return.—That part of any work which falls away from the line in front.

Ridge.—The highest part of a roof, or the timber against which the rafters pitch.

Riser.—That board in stairs set on edge under the tread or step of the stair.

Rustic.—This term is applied to those courses of stone-work, the face of which is jagged or pecked so as to present a rough surface. That work also is called rustic in which horizontal and vertical channels are cut in the joinings of the stones, so that when placed together an angular channel is formed at each joint.

S.

Sash.—The framework which holds the squares of glass in a window.

Sash-frame.—The frame which receives the sash.

Scantling.—The measure to which a material is to be or has been cut.

Scotia.—A semicircular concave moulding, chiefly used between the tori in the base of a column.

Scribing.—Fitting wood-work to an irregular surface.

Scroll.—A carved curvilinear ornament, somewhat resembling in profile the turnings of a ram's horn.

Sill.—The horizontal piece of timber at the bottom of framing; the term is chiefly applied to those pieces of timber or stone at the bottom of doors or windows.

Shaft.—The body of a column; that part between the base and capital.

Shore.—A piece of timber placed in an oblique direction to support a building or wall.

Skirting.—The narrow boards placed round an apartment against the walls, and standing vertically on the floor.

Sleepers.—Pieces of timber placed on the ground to support the ground-joists, or other woodwork.

Soffit.—A term applied to a frame or paneling overhead, or to a lining, such as that which is fixed in the underside of the tops of windows.

Stiles.—The upright pieces in framing or paneling.

Struts.—Pieces of timber which support the rafters.

Summer.—A large piece of timber supported by piers or posts; when it supports a wall, it is called a breast-summer, or bres-summer.

T.

Tenon.—A piece of wood so formed as to be received into a hole in another piece called a mortice.

Throat.—That hollow which terminates the upper end of the shaft of a column.

Tongue.—That projecting piece at the end of a board which is formed to be inserted into a groove.

Torus.—A moulding that has a convex semicircular or semi-elliptical profile.

Transom.—A piece that is framed across a double window-light.

Trellis.—An open framing, pieces crossing each other so as to form diamond or lozenge-shaped openings.

Tryglyphs.—Ornaments in the Doric frieze consisting of a square projection with two angular channels, the edges of each forming half a channel. They are placed immediately over the centre of a column; their width is generally one module.

Trimmers.—Pieces of timber framed at right angles to the joist for chimneys, and the well-holes of stairs.

Tympanum.—The space inclosed by the inclined and horizontal sides of a pediment.

V.

Valley.—The space between two inclined sides of a roof.

Vaults.—Underground buildings with arched ceilings, whether circular or elliptical.

Vertex.—The top or summit of a pointed body, as of a cone.

Volute.—The scroll in the capital of the Ionic order.

Voussoirs.—The stones which compose the face of an arch, having a somewhat wedge-shaped form.

W.

Wall-plates.—The timbers built up with a wall, to carry the joists.

Weather-boarding.—Weather-edge boards, fixed vertically, so as to lap over one another.

Well-hole.—The aperture left in floors to bring up the stairs.

GLUES.

Parchment Glue.

Parchment shavings 1 pound; water 6 quarts. Boil until dissolved, then strain and evaporate slowly to the proper consistence. Use a water bath if you want it very light colored.

Japanese Cement, or Rice Glue.

Rice flour; water, sufficient quantity. Mix together cold, then bring the mixture to a boil, stirring it all the time. Observe to boil it in a vessel that will not color it.

Japanners' Gold Size.

Gum ammoniac 1 pound; boiled oil 8 ounces; spirits of turpentine 12 ounces. Melt the gum, then add the oil, and lastly the spirits of turpentine.

Gold Size.

Yellow ochre 1 part; copal varnish 2 parts; linseed oil 3 parts; turpentine 4 parts; boiled oil 5 parts. Mix. The ochre must be in the state of the finest powder, and ground with a little of the oil before mixing.

Glue Liquid.

Glue, water, vinegar, each 2 parts. Dissolve in a water-bath, then add alcohol 1 part. An excellent cement.

Transparent Liquid Japan for Metal.

Copal varnish 35 parts; camphor 1 part; boiled oil 2 parts. Mix.

Portable Glue for Draughtsmen, &c.

Glue 5 parts; sugar 2 parts; water 8 parts. Melt in a water-bath, and cast it in moulds. For use, dissolve in warm water.

Waterproof Glue.

1. Glue 1 part; skimmed milk 8 parts. Melt and evaporate in a water-bath to the consistence of strong glue.
2. Glue 12 parts; water sufficient to dissolve. Then add yellow resin 3 parts, and when melted add turpentine 4 parts. Mix thoroughly together. This should be done in a water-bath.

PAPERS.

Fire-proof Paper.

Take a solution of alum and dip the paper into it, then throw it over a line to dry. This is suitable to all sorts of paper, whether plain or colored, as well as textile fabrics. You must try a slip of the paper in the flame of a candle, and if not sufficiently prepared dip and try it a second time.

Black Edge Paper.

Blacklead 11 parts; common ink 22 parts; dissolved gum-arabic 1 part. Mix. Then with a sponge lay the color on the edge of the paper, previously placed in the cutting-press, rub it in with a piece of cloth, and burnish. The edge of the paper must be rendered perfectly smooth before applying the black.

To Stain Paper or Parchment.

Red.—Brazil 12 parts; water 70 parts; alum 5 parts. Boil.

1. *Blue.*—Sulphate of indigo. Water to dilute.

2. Prussian blue 2 parts; muriatic acid 1 part. Water to dilute.

3. Logwood 4 parts; water 30 parts; sulphate of copper 1 part. Mix.

Green.—Crystals of verdigris 2 parts; vinegar 1 part. Water to dilute.

Yellow.—French berries, water, and a little alum. Boil.

Purple.—Logwood 2 parts; alum 1 part; water 20 parts. Boil. The addition of a little gum to the above renders them suitable for coloring maps, &c.

Paper for Draughtsmen, &c.

Powdered tragacanth 1 part; water 10 parts. Dissolve and strain through clean gauze, then lay it smoothly with a painter's brush on the paper, previously stretched on a board. This paper will take either oil or water colors.

Copying Paper.

Lay open your quire of paper (clean white, of large size), take the brush and cover it with the following varnish, then hang it up on the line; take another sheet and repeat the operation, until you have finished your quantity. If not clear enough, give each sheet another coat when dry:—Canada balsam, turpentine, equal parts. Mix.

Liquid Gold, for Vellum, &c.

Take gold-leaf and grind it with gum-water; then add a small quantity of bichloride of mercury, and bottle for use.

Liquid Silver, for Vellum, &c.

Take silver-leaf and grind it, with gum-water or glair of egg.

Paper that Resists Moisture.

Take unsized paper, lay it flat on a clean surface, and brush it over with a solution of mastic in oil of turpentine; or plunge it into the solution and hang it up to dry. This paper possesses all the usual qualities of writing paper, with the advantage of resisting moisture.

To Detect the presence of Plaster in Paper.

Calcine the paper in a close vessel, and dilute the residue with

vinegar, in a silver spoon; if sulphuretted hydrogen is disengaged, which blackens the spoon, the presence of a sulphate (plaster) will be shown. This adulteration has lately become very common among the paper-makers, with the view of increasing the weight.

Waxed Paper.

Take cartridge or other paper, place it on a hot iron and rub it with beeswax, or make a solution of the wax in turpentine, and apply it with a brush. Useful for making water or air-proof pipes, for chemical experiments, &c.

To extract Grease Spots from Paper.

Apply a little powdered pipe-clay, on which place a sheet of paper, then use a hot iron. Remove the adhering powder with a piece of India-rubber.

Papier Mâché.

Take paper, any quantity. Boil it well, then pound it to a paste, and mould. Used in making toys, snuff-boxes, &c.

To Gild the Edges of Paper.

Armenian bole 4 parts; sugar-candy 1 part. White of egg to mix. Apply this composition to the edge of the leaves, previously firmly screwed in the cutting-press; when nearly dry smooth the surface with the burnisher; then take a damp sponge and pass over it, and with a piece of cotton-wool take the leaf from the cushion and apply it to the work; when quite dry burnish, observing to place a piece of silver or India paper between the gold and the agate.

Tracing Paper.

Nut oil 4 parts; turpentine 5 parts. Mix, and apply it to the paper, then rub it dry with wheat flour, and brush it over with ox-gall. This will bear writing on.

Lithographic Paper.

Give the paper 3 coats of thin size, 1 of starch, and 1 of solution of gamboge. Each to be applied with a sponge, and allowed to dry before the next is applied.

Hydrographic Paper.

This name has been given to paper which may be written on with water. It may be made by rubbing paper over with a mixture of finely powdered galls and sulphate of iron heated till it becomes white. The powder may be pressed into the paper by passing it between rollers, or passing a heavy iron over it. A mixture of dried sulphate of iron and ferro-prussiate of potash may be used for blue writing. Or the paper may be imbued with a strong solution of one ingredient thoroughly dried, and the other applied in powder. Paper which has been wet with a solution of ferro-

prussiate of potash also serves for writing on with a colorless solution of persulphate of iron.

Iridescent Paper.

Nut-galls 8 parts; sulphate of iron 5; sal-ammoniac 1; sulphate of indigo 1; gum-arabic $\frac{1}{8}$. To be boiled in water, and the paper washed with it exposed to ammonia.

To give Paper the Appearance and Toughness of Parchment.

Dip white unsized paper for half a minute in strong sulphuric acid, and afterwards in water containing a little ammonia. When dried it will look like, and be as strong as parchment.

Photographic Papers.

The following papers should be the finest satin post, of uniform texture, free from the maker's mark, specks, and all imperfections. The papers must be prepared by candle-light, and kept in the dark till used.

1. *Simple Nitrated Paper.*—This is merely paper brushed over with a strong solution of nitrate of silver. In brushing over the paper it must be crossed. Its sensitiveness is increased by using spirit of wine instead of water. This paper only requires washing in water to fix the drawing.

2. *Muriated Paper.*—The paper is first soaked in solution of copper salt, pressed with a linen cloth or blotting paper, and dried. It is then brushed over on one side (which should be marked near the edge) with the solution of nitrate of silver, and dried at the fire. The stronger the solution the more sensitive the paper. If dipped in a solution consisting of 35 grains of chloride of barium and 2 oz. of distilled water, richer shades of color are obtained.

3. *Iodized Paper.*—Brush over the paper on one side (which should be marked) with strong solution of nitrate of silver (100 gr. to 1 oz); then dip it in a solution consisting of 100 gr. of iodide of potassium dissolved in 4 oz. of distilled water. Wash it in distilled water, drain, and dry it.

4. *Bromide Paper.*—Soak the paper in a solution composed of 40 gr. bromide of potassium dissolved in 1 oz. of distilled water; then brush it over with a strong solution of nitrate of silver, and dry in the dark.

5. *Calotype Paper.*—Dissolve 100 gr. of crystallized nitrate of silver in 2 oz. of distilled water, and add 2 fluid dr. and 40 minims of acetic acid. Mix these at the time of using with an equal measure of cold saturated recently prepared solution of gallic acid. Brush iodized paper with this solution, and mark the side; in half a minute dip it into water, and press it between blotting paper. It is then ready for the camera, where it remains from half a minute to 5 minutes. When removed from the camera dip it into water,

press it between blotting paper, and wash it with a solution of 100 gr. of bromide of potassium in 8 or 10 oz. of water.

6. *Chromotype Paper.*—Soak the paper in a solution of bichromate of potash (in which solution a little sulphate of indigo is sometimes added to vary the color), and dry it at a brisk fire. To fix the drawing careful immersion in warm water is all that is required. It is not sufficiently sensitive for the camera.

7. *Compound Chromotype Paper.*—Dissolve 10 gr. of bichromate of potash, and 20 gr. of sulphate of copper, in an ounce of water. Wash the paper in this solution, and dry it. After the paper has been exposed to the sun, with the article to be copied superposed upon it, it is washed over in the dark with a solution of nitrate of silver of moderate strength. A vivid picture makes its appearance, which is sufficiently fixed by washing in pure water. This is for copying engravings, &c. Another method is to brush writing paper over with a solution of 1 dr. of sulphate of copper in 1 oz. of water; and when dry with a strong, but not saturated, solution of bichromate of potash.

8. *Cyanotype Paper.*—Brush the paper over with a solution of ammonio-citrate of iron. Expose the paper in the usual way, then wash it over with a solution of ferro-cyanide of potassium.

9. *Crysotype Paper.*—Wash the paper with solution of ammonio-citrate of iron, dry it, and afterwards brush it over with a solution of ferro-cyanide of potassium. Dry it in a dark room. The image is brought out by brushing it over with a neutral solution of gold or silver.

10. *Catalisotype.*—Steep paper in water, with a drop or two of hydrochloric acid; absorb the superfluous moisture with blotting paper; brush over with a mixture of ½ dr. syrup of iodide of iron, 2½ dr. of water, and a drop or two of tincture of iodine. Dry with blotting paper, and brush over with a solution of 12 gr. of nitrate of silver to 1 oz. of distilled water. It is then ready for the camera. The picture is fixed by washing in water, and afterwards in a solution of 20 gr. of bromide to 1 oz. of potassium.

11. *Paper for Positive Photographs.*—Most of the preceding give negative pictures, the lights and shadows being reversed; in the following they are correct: Dissolve 40 gr. of muriate of ammonia in 4 oz. of water. Wash highly glazed paper in this solution, dry it, and brush it over with the following solution: Dissolve 120 gr. of crystallized nitrate of silver in 1½ oz. of distilled water; and add 1½ oz. of alcohol; after it has stood a few hours filter it. Expose the paper thus washed to the sunshine, till it is darkened; if mottled, wash it a second time, and expose it again. Before using the paper make up the following solution: Hydriodate of barytes 40 gr.; water 1 oz.; pure sulphate of iron 5 gr. Mix, filter, add a drop or two of diluted sulphuric acid, and when settled decant the clear liquor for use. Wash the paper over in this solution, expose

it in the damp state, with the engraving or other object on it to the light, and fix the drawing by washing with water only.

Photographs.

To copy objects, lay them on a plate of clear glass, fixed in a frame; place the prepared paper over them; and fix a back, with a cushion attached to it, so as to press the paper closely on the glass. The glass is then exposed to the light, and the drawing afterwards fixed, as described above. For feathers, lace-work, and other objects which freely admit light through them, the nitrated paper and less sensitive muriated papers may be used. For copying engravings leaves, and other botanical objects, or entomological specimens, the more sensitive muriated papers, or the bromide paper, or other sensitive kinds, may be used. Engravings should be wetted, and placed with their face to the prepared side of the paper, and kept in close contact with it. Leaves should have their under surface next the glass. For the camera, the most sensitive samples of the muriated papers, made with not less than 100 gr. of nitrate of silver to the ounce, are selected. The calotype is still more certain. The papers intended for the camera require to be very carefully prepared. Glass is used instead of paper, after being coated with white of egg, or collodion, with which the compounds of silver are mixed, or over which they are brushed.

BRONZING.

Bronzing Sculpture, Wood, &c.

Bronze of a good quality acquires, by oxidation, a fine green tint, called *patina antiqua*. Corinthian brass receives, in this way, a beautiful clear green color. This appearance is imitated by an artificial process, called bronzing. A solution of sal-ammoniac and salt of sorrel in vinegar is used for bronzing metals. Any number of layers may be applied, and the shade becomes deeper in proportion to the number applied. For bronzing sculptures of wood, plaster figures, &c., a composition of yellow ochre, Prussian blue and lamp-black, dissolved in glue-water, is employed.

Bronze.

1. Copper 83 parts; zinc 11 parts; tin 4 parts; lead 2 parts Mix.
2. Copper 14 parts; melt, and add zinc 6 parts; tin 4 parts.

Ancient Bronze.

Copper 100 parts; lead and tin each 7 parts. Mix.

To give an Antique Appearance to Bronze Figures.

Salt of sorrel 1 part; sal ammoniac 4 parts; white vinegar 224 parts. Dissolve, and apply with a camel-hair pencil, just sufficient to damp the bronze, previously warmed. Repeat the operation if required.

Keller's Bronze.

Copper 91 parts; tin 2 parts; zinc 6 parts; lead 1 part. Mix.

Bronze Powder.

Bichloride of mercury 1 part; borax and nitre each 8 parts; tutty 16 parts; verdigris 32 parts; oil to make into a paste. Melt.

Beautiful Red Bronze Powder.

Sulphate of copper 100 parts; carbonate of soda 60 parts. Apply heat until they unite into a mass, then cool, powder, and add copper filings 15 parts. Well mix, and keep them at a white heat for twenty minutes, then cool, powder, and wash and dry.

Bronzing Fluid for Guns, &c.

Nitric acid sp. gr. 1·2, nitric ether, alcohol, muriate of iron, each 1 part. Mix, then add sulphate of copper 2 parts; dissolved in water 10 parts.

ENAMELS.

White Enamel.

Tin 2 parts; lead 1 part. Calcine, then take of the above oxides 1 part; crystal 2 parts; manganese a small portion. Grind well together, fuse, and pour the mass into cold water; dry, grind again to powder, and fuse; repeat the process four or five times, observing great care to prevent any contamination from smoke, or iron, or copper.

Another.

Arsenic 14 parts; potash 25 parts; nitre 12 parts; glass 13 parts; flint 5 parts; litharge 3 parts.

Blue Enamel.

Fine paste (not metallic) 10 parts; nitre 3 parts. Oxide of cobalt to color.

Green Enamel.

Frit 1 pound; oxide of copper ½ ounce; red oxide of iron 12 grains.

Fluxes of Enamel Colors.

1. Flint powder 1 part; calcined borax 1 part; flint glass 8 parts; red lead 4 parts. Keep them in a state of fusion, in a Hessian crucible, for three hours; then pour into cold water, dry and powder.

2. Glass powder 11 parts; white arsenic 1 part; nitre 1 part Mix.

Yellow Enamel.

White oxide of antimony 1 part; white lead 2 parts; alum and sal-ammoniac each 1 part. Mix in fine powder, and apply just sufficient heat to decompose the ammoniac.

Black Enamel.

Clay 2 parts; protoxide of iron 1 part. Mix.

MARBLE STAINING.

To Stain Marble.

It is necessary to heat the marble hot, but not sufficiently so as to injure it, the proper heat being that at which the colors nearly boil.

Blue.—Alkaline indigo dye, or turnsole with alkali.

Red.—Dragon's blood in spirits of wine.

Yellow.—Gamboge in spirits of wine.

Gold Color.—Sal-ammoniac, sulphate of zinc, and verdigris, equa parts.

Green.—Sap green, in spirits, with potash.

Brown.—Tincture of logwood.

Crimson.—Alkanet root in turpentine.

The marble may be veined according to taste. To stain marbl *well* is a tedious and difficult operation.

To Stain White Marble.

Apply with a brush a strong alcohol tincture, made from the roo *alkanet.*

To Clean Marble.

Chalk (in fine powder) 1 part; pumice 1 part; common soda parts. Mix. Wash the spots with this powder, mixed with a littl water; then clean the whole of the stone, and wash off with soa and water.

To Extract Oil from Stone or Marble.

Soft soap 1 part; Fuller's earth 2 parts; potash 1 part; boilin water to mix. Lay it on the spots of grease, and let it remain for few hours.

COMPOUND COLORS IN DYEING,

Are produced by mixing together two simple ones; or, which is he same thing, by dyeing cloth first of the simple color, and then y another. These colors vary to infinity, according to the proportions of the ingredients employed. From blue, red, and yellow, ed-olives, and greenish-greys are made.

From blue, red, and brown, olives are made from the lightest to he darkest shades; and by giving a greater shade of red, the lated and lavender-greys are made.

From blue, red, and black, greys of all shades are made, such as age, pigeon, slate, and lead-greys. The king's or prince's color is luller than usual; this mixture produces a variety of hues or colors lmost to infinity.

From yellow, blue, and brown, are made the goose-dung and lives of all kinds.

From brown, blue, and black, are produced brown-olives, and heir shades.

From the red, yellow, and brown, are derived the orange, gold color, feuille-mort or faded leaf, dead carnations, cinnamon, fawn, and tobacco, by using two or three of the colors as required.

From yellow, red, and black, browns of every shade are made.

From blue and yellow, greens of all shades.

From red and blue, purples of all kinds are formed.

Dyer's Spirit.

Aquafortis 10 parts; sal-ammoniac 5 parts; tin 2 parts. Dissolve.

Japan Grounds.

Red.—Vermillion makes a fine scarlet, but its appearance in apanned work is much improved by glazing it with a thin coat of ake, or even rose pink.

Yellow.—King's yellow, turpeth mineral, and Dutch pink, all form very bright yellows, and the latter is very cheap. Seed-lac varnish ussimilates with yellow very well; and when they are required very bright, an improvement may be effected by infusing turmeric n the varnish which covers the ground.

Green.—Distilled verdigris laid on a ground of leaf gold produces the brightest of all greens; other greens may be formed by nixing King's yellow and bright Prussian blue, or turpeth mineral and Prussian blue, or Dutch pink and verdigris.

Blue.—Prussian blue, or verditer glazed with Prussian blue or smalt.

White.—White grounds are obtained with greater difficulty than any other. One of the best is prepared by grinding up flock-white, or zinc-white, with one sixth of its weight of starch, and drying it; t is then tempered, like the other colors, using the mastic varnish for common uses; and that of the best copal for the finest. Par-

ticular care should be taken that the copal for this use be made of the clearest and whitest pieces. Seed-lac may be used as the uppermost coat, where a very delicate white is not required, taking care to use such as is least colored.

Black.—Ivory-black, or lamp-black; but if the lamp-black be used it should be previously calcined in a closed crucible. Black grounds may be formed on metal, by drying linseed oil only, when mixed with a little lamp-black. The work is then exposed in a stove, to a heat which will render the oil black. The heat should be low at first, and increased very gradually, or it will blister. This kind of japan requires no polishing. It is extensively used for defending iron articles from rust.

POLISHES.

To Polish Brass Inlaid Work.

File the brass very clean with a smooth file; then take some tripoli powdered very fine, and mix it with the linseed oil. Dip in this a rubber of hat, with which polish the work until the desired effect is obtained.

If the work is ebony, or black rosewood, take some elder-coal powdered very fine, and apply it dry after you have done with the tripoli, and it will produce a superior polish.

The French mode of ornamenting with brass differs widely from ours, theirs being chiefly water-gilt (*or molu*), excepting the flutes of columns, &c., which are polished very high with rotten stone, and finished with elder-coal.

To Brass Plates of Copper.

The plates previously sufficiently heated, expose them to the fumes of zinc.

To Clean Brass.

1. Finely powdered sal-ammoniac; water to moisten.
2. Roche alum 1 part; water 16 parts. Mix. The articles to be cleaned must be made warm, then rubbed with either of the above mixtures, and finished with fine tripoli. This process will give them the brilliancy of gold.

To Brass Vessels of Copper.

Argol 1 part; amalgam of zinc 1 part; muriatic acid 2 parts; water to fill the vessel. Boil.

Method of Cleaning Brass Ornaments.

Brass ornaments that have not been gilt or lacquered may be cleaned, and a very brilliant color given to them, by washing them

with alum boiled in strong ley, in the proportion of an ounce to a pint, and afterwards rubbing them with strong tripoli.

French Polish.

Alcohol 260 parts; copal varnish 13 parts; sandarach (powdered) 1 part; mastic (powdered) 1 part; shell-lac (powdered) 24 parts. Mix, and digest in a moderate heat, in a strong close vessel.

To French Polish.

The varnish being prepared (shell-lac), the article to be polished being finished off as smooth as possible with glass paper, and your rubber being prepared as directed below, proceed to the operations as follows: The varnish, in a narrow necked bottle, is to be applied to the middle of the flat face of the rubber, by laying the rubber on the mouth of the bottle and shaking up the varnish once, as by this means the rubber will imbibe the proper quantity to varnish a considerable extent of surface. The rubber is then to be inclosed in a soft linen cloth, doubled, the rest of the cloth being gathered up at the back of the rubber, to form a handle. Moisten the face of the linen with a little raw linseed oil, applied with the finger to the middle of it. Placing your work opposite the light, pass your rubber quickly and lightly over its surface until the varnish becomes dry, or nearly so; charge your rubber as before with varnish (omitting the oil), and repeat the rubbing, until three coats are laid on, when a little oil may be applied to the rubber, and two coats more given to it. Proceeding in this way, until the varnish has acquired some thickness, wet the inside of the linen cloth, before applying the varnish, with alcohol, and rub quickly, lightly, and uniformly the whole surface. Lastly, wet the linen cloth with a little oil and alcohol without varnish, and rub as before till dry.

To make the Rubber.—Roll up a strip of thick woollen cloth which has been torn off, so as to form a soft elastic edge. It should form a coil from one to three inches in diameter, according to the size of the work.

BOOKBINDERS' RECIPES.

Japan Coloring, for Leather Book-Covers, &c.

After the book is covered and dry, color the cover with potash-water mixed with a little paste, give it two good coats of Brazil wash, and glair it. Put the book between wands, allowing the boards to slope a little. Dash on copperas water, then with a sponge full of red liquid, press out on the back and on different parts large drops, which will run down each board, and make a fine shaded red. When the cover is dry wash it over two or three times with Brazil wash, to give it a brighter color.

Blue Sprinkle for Bookbinders.

Strong sulphuric acid 8 ounces; Spanish indigo, powdered, 2 oz. Mix in a bottle that will hold a quart, and place it in a water-bath to promote solution. For use, dilute a little to the required color in a teacup.

Blue Marble for Books, &c.

Color the edges with King's yellow, and when dry tie the book between boards. Throw on blue spots in the gum trough, wave them with the iron pin, and apply the edges thereon.

Brown Color for Marbling or Sprinkling Books.

1. Logwood chips 1 part; annatto 1 part; boil in water 6 parts. If too light, add a piece of copperas about the size of a pea.

2. Umber, any quantity. Grind it on a slab with ox gall and a little lampblack. Dilute with ale.

Gold Sprinkle for Books.

Put into a marble mortar half an ounce of pure honey and one book of gold leaf, rub them well together until they are very fine, add half a pint of clear water, and mix them well together: when the water clears, pour it off, and put in more, till the honey is all extracted, and nothing remains but the gold. Mix one grain of corrosive sublimate in a teaspoonful of spirits of wine, and when dissolved, put the same, together with a little gum-water, to the gold, and bottle it close for use. The edges of the book may be sprinkled or colored very dark, with green, blue, or purple, and lastly with the gold liquid, in small or large spots, very regular, shaking the bottle before using. Burnish the edges when dry, and cover them with paper to prevent the dust falling thereon. This sprinkle will have a most beautiful appearance on extra work; ladies may use it for ornamenting their fancy work, by putting it on with a pen or camel's-hair brush, and when dry burnish it with a dog's tooth.

Marble for Leather Book-covers.

Wash the cover and glair it, take a sponge charged with water, having the book between wands, and drop the water from the sponge on the different parts of the cover; sprinkle very fine with vinegar black, then with brown, and lastly with vitriol water. Observe to sprinkle on the colors immediately after each other, and to wash the cover over with a clean sponge and water.

Chinese Edge for Books.

1. Color the edge with light liquid blue and dry; then take a sponge charged with vermillion, and dab on spots according to fancy; next throw on rice, and finish the edge with dark liquid blue.

2. Color light blue on different parts of the edge with a sponge; do the same where there are vacancies with yellow and Brazil red;

dry and dab on a little vermillion in spots; then throw on rice, and finish with a bold sprinkle of dark blue. Burnish.

Wax Marble for Leather Book-covers, &c.

This marbling must be done on the fore edge, before the back of the book is rounded, or becomes round, when in boards, and finished on the head and foot. Take beeswax and dissolve it over the fire in an earthen vessel; take quills stripped of their feathers, and tie them together; dip the quill-tops in the wax, and spot the edge, with large and small spots; take a sponge charged with blue, green, or red, and smear over the edge; when done, dash off the wax, and it will be marbled. This will be useful for stationery work, or for folios and quartos.

Egyptian Marble for Leather Book-covers.

1. *Yellow.*—Boil quercitron bark with water and a little powdered alum, over a slow fire, until it is a good strong yellow. Pour the liquid into a broad vessel, sufficiently large to contain the cover when extended. Before the liquid is cool, take the dry cover, and lay the grain side flat on the color; press it lightly that the whole may receive the liquid; let it soak some time, and then take it from the vessel. The book must be covered in the usual manner, and permitted to dry from the fire. Glair the book; when dry, place it between the wands; take a sponge and water, and press large spots thereon; dip a quill-top into the vinegar black, with it touch the water on the cover in different parts, which will have a fine effect when managed with care. Let it stand a few minutes, then take off the water with a clean sponge.

2.—*Green.*—Color the cover in a large vessel, as mentioned before, with Scott's liquid blue; when done, put it into a vessel of clear water for an hour. Take it out and press out the water, then cover the book. Glair the cover; when dry, place it between wands, and drop weak potash water from a sponge thereon; dip the quill-top into the strong black, and touch the water with it. This must be repeated till you have a good black. When dry, clear it with a sponge and water.

3. *Red*—Boil Brazil dust in rain-water on a slow fire, with a little powdered alum and a few drops of solution of tin, till a good color is produced. Dip a piece of calf leather into the liquid, and you may ascertain the color wanted. If too light, let it boil till it is reduced to one half of the quantity; take it from the fire, add a few more drops of the solution of tin, and pour it into a large vessel. Put the dry cover on the liquid, and let it remain for a quarter of an hour, then press out the water. Color it over with a sponge and the quercitron bark water, and cover the book. Glair the cover, place it between wands, dash on water with a brush, also potash water; and, lastly, finish it with the strong vinegar black, with the quill-top. Observe that too much black is not put

on; the intention of the marble is to show the red as transparently as possible.

French Marble for Books.

Provide a wooden trough, two inches deep, six inches wide, and the length of a super-royal sheet. Boil in a brass or copper pan any quantity of linseed and water, until a thick mucilage is formed; strain it into the trough and let it cool; then grind on a marble slab any of the following colors in small-beer: Prussian blue, king's yellow, rose pink, vermillion, flake white, lamp-black, brown umber, green, blue, and yellow, orange, red, and yellow, purple, red, and blue, brown, black, and yellow, or red.

The lamp-black and umber must be burnt over the fire to deprive them of their greasy nature.

For each color you must have two cups, one for the color after grinding, the other to mix it with ox-gall, which must be used to thin the colors at discretion. If too much gall is used the colors will spread; when they keep their place on the surface of the trough, when moved with a quill, they are fit for use.

To prevent the water entering between the leaves of the book, tie it tight between cutting-boards of the same size, and place the trough in a steady situation, to prevent the colors from moving.

Having all things in perfect readiness for marbling, supposing you begin with the blue, throw on with the brush bold spots of blue, sprinkle very fine with the white on the blue spots, fill up the spaces with red and yellow, by dipping first the quill-top into the yellow, and touching the gum therewith, then with the red. The red and yellow may be waved or drawn round the blue spots with an iron pin, or as the marbler may think proper, according to fancy.

Hold the book with its edge downwards, and press it lightly on the colors so disposed on the gum, and the edge will be immediately marbled. The colors that may remain on the gum must be taken off, by applying paper thereon, before you prepare for marbling again. In this manner you may marble the edges to resemble the end-papers, which will have a pleasing effect.

Chinese Marble for Leather Book-covers, &c.

Color the cover of the book dark brown, and when dry put it into the cutting-press, with the boards perfectly flat; mix whiting and water of a thick consistence and throw it on, in spots or streaks, some large and some small, which must remain till dry. Spot or sprinkle the cover with liquid blue, and lastly throw on large spots of liquid red. The colors must be dry before washing off the whiting.

Orange Sprinkle for Books.

Color the edge with King's yellow, mixed in weak gum-water, then sprinkle with vermillion mixed in the same manner.

Green Sprinkle for Books.

1. Yellow the edge, then sprinkle with dark blue.
2. French berries 1 part; soft water 8 parts. Boil, and add a little powdered alum; then bring it to the required shade of green, by adding liquid blue.

Green Marble for Leather Book-covers, &c.

The edge must be marbled with a good bright green only. When the color is prepared with the ox-gall, and ready for use, a few drops of sweet oil must be mixed therein, the color thrown on with a brush, in large spots, till the gum is perfectly covered. The oil will make a light edge round each spot, and have a good effect.

Blue, green, and brown may be also used separately in like manner.

Sheets of paper may be done, having a trough large enough, and the sheets damped as for printing, before marbling.

Spirits of turpentine may be sprinkled on the colors, which will make white spots.

Binder's Thread Marble.

Yellow the edge; when dry, cut pieces of thick thread over the edge, which will fall on different parts irregularly; give it a fine dark sprinkle, and shake off the thread.

Rice Marble, for Leather Book-covers, &c.

Color the cover with spirits of wine and turmeric, then place on rice in a regular manner; throw on a very fine sprinkle of copperas-water till the cover is nearly black, and let it remain till dry. The cover may be spotted with the red liquid or potash-water, very freely, before the rice is thrown off the boards.

Orange Color for Marbling or Sprinkling Books, &c.

Ground Brazil-wood 16 parts; annotto 4 parts; alum, sugar, and gum-arabic, each 1 part; water 70 parts. Boil, strain, and bottle.

Tree Marble, for Leather Book-covers.

A marble in the form of trees may be done by bending the boards a little on the centre, using the same method as the common marble, having the cover previously prepared. The end of a candle may be rubbed on different parts of the boards, which will form knots.

Vinegar Black for Bookbinders, &c.

Steep iron filings or rusty iron in good vinegar for two or three days, then strain off the liquor.

To Sprinkle Books.

Take a stiff brush made of hogs' bristles, perfectly clean, dip it in the color; squeeze out the superfluous liquid; then rub a

folding-stick across the brush, and a fine sprinkle will fall on the edge of the book, which should be previously screwed tight in the cutting-press. Repeat the operation until the color is thrown equally on every part of the leaves. The brush should be held in the left hand, and the stick in the right.

Purple Sprinkle for Bookbinders.

Logwood chips 4 parts; powdered alum 1 part; soft water 24 parts. Boil until reduced to sixteen parts, and bottle for use.

2. Brazil dust (fine), and mix it with potash-water for use.

Soap Marble for Books.

This is applicable for marbling stationery, book edges, or sheets of paper for ladies' fancy work.

Grind, on a marble slab, Prussian blue, with water, and a little brown soap, to a fine pliable consistence, that it may be thrown on with a small brush.

Grind King's yellow, in the same manner, with water and white soap.

When green is intended for the ground color, grind it with brown soap, and King's yellow with white soap. Lake may be used for a ground color, and Prussian blue ground with white soap; brown umber for a ground color, and flake-white ground with white soap. Any color of a light substance may be ground for marbling.

Spotted Marble for Books, &c.

After the fore-edge of the book is cut, let it remain in the press, and throw on linseeds in a regular manner; sprinkle the edge with any dark color, till the white paper is covered, then shake off the seeds. Various colors may be used. The edge may be colored with yellow or red before throwing on the seeds and sprinkling with blue. The seeds will make a fine fancy edge when placed very thick on different parts, with a few slightly thrown on the spaces between.

Brown Sprinkle for Leather Book-covers, &c.

Pearlash or potash 1 part; soft water 4 parts. Dissolve and strain.

Red Sprinkle for Binders.

Brazil-wood (ground) 4 parts; alum 1 part; vinegar 4 parts; water 4 parts. Boil until reduced to seven parts, then add a small quantity of loaf-sugar and gum. Bottle for use.

Black Sprinkle for Leather Book-covers, &c.

Green copperas 1 part; soft water, hot, 6 parts. Dissolve.

Stone Marble for Leather Book-covers, &c.

Glair the cover, and when dry put the book into the cutting-press, with the boards sloping, to cause the colors to run gently

down. Throw on weak copperas-water with a brush; dip a sponge into the strong potash-water, and press out the color from the sponge on different parts of the back, so that the colors may run down each side from the back. Where the brown has left a vacancy apply vitriol-water in the same manner. The book must remain till perfectly dry before washing it.

CRAYONS.

Lithographic Crayons.

1. Take white wax 4 parts; gum-lac 2 parts. Melt over a gentle fire, then add dry tallow soap in shavings 2 parts. Stir until dissolved. Next add white tallow 2 parts; copal varnish 1 part; lampblack 1 part. Mix well, and continue the heat and stirring until, on trial by cooling a little, it appears of a proper quality, which should be that it will bear cutting to a fine point, and trace delicate lines without breaking.

2. Take dry white tallow soap 6 parts; white wax 6 parts; lampblack 1 part. Fuse in a covered vessel.

3. Take lampblack 1 part; tallow soap 2 parts; shell-lac 2 parts; wax 4 parts. Mix, with heat, and mould.

4. Take dried tallow soap 5 parts; wax 4 parts; lampblack 1 part. Mix as before.

Crayons.

1. Shell-lac 6 parts; spirit 4 parts; turpentine 2 parts; color 12 parts; pale clay 12 parts. Mix.

2. Pipe-clay, color as required, water to mix. Form into a stiff paste, and roll it into crayons.

To Fix Crayon Colors.

Paste your paper on canvass, in a frame, in the usual way, then brush over the back two or three times with the following mixture, and when the last coat is dry give the face of the picture one or two coats in the same way. This will make it resemble an oil painting. Spirits of turpentine 10 parts; boiled oil 6 parts. Mix.

To render permanent Chalk or Pencil Drawings.

Lay the drawing on its face, and give the back two or three thin coats of the following (No. 1) mixture; let it dry, and turn it with the chalk upwards, and give that side one or two coats also; lastly, if you choose, give it one or two coats of No. 2.

1. Isinglass or gum-arabic 5 parts; water 12 parts. Mix.
2. Canada balsam 4 parts; turpentine 5 parts. Mix.

Wash to fix Blacklead Pencil Drawings.

1. Isinglass 1 part; water 50 parts. Dissolve with heat, and filter.

2. Take skimmed milk, and strain. For use, pour the liquid on a surface sufficiently large, and take the drawing by the corners, lay it flat on the wash, then carefully remove it, and place it on a slanting surface to drain and dry. This will also answer for chalk drawings.

GILDING.

To Gild or Silver Leather.

Finely powder resin, and dust it over the surface of the leather, then lay on the leaf, and apply (hot) the letters or impression you wish to transfer; lastly, dust off the loose metal with a cloth. The cloths used for this purpose become, in time, very valuable, and are often sold to the refiners for \$5 to \$7.

To gild on Calf and Sheep Skin.

Wet the leather with the white of eggs; when dry rub it with your hand and a little olive oil, then put the gold leaf, and apply the hot iron to it. Whatever the hot iron shall not have touched will go off by brushing.

To gild Copper, Brass, &c. (Patent.)

Fine gold 5 parts; nitric acid (sp. g. 1·45) 21 parts; hydrochloric acid (sp. g. 1.15) 17 parts; pure water 14 parts. Digest with heat in a glass vessel until all the gold is dissolved, and till red or yellow fumes cease to rise. Decant the clear liquid into some convenient vessel, and add water, 500 to 600 parts. Boil for two hours, let it stand to settle, and pour off the clear into a suitable vessel. For use, heat the liquid and suspend the articles (previously well cleaned) by means of a hair or fine wire, until sufficiently coated with gold, then well wash them in pure water.

To gild Glass and Porcelain.

1. Apply to the part a surface of gold size; when nearly dry lay on the leaf.

2. Gold powder 2 parts; borax 1 part; turpentine to mix. Mix and apply to the surface to be gilded with a camel-hair pencil; when quite dry, heat it in a stove until the borax vitrifies. Burnish. Platina, silver, tin, bronze, &c., may be applied in a similar manner.

To give Iron the color of Copper.

Take 1 oz. of copper-plates, cleansed in the fire; 3 oz. of aquafortis; dissolve the copper, and when it is cold use it by washing your iron with it by the help of a feather; it is presently cleansed

and smooth, and will be of a copper color; by much using or rubbing it will wear off, but may be renewed by the same process.

A way of Gilding with Gold upon Silver.

Beat a ducat thin, and dissolve in it two ounces of aqua-regia; dip clean rags in it, and let them dry; burn the rags, and, with the tinder thereof, rub the silver with a little spittle; be sure first that the silver be cleansed from grease.

Gilder's Wax.

1. Yellow wax 3 pounds; verdigris 1 pound; sulphate of zinc 1 pound; red oxide of iron 2½ pounds. Powder the last three articles very fine.

2. Yellow wax 7 pounds; colcothar 7 pounds; verdigris 3 pounds; borax ½ pound; alum ½ pound.

To dye in Gold Silver Medals, or Laminas, through and through.

Take glauber salt, dissolve it in warm water, so as to form a saturated solution. In this solution put a small proportionate quantity of calx, or magister of gold. Then put and digest in it silver laminas cut small and thin, and let them lie twenty-four hours over a gentle fire. At the end of this term you will find them thoroughly dyed gold color inside and out.

To gild Silks, Satins, &c.

Nitromuriate of gold, in solution, 1 part; distilled water 3 parts. Mix. Lay out any design with this fluid, and expose it, while wet, to a stream of hydrogen gas; then wash it with clear water.

To make Transparent Silver.

Refined silver one ounce; dissolve it in two ounces of aqua-fortis; precipitate it with a pugil (a quantity that may be taken up between the thumb and finger) of salt, then strain it through a paper, and the remainder melt in a crucible for about half an hour, and pour it out, and it will be transparent.

To make Copper into a Metal like Gold.

Distilled verdigris 4 oz.; Tutiæ Alexandrinæ præparatæ two oz.; saltpetre 1 oz.; borax ½ oz. Mix all together with oil, till they be as thick as pap; then melt it in a crucible, and pour it into a fire-shovel, first well warmed.

Mercurial Plating.

Quicksilver 4 parts; nitric acid 4 parts; finely powdered cream of tartar 2 parts; finely powdered salt of sorrel 1 part. Dissolve the silver in the acid, then add the rest, and stir until dissolved. This imparts a pleasing silvery appearance to articles formed of copper, by merely applying it with the finger.

Grecian Gilding.

Take sal-ammoniac and bichloride of mercury, equal parts,

dissolve in nitric acid, and make a solution of gold with this fluid, lay it on the silver, and expose it to a red heat; it will then be gilded.

To gild or silver Writing.

Let there be a little gum and lump-sugar in the ink you write with; when dry, breathe on it and apply the leaf.

To whiten Copper throughout.

Take thin plates of copper, as thin as a knife, heat them six or seven times, and quench them in water; then melt them, and to each pound add 4 ounces of saltpetre and 4 ounces of arsenic, well powdered and mixed, and first melted apart in another crucible, by gentle degrees; then take them out, and powder them; then take Venetian borax and white tartar, of each an ounce and a half; then melt these with the former powder, in a crucible, and pour them out into some iron receiver; it will appear as clear as crystal, and is called *crystallinum fixum arsenicum.* Of this clear matter, broken into little pieces, throw into the melted copper (by small pieces at a time, staying five or six minutes between each injection) 4 ounces; when all is thrown in, increase the fire, till all be well melted together for a quarter of an hour; then pour it out into an ingot.

To gild Steel.

Apply an etherial solution of gold. This is equally adapted to lettering, as wholly covering the object. It may be applied with a pen, or otherwise.

GLASS STAINS.

Red Stain for Glass.

1. Rust of iron 100 parts; glass of antimony 99 parts; yellow glass of lead 98 parts; sulphuret of silver 3 parts. Mix.
2. White hard enamel 100 parts; red chalk 50 parts; peroxide of copper 5 parts. Reduce to fine powder, and mix.

Blue Glass.

Plain paste 300 parts; zaffre 3 parts; manganese 1 part. If the glass should be of too deep a blue, use less zaffre and manganese; if too purple, omit the manganese altogether.

Black Stain for Glass.

1. Black scales of iron 29 parts; white crystal glass 4 parts; antimony 2 parts; manganese 1 part; vinegar to mix.
2. Glass of antimony 1 part; oxide of copper 2 parts; crystal glass 3 parts. Mix.

Orange Stain for Glass.

Precipitated silver powder, yellow ochre, red ochre, equal parts. Turpentine to mix.

Brown Stain for Glass.

White glass 2 parts; manganese 1 part. Mix.

Flesh Color for staining Glass.

Red lead 1 part; red enamel 2 parts. Mix with alcohol.

Yellow Stain for Glass.

Chloride of silver 1 part; burnt pipeclay 3 parts. Reduce to fine powder, and mix. This stain must be applied to the back of the glass.

To Marble a Glass Globe.

Grind well on a stone, minium for red, turmeric, or rather cerussa citrina, for yellow, smalt for blue, verdigris for green, ceruse, or chalk for white. Work each in oil separate, and with a hog's hair pencil, single or mixed, as you think fit, scatter the same into the glass, and roll it, or dispose the colors, as you like. Then, last of all, fling a little mead amongst them, which covers all.

For the Magic Lantern, paint the glasses with transparent colors, tempered with oil of spike.

FACTITIOUS STONES.

Factitious Amethyst.

1. Take strass 5000 parts; oxide of manganese 37 parts; oxide of cobalt 25 parts; purple of Cassius 1 part. Fuse for twenty-six hours, and cool slowly.

2. Take paste or strass 10,000 parts; oxide of manganese 25 parts; oxide of cobalt 1 part.

Factitious Emerald.

1. Oxide of chrome 1 part; green oxide of copper 20 parts; strass 2300 parts. Fuse with care for twenty-six hours, then cool slowly.

2. Strass 10,000 parts; acetate of copper 150 parts; protoxide of iron 3 parts. As before.

3. Strass 6600 parts; carbonate of copper 60 parts; glass of antimony 6 parts. Fuse with care.

4. Strass 500 parts; glass of antimony 20 parts; oxide of cobalt 3 parts. As before.

Artificial Coral.

Yellow resin 4 parts; vermillion 1 part. Melt. This gives a very pretty effect to glass, twigs, cinders, stones, &c., dipped into

it. It is also useful for a cement for ladies' fancy work, such a grottoes, &c.

Paste resembling the Red Cornelian.

Plain paste 1000 parts; glass of antimony 500 parts; calcine vitriol 63 parts or less; manganese 4 parts. Melt together.

Paste resembling the White Cornelian.

Plain paste 1000 parts; yellow ochre 8 parts; calcined bones 3 parts. As before.

Factitious Opal.

1. Strass 500 parts; horn silver 10 parts; calcined magnetic or 2 parts; chalk marl 25 parts. Mix in fine powder, and fuse wit great care.
2. Plain paste 100 parts; calcined bones 6 parts.

Factitious Oriental Ruby.

Strass 7000 parts; precipitate of Cassius and nitric peroxide o iron each 165 parts; golden sulphuret of antimony 160 parts manganese calcined with nitre 150 parts; rock crystal 1000 parts. Mix in fine powder, and carefully melt.

Factitious Sapphire.

1. Oxide of cobalt 1 part; strass 80 parts.
2. Paste or strass 2300 parts; oxide of cobalt 34 parts. Fuse carefully for thirty hours.
3. Plain paste 100 parts; smalts 12 parts; manganese 1 part. As before.
4. Plain paste 10 pounds; zaffre 3 drachms; precipitate of gold and tin 1 drachm. As before.

Factitious Topaz.

1. Strass 1000 parts; glass of antimony 42 parts; purple of Cassius 1 part. Fuse for twenty-four hours, and cool slowly.
2. Strass 4000 parts; saffron of Mars 40 parts. As before.

To solder together Rubies.

Apply them to a strong flame by means of the blow-pipe, and when sufficiently soft unite them with care; they will neither lose their color nor weight.

Factitious Ruby.

Strass 40 parts; oxide of manganese 1 part. Mix, and treat as for topaz.

White Crystal, or Factitious Diamond.

Manganese 1 part; rock crystal 2800 parts; borax 1900 parts; white lead 5700 parts. Mix in fine powder, then fuse in a clean crucible, pour it into water, dry, powder, and repeat the process two or three times.

Composition for Fixed Brilliants.

Meal gunpowder 16 parts; zinc, or steel, or cast-iron borings 6 parts. Mix.

Paste resembling Vinegar Garnet.

Plain paste 1000 parts; glass of antimony 500 parts; calcined iron 16 parts. Add the antimony last.

Gold or Yellow Paste.

Take plain paste (made without the saltpetre) 100 parts; oxide of iron 1 part. Fuse.

Factitious Lapiz Lazuli.

Plain paste 1000 parts; calcined bones 73 parts; zaffre 7 parts; magnesia 5 parts. If it is desired to vein it with gold—gold powder and borax, equal parts; vein the cakes to taste, and then heat them sufficiently hot for cementation.

Foils for Crystals, Pastes,

Put two or three layers of tin-foil into the socket made for the stone, heat it gently, and fill it with quicksilver, let it rest two or three minutes, then pour it out, and place in the stone.

Factitious Yellow Diamond.

Strass 500 parts; glass of antimony 10 parts. Fuse.

Another.

Strass 500 parts; chloride of silver 25 parts. Mix, and fuse.

Strass, or Mayence Base.

1. Pure rock crystal, or flint, 8 parts; salt of tartar 25 parts. Powder, mix well, bake, and cool, then put it into a basin of water, and add dilute nitric acid until effervescence ceases; collect, wash, and dry the powder; next add fine white-lead 12 parts. Levigate and well wash it with pure water, then of the above mixture dried 12 parts; calcined borax 1 part. Triturate them together, melt in a clean crucible, and pour the mixture into cold water; dry, powder, and melt it in the same manner, a third time, always in a fresh crucible, observing to separate any lead that may be revived. To the third frit, ground to powder, add purified nitre $\frac{1}{8}$ part. Remelt, and a mass of crystal will be found in the crucible of a beautiful and diamond-like lustre.

2. Arsenic 1 part; borax 23 parts; pure pearlash 180 parts; minium 525 parts; rock crystal 338 parts. Mix, as before.

3. Arsenic 1 part; borax 30 parts; potash 105 parts; carbonate of lead 709 parts; fine white sand 315 parts. Mix with care.

4. Arsenic 1 part; borax 35 parts; potass 325 parts; minium 900 parts; rock crystal 580 parts. Treat as before.

5. Rock crystal 400 parts; pure white lead 945 parts; pure potash 140 parts; borax 41 parts.

6. Pure potash 2 parts; fine white sand 15 parts; litharge 20 parts. See also *Paste*.

INKS.

Indestructible Ink.

1. Powdered copal 25 parts; oil of lavender 200 parts; lamp-black 2 parts; indigo 1 part. Dissolve.

2. Asphaltum 1 part; lamp-black ¼ part. Melt, then add oil prepared for printers' ink, by boiling and burning until sufficiently stringy, 1½ part. Mix together, and add spirits of turpentine 3 or 4 parts. We would propose this ink, made with less turpentine, so as to be sufficiently thick for stamping, as the most perfect preventive of fraud, as when applied to the surface of an engraving, or letter-press, nothing will remove it that will not also discharge the ink of the stamp. It will stand the action of the alkalies, chlorine, acids, &c., even in a heated state, when they will at once destroy the texture of the paper.

Lithographic Ink.

1. Take Venice turpentine 1 part; lamp-black 2 parts; tallow 6 parts; hard tallow soap 6 parts; mastic in tears 8 parts; shell-lac 12 parts; wax 16 parts. Melt, and pour it out on a slab.

2. Take dry tallow soap 5 parts; mastic in tears 5 parts; Scotch soda 5 parts; shell-lac 25 parts; lamp-black 2 parts. Fuse the soap and lac, then add the remainder.

For use, this ink must be rubbed down with water in a saucer (warmed), until an emulsion is formed of a proper consistence to flow easily from a pen or pencil.

Blue Writing Fluid.

1. Ferrocyanide of iron, powdered, and strong hydrochloric acid, each 2 parts. Dissolve, and dilute with soft water.

2. *Indestructible.*—Shell-lac 4 parts; borax 2 parts; soft water 36 parts; boil in a close vessel till dissolved; then filter, and take of gum-arabic 2 parts; soft water 4 parts. Dissolve, and mix the two solutions together, and boil for five minutes as before, occasionally stirring to promote their union; when cold, add a sufficient quantity of finely powdered indigo and lamp-black to color; lastly, let it stand for two or three hours, until the coarser powder has subsided, and bottle for use. Use this fluid with a clean pen, and keep it in glass or earthen inkstands, as many substances will decompose it while in the liquid state. When dry, it will resist the action of water, oil, turpentine, alcohol, diluted sulphuric acid, diluted hydrochloric acid, oxalic acid, chlorine, and the caustic alkalies and alkaline earths.

Red Ink for Writing.

Boil over a slow fire 4 ounces of Brazil wood, in small raspings or chips, in a quart of water, till a third part of the water is evaporated. Add during the boiling 2 drachms of alum in powder. When the ink is cold steam it through a fine cloth. Vinegar or stale urine is often used instead of water. In case of using water adding a very small quantity of sal-ammoniac would improve this ink.

Fine Black Writing Ink.

Take 2 gallons of a strong decoction of logwood, well strained, and then add 1½ pounds blue galls in coarse powder; 6 ounces sulphate of iron; 1 ounce acetate of copper; 6 ounces of well ground sugar; and 12 ounces of gum-arabic. Set the above on the fire until it begins to boil, then set it away until it has acquired the desired black.

Black Ink improved.

To 1 pint of common black ink add 1 drachm of impure carbonate of potassa, and in a few minutes it will be a jet black. Be careful that the ink does not run over, during the effervescence caused by the potassa.

Green Ink.

1. Cream of tartar 1 part; verdigris 2 parts; water 8 parts. Boil until reduced to a proper color.
2. Crystallized acetate of copper 1 ounce; soft water 1 pint. Mix.

Marking Ink.

Lunar caustic 2 parts; sap green and gum-arabic each 1 part; distilled water. Dissolve.

The Preparation.—Soda 1 ounce; water 1 pint; sap green ½ drachm. Dissolve, and wet the linen (where you intend to write) with this mordant, then well dry it.

Printing Ink.

1. (Very fine.)—Balsam of capaivi 9 parts; fine lamp-black 4 parts; indigo 1 part; dry yellow soap 3 parts. Grind perfectly smooth.
2. (Extemporaneous.)—Balsam of capaivi, lamp-black to color. Grind well together with a little soap.
3. Take linseed oil; heat it in a proper vessel until it begins to boil, then remove it from the fire, and kindle the vapor; allow it to burn till it becomes stringy when tried between the fingers, then add gradually to every quart black resin 1 pound. Dissolve, and add very cautiously dry brown soap in shavings, 4½ ounces to every quart. Set it upon the fire, and stir the mixture until the combination is complete; next, put into a suitable pot, finely ground indigo 1 ounce; fine Prussian blue 1 ounce; fine lamp-black 18

ounces. For every pound of resin employed pour the liquid on the color, well mix, and lastly, subject it to the action of a mill.

Indelible Ink for Marking Linen.

1. The juice of sloes 1 pint; gum ½ ounce. This requires no mordant, and is very durable.

2. Nitrate of silver 1 part; water 6 parts; gum 1 part. Dissolve. If too thick dilute with warm soft water.

Autographic Ink for Lithographers.

White soap 25 parts; white wax 25 parts; mutton suet 6 parts; lamp-black 6 parts; shell-lac 10 parts; mastic 10 parts. Mix with heat, and proceed as for lithographic ink.

To restore Writing effaced with Chlorine.

1. Expose it to the vapor of sulphuret of ammonia, or dip it into a solution of the sulphuret.

2. Ferrocyanide of potass 5 parts; water 85 parts. Dissolve, and immerse the paper in the fluid, then slightly acidulate the solution with sulphuric acid.

To give an appearance of Age to Writing.

Infuse a drachm of saffron in half a pint of ink, then write with it.

Perpetual Ink for Tombstones, Marble, &c.

Pitch 11 parts; lamp-black 1 part; turpentine sufficient. Mix, with heat.

Blue Ink.

Take sulphate of indigo, dilute it with water till it produces the color required. It is with sulphate very largely diluted, that the faint blue lines of ledgers and other account books are ruled. If the ink were used strong, it would be necessary to add chalk to it to neutralize the acid. The sulphate of indigo may be had of the woollen dyers.

Copying Ink.

Add 1 ounce of moist sugar to every pint of common ink.

Red Permanent Ink.

Vermillion 4 parts; sulphate of iron 1 part; drying oil to mix. Any other color will answer besides red. This ink will resist most of the usual reagents.

Black Permanent Ink.

Nitrate of silver 2 parts; distilled water 28 parts; sap green 1 part. Dissolve.

For the Mordant.—Common soda 2 parts; gum-arabic 1 part; soft water 8 parts. Mix, and moisten the linen with this fluid, and well dry before using the ink.

Yellow Ink.

1. French berries 1 pound; alum 2 ounces; water 1 gallon. Boil and strain, then add gum-arabic 4 ounces.

2. Water 30 parts; Avignon berries 7 parts; gum and alum each 5 parts. Boil for one hour, and strain.

Blue Ink for Ruling.

Take 4 ounces of vitriol, best quality, to 1 ounce of indigo; pulverize the indigo very fine; put the indigo on the vitriol, let them stand exposed to the air for six days, or until dissolved; then fill the pot with chalk, add half a gill of fresh gall, boiling it before use.

Black Ink for Ruling.

Take good black ink, and add gall as for blue; do not cork it, as it will prevent it from turning black.

Red Ink for Ruling.

One pound of Brazil wood to one gallon of the best vinegar; let the vinegar simmer before you add the wood, then let them simmer together for half an hour, then add three quarters of a pound of alum to set the color; strain it through a woollen or cotton cloth, cork it tight in a stone or glass bottle. For ruling, add half a gill of fresh gall to 1 quart of red ink, then cork it up in a bottle for use.

Indian Ink.

1. Take finest lamp-black, and make it into a thick paste with thin isinglass; size, then mould it; attach the gold leaf, and scent with a little essence of musk.

2. Take lamp-black, make it into a thick paste with gum-water, and mould it.

Carbon Ink.

Dissolve real India ink in common black ink; or add a small quantity of lamp-black, previously heated to redness, and ground perfectly smooth with a small portion of the ink.

Gold and Silver Ink.

Fine bronze powder, or gold or silver leaf, ground with a little sulphate of potash, and washed from the salt, is mixed with water and a sufficient quantity of gum.

Gluten Ink.

Dissolve wheat gluten, free from starch, in weak acetic acid of the strength of common vinegar; mix 10 gr. of lamp-black and 2 gr. of indigo with 4 oz. of the solution, and a drop or two of oil of cloves.

Ink for writing on Zinc Labels—Horticultural Ink.

1. Dissolve 100 gr. of chloride of platina in a pint of water. A little mucilage and lamp-black may be added.

2. Sal-ammoniac 1 dr., verdigris 1 dr., lamp-black ½ dr., water 10 dr. Mix.

Chrome Ink.

Extract of logwood ½ oz.; gum ¼ oz.; water a pint. Dissolve also in 12 oz. of water ½ oz. of yellow chromate of potash (or ¼ oz. each of bichromate and bicarbonate of potash). Mix the two solutions. The ink is ready for immediate use.

Ink for writing on Steel, Tin Plate, or Sheet Zinc.

Mix 1 ounce of powdered sulphate of copper and ½ ounce of powdered sal-ammoniac, with 2 ounces of diluted acetic acid; adding lamp-black or vermillion.

WAXES.

Black Sealing-wax.

1. Shell-lac 2 parts; yellow resin 3 parts; ivory black 2 parts. Powder fine, and mix by melting carefully.

2. Yellow resin 15 pounds; lard 1 pound; beeswax 1 pound; lamp-black 3 pounds. Mix with heat.

Soft Sealing-wax.

Yellow resin 1 part; beeswax 4 parts; lard 1 part; Venice turpentine 1 part; color to fancy. Mix with a gentle heat.

Gold Colored Sealing-wax.

1. Bleached shell-lac 1 pound; Venice turpentine 4 ounces. Melt, and add gold colored talc as required.

2. Bleached shell-lac 3 pounds; turpentine 1 pound; Dutch leaf ground fine, 1 pound or less. Mix with a gentle heat. The leaf should be ground or powdered sufficiently fine without being reduced to dust.

Green Sealing-wax.

Shell-lac 2 parts; yellow resin 1 part; verdigris 1 part. Powder and mix by heating slowly.

Scented Sealing-wax.

1. Balsam of Peru 2 parts; sealing-wax composition 130 parts. Mix, with a gentle heat

2. Sealing-wax composition 99 parts; essence of musk 3 parts. Add the latter when the wax is cooling, and stir well.

3. Wax composition 96 parts; oil of lavender 4 parts; oil of lemon 3 parts. As before.

Blue Sealing-wax.

Shell-lac 2 parts; smalts 1 part; yellow resin 2 parts. Powder, and mix carefully with heat.

Red Sealing-wax.

1. Shell-lac 2 parts; resin 1 part; vermillion 1 part. Powder fine, and melt over a slow fire.
2. Yellow resin 14 parts; Venetian turpentine 4 parts; beeswax 1 part; red or orange lead 5 parts. Mix, with heat.
3. Oil of turpentine 1 part; lard 1 part; vermillion 2 parts; gum-lac 12 parts. Mix, with a gentle heat.
4. (Very fine.)—Shell-lac 4 parts; Venice turpentine 1 part; vermillion 3 parts. Mix.

Engravers' Border Wax.

Beeswax 1 part; pitch 2 parts; tallow 1. Mix.

Black Bottle Wax.

Common resin 20 pounds; tallow 5 pounds; lamp-black 4 pounds. Mix, with heat.

Red Bottle Wax.

Common resin 15 pounds; tallow 4 pounds; red lead 5 pounds. Mix, with heat. Any color may be employed.

Marbled Sealing-wax.

Take wax of different colors and melt them in separate vessels, and when they begin to cool a little stir them all together, and form the mass into sticks.

THE
ENGINEER'S FIELD BOOK:

CONTAINING FORMULÆ

FOR THE VARIOUS METHODS OF RUNNING AND CHANGING LINES, LOCATING SIDE TRACKS AND SWITCHES, ETC.

AND

TABLES

OF RADII AND THEIR LOGARITHMS, NATURAL AND LOGARITHMIC VERSED SINES, AND EXTERNAL SECANTS, &c.

TOGETHER WITH A TABLE OF

NATURAL SINES AND TANGENTS, ETC.,

TO EVERY DEGREE AND MINUTE OF THE QUADRANT.

AND LOGARITHMS OF NUMBERS FROM 1 TO 10,000.

BY

CHARLES HASLETT,

Civil Engineer.

RECOMMENDATIONS.

Office of the O. & M. R. R. Co. Cincinnati, May, 1855.

Having examined Mr. Haslett's Field Book for Railroad Engineers, and made use of the rules he has laid down, I am satisfied of its superiority to any similar work yet published, in comprehensiveness and the ready application of the rules. The introduction of versed sines and external secants into the calculations very much reduces the time and labor required by the usual method of calculations for locating lines.

J. B. CUMMINGS,
Engineer Eastern Div. Ohio and Mississippi R. R.

I most fully concur in recommending Mr. Haslett's work to the attention of Engineers, believing it better than anything of the kind yet published.

N. A. GURNEY,
Chief Engineer, Indiana South Western R. R.

C. A. HASLETT, Esq.—Dear Sir:—I have examined with considerable care the work you propose to publish for the use of engineers in the field, and I have no hesitancy in saying that it will be the most useful of any work of its character yet offered to the public.

Yours very truly,
A. L. OSGOOD,
Division Engineer, Ohio and Mississippi R. R.

I concur with Mr. Cummings in the opinion that Mr. Haslett's mode of locating lines very much reduces the time and labor required by the usual method.

S. S. POST,
Chief Engineer, Ohio and Mississippi R. R.

From statements received from engineers of the Ohio and Mississippi Railroad who have used Mr. Haslett's method, I have every reason to believe it to be an improvement in simplicity and accuracy over the old methods in use.

O. M. MITCHELL,
Con. Engineer, Ohio and Mississippi R. R.

I have examined the mathematical tables recently prepared by Mr. Chas. Haslett, for the purpose of facilitating the calculations of Railway Engineers in adjusting curves, &c. I think they are very useful and well adapted to the wants of the profession generally. From my personal acquaintance with the author, and knowing well his mathematical attainments, I have the fullest confidence in the accuracy of the tables above referred to.

S. A. RICHARDSON,
Division Engineer, Virginia Central R. R.

PREFACE.

In presenting this work to the public, the Author claims for it the adaptation of a new principle in trigonometrical analysis of the formulas generally used in field calculations. Experience has shown, that versed sines and external secants as frequently enter into calculations on curves as sines and tangents; and by their use, as illustrated in the examples given in this work, it is believed that many of the rules in general use are much simplified, and many calculations concerning curves and running lines made less intricate, and results obtained with more accuracy, and far less trouble, than by any methods laid down in works of this kind.

The examples given have all been suggested by actual practice, and will explain themselves. It has not been thought necessary to enter into all the details of demonstration, as this book is intended expressly for use in the field; and engineers seldom have time to enter into tedious geometrical demonstrations, when direct application of rules is required.

As a book for practical use in field work, it is confidently believed that this is more direct in the application of rules and facility of calculation than any work now in use.

In addition to the tables generally found in books of this kind, the author has prepared, with great labor, a Table of Natural and Logarithmic Versed Sines and External Secants, calculated to 60 degrees, for every minute; also, a Table of Radii and their Logarithms, from 1° to 15°. Rules and examples are also given for running curves without the use of an instrument; also for locating turnouts, side tracks, switches, &c.

Having been for several years engaged in surveys and locations of railroads, and practically convinced of the great saving of time

and trouble gained by using the rules and principles given in this book, the Author submits it, without further preface, to the profession, fully confident that its use will be practical proof of its merits.

The tables and examples have been prepared with great care, and their accuracy may be relied upon.

While the Author claims a fair share of originality in the following work, he would acknowledge many valuable suggestions derived from Mifflin's Diagrams, as also from Henck on Compound and Reversed Curves, authors to whom he would refer those wishing to follow the subject at greater length. On the manner of working an instrument Mifflin is very clear and concise. *This work is designed especially for practical field engineers, already familiar with minor details.*

C. H.

Cincinnati, 1855.

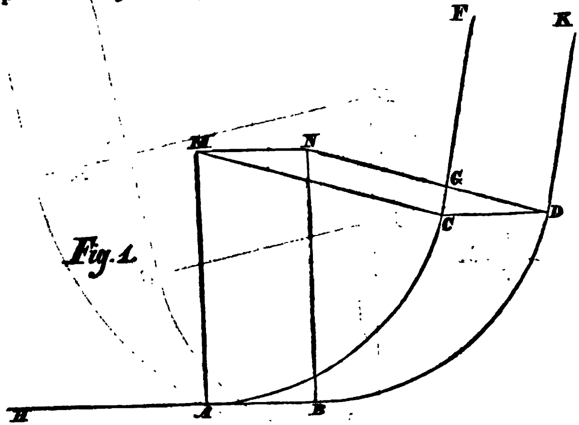
THE

ENGINEER'S FIELD BOOK.

FORMULÆ FOR RUNNING LINES, LOCATING SIDE-TRACKS, &c.

PROPOSITION I. Fig. 1.*

To change the origin of a curve so that it shall terminate in a tangent parallel to a given tangent.



Suppose the curve A C to have been described containing 60° of curvature, and that the distance G D equal 50 feet.

We have by logarithms:

Sine 60° (total amount of curvature), . .	9·937531
Is to R.	10·000000
So is G D, 50 feet, ...	1·698970
To A B = 57·73 feet, ...	1·761439

$$\text{Or by nat. sines} = \frac{\text{G D}}{\text{sin. } 60^\circ} = \frac{50}{\cdot 86603} = 57\cdot 73.$$

Produce the tangent from A to B = 57·73 feet; then make th

* The diagrams in this work are not drawn to any exact scale, but are designed to represent merely the *abstract* geometrical relation of lines.

curve B D equal A C; that is A M C = B N D; then the tangents will be parallel.

This rule will apply to the origin of a compound curve, using the total amount of curvature run.

PROPOSITION II. Fig. 2.

Having a curve A B terminating in a tangent D F, it is required to find the radius of a curve that will give a tangent C G parallel to D F at any given distance therefrom, as at D C say 30 feet.

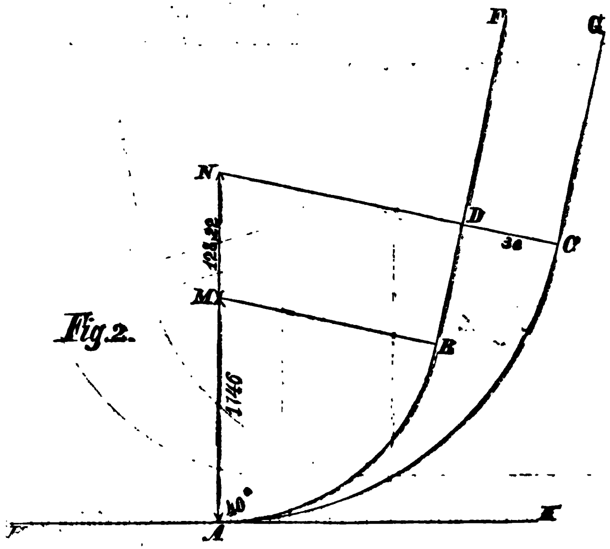


Fig. 2.

Let A M be the given radius = 1146 feet, the arc A B = 800 feet, containing 40°, and D C perpendicular distance 30 feet.

By logarithms:

As versed sine 40°	9·369133
Is to R.	10·000000
So is D C = 30 feet	1·477121
To M N = difference of radii given and required = 128·22	2·107988

Then we have 1146 + 128 = 1272 = radius of a 4° 30′ curve. Then say: 1146 : 1272 :: 800 : 888 = arc A C.

This case is equally applicable to changing the last radius used in a compound curve terminating in a parallel tangent.

PROPOSITION III. FIG. 2.

In case the preceding method should consume too much of the tangent C G, it is required to change the origin of the curve, also the length of radius, so that the required tangent may commence opposite to B, running parallel to B H.

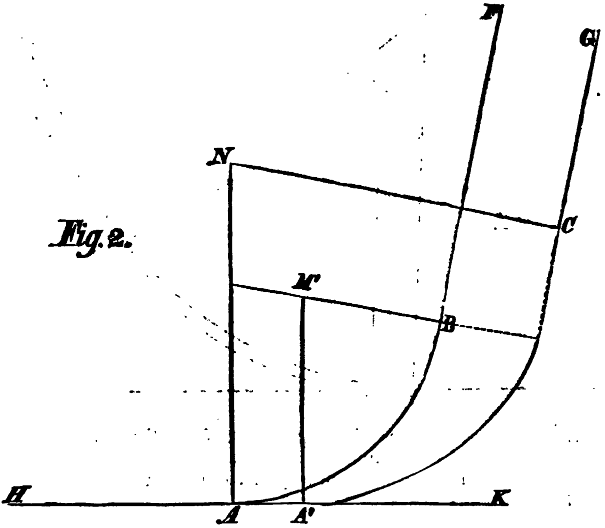


Fig. 2.

In this case the radiating point will be changed from N towards A and B, the radius shortened, and the point A moved towards K.

Let the required distance between tangents, the given radius, and curvature be as in Proposition II., then we have by logarithms:

As the external secant of 40° . .	9·484879
Is to radius	10·000000
So is 30 feet = ...	1·477121
To difference of radii = 98·23 . .	1·992242

By natural external secants $= \dfrac{30}{\cdot 305407} = 98 \cdot$ —

And 1146 — 98 = 1048 = radius of a 5° 28′ curve.

Then, as 1146 : 1048 :: 800 : 732 = length of 5° 28′ curve.

98 (natural tangent of 40° = ·83910) = 82 feet.

Produce tangent 82 feet from A to K, and curve from thence 732 feet of a 5° 28′ curve. M' A' will be the new radius.

PROPOSITION IV. FIG. 3.

Having located a curve with a given radius, terminating in a given point, it is required to change the origin of the curve, also the radius, so as to pass through the same terminating point, with a different direction of tangent.

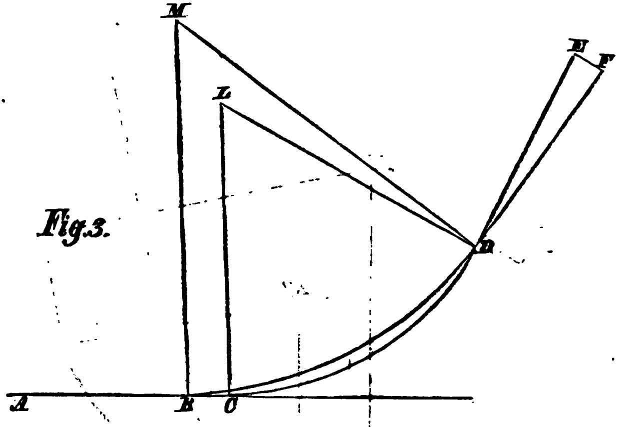


Fig. 3.

Let the given radius M B equal 2292 feet; the given arc B D equal 1000 feet, containing 25° of curvature; the given tangents D F and D E make an angle of (say) 4°, D F being 400 feet, and E F = 28 feet.

We have $\frac{28}{4 \times 1{\cdot}75} = 4° =$ angle E D F, consequently the angle C L D = 25° + 4° = 29°.

By logarithms:

As versed sine 29°	9·098229
Is to versed sine 25° . . .	8·971703
So is radius given B M = 2292 .	3·360217
To radius required C L = 1714 feet.	3·233991

By tables 1714 feet = radius of 3° 20½′ curve.

PROPOSITION V. FIG. 4.

Having produced the two tangents to their intersection, it is required to connect them by a curve passing a given distance from the vertical point.

Given the angle L C B = 31° 44′, and C E = 50 feet, to find the

radius M A. By geometry, the angle A M E = ½ L C B = 15° 52′.

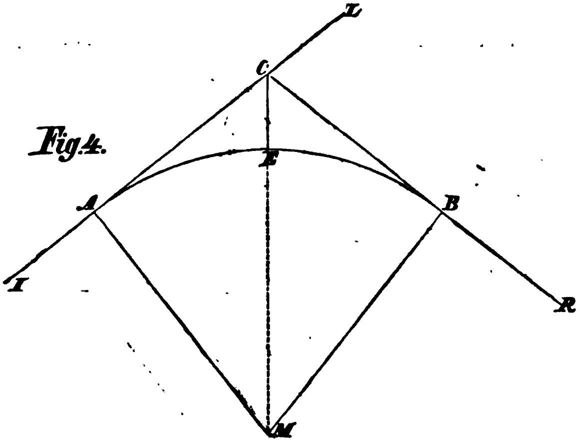


Fig. 4.

By logarithms we have:

As external secant 15° 52′ = ½ L C B	8·597789
Is to 50	1·698970
So is R.	10·000000
To M A = 1262 = R. of a 4° 32½′ curve	3·101181

By natural external secants $\frac{50}{\text{ex. sec. } 15° \ 52'} = \frac{50}{\cdot 039603} = 1262$ ft.

CASE 2D.

To find the tangent A C, or C B; or point of curve.
By logarithms:

As R.	10 000000
Is to A M = 1262	3·101181
So is tangent 15° 52′ . . .	9·453668
To A C = 388·8	2·554849

By natural tangents:
1262 × (natural tangent 15° 52′ = ·26546) = 388 feet
= C A = C B.

PROPOSITION VI. Fig. 5.

Having located a curve connecting two tangents, it is required to move the middle of the curve any given distance, either towards or from the vertex.

Let the angle T L G = 36° = whole amount of curvature; the

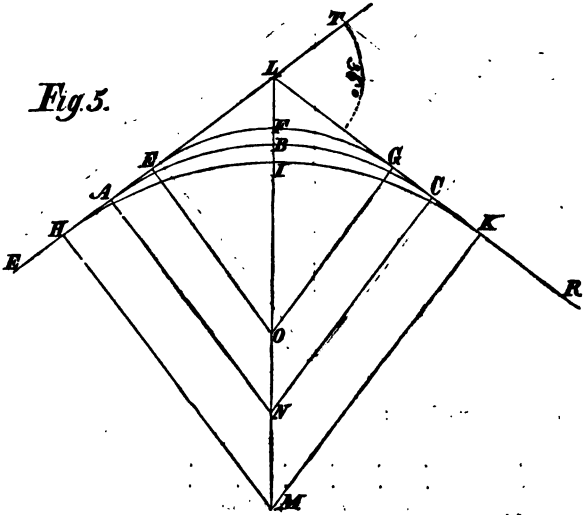


Fig. 5.

arc A B C = 1200 feet; the radius A N = C N = 1910 feet, and I B = B F = 10 feet.

It is required to find the radii H M and E O.

We have by logarithms:

External secant 18° = half of 36° = A N L	8·737153
Is to 10	1·000000
So is R.	10·000000
To difference of radii = 183 feet . . .	2·262847

By natural external secants: $\frac{10}{\cdot 054595} = 183$ ft.

1910 + 183 = 2093 = M H = radius of a 2° 44′ curve;
and 1910 — 183 = 1727 = O E = radius of a 3° 19′ curve.

By natural tangents:
183 × (natural tangent 18° = ·32492) = 59·4 = H A = A E.

PROPOSITION VII. Fig. 6.

It is required to locate a tangent from an inaccessible point on curve.

Let A B C be the given curve with a R. of 1637 feet curving 3° 3(per 100 feet; C the inaccessible point. Assume a point B, if cor venient, at a given distance, say 300 feet, from C. Throw off tangent, and measure, at right angles therefrom, B E = externa

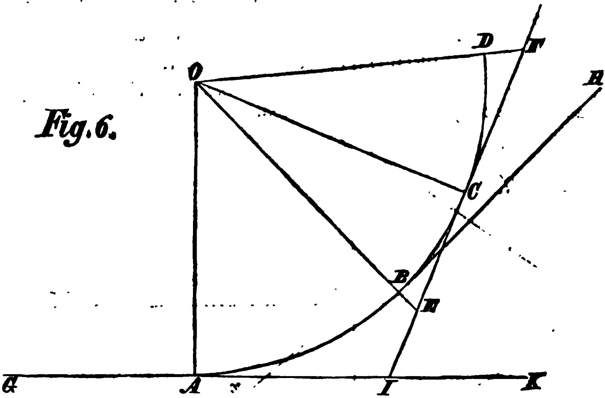


Fig. 6.

secant of arc B C; then to find by logarithms the distance B E, we have:

As radius	10·000000
Is to O C = 1637	3·214122
So is external secant 10° 30′ = angle C O B .	8·231221
To B E = 27·88	1·445343

By natural external secants:

$$1637 \times (\text{nat. ex. sec. } 10^\circ\ 30' = \cdot 017030) = 27\cdot 88.$$

Measure the line B E = 27·88 feet at right angles to B H. Set the instrument over E, and turn off the angle B E C = 79° 30′ = complement of 10° 30′. E C F will be the direction of the tangent required.

CASE 2D.

Suppose there be no convenient accessible point between A and C, produce the curve to D, measure the external secant D F as before, place the instrument at F, and turn off the angle D F C. This will give the direction of the tangent F C as before.

CASE 3D.

Should the lines A I and I C be more practicable for operating

than the curve A B C, calculate and produce the tangent from A to I, the vertex of the curve A B C, and turn off the angle K I F = A O C, and make I C = A I, as calculated.

CASE 4TH.

Again, should the last method be found impracticable, and the chord A D clear from obstructions, measure the chord A D, and turn off tangent from D.

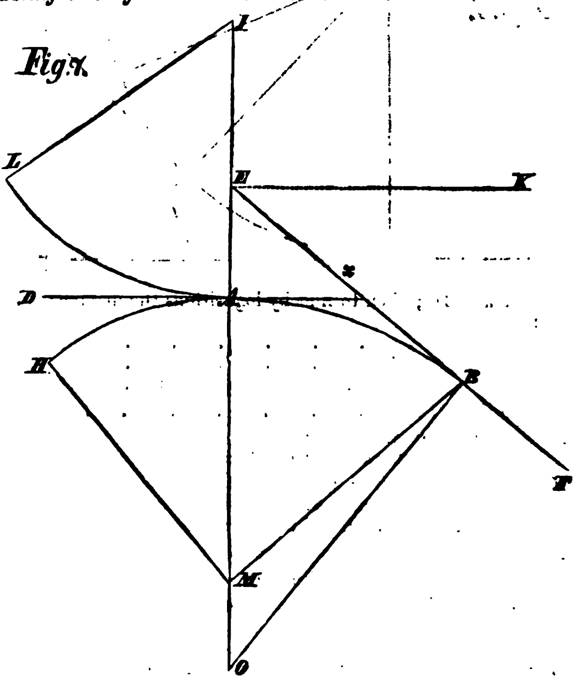
Suppose angle K A D = 25°, then we have 1637 × (nat. sine 25° = ·42262) 2 = A D = 1384 feet.

Note.—The arc A B C D contains 50° curvature.

PROPOSITION VIII. FIG. 7.

It is required to find a curve which will connect two lines without producing the tangents to an intersection.

Fig. 7.



The principle involved in this diagram affords an easy mode of solving a very interesting geographical problem. Suppose A E is a mountain near the sea or a very extensive level. Measure with an instrument for taking vertical angles the depression or "dip" of the horizon K E B = B O A; then external secant K E B × radius of earth = A E = height of mountain.

Let the line be either a curve L A, H A, or a tangent D A, as the case may be. Suppose it impracticable, by reason of buildings or other obstructions, to produce the tangent to a vertex x.

At A lay off with the instrument a right angle to tangent, and produce it till it meets F B produced in E. Measure this distance, and the angle A E B; then its complement A O B will be the amount of curvature required to curve on to the tangent B F.

Suppose the angle A E B = 65°, then A O B = 25°. Let A E be = 120 feet, then we have by logarithms:

As external secant 25° . . .	9·014427
Is to 120 feet	2·079181
So is R.	10·000000
To O B = 1160·8 = a 4° 56½′ curve . .	3·064754

And 1160·8 × (tangent 25° = ·46631) = E B = 541·28 feet.

Then will be 25° of curvature ÷ 4° 56½′ = the rate of curvature, give the length of curve between the two given points A and B = 506·2 feet.

PROPOSITION IX. FIG. 8.

To draw a tangent to two curves already located.

Let the curve C R A G H, of 2000 feet radius, be located from tangent C K; and let E S B D be a curve of 2605 feet radius, located from tangent E F. We are required to find the points A and B having a tangent common to both.

Suppose R to be the point in the first curve, and S the point in the second. There being obstructions in the way, we will run the zigzag line R L P S, making R L tangent to R, and P S tangent to S.

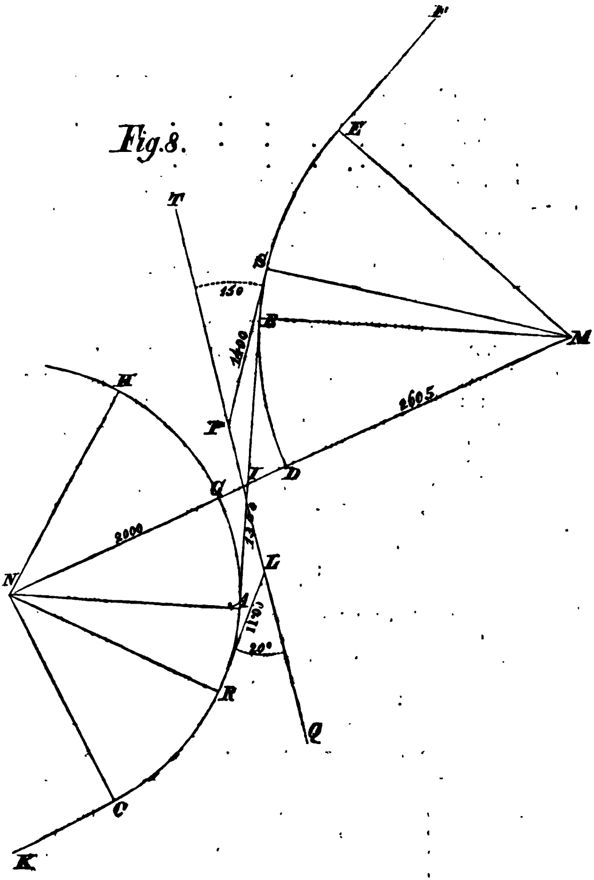
Suppose R L Q = 20°
and T P S = 15°;
let R L = 1100 feet, L P = 1300, and P S = 1400.

Assume radius N R as a meridian; that is, suppose N R to be due north. Then will R L be due west, L P south 70° west, P S south 85° west, and radius S M north 5° west. These artificial courses, then, will show the *relative* bearings, with which we obtain the following traverse:

Course.	Distance.	Northing.	Southing.	Easting.	Westing.
North . .	2000	2000	0000	0000	0000
West . .	1100	0000	0000	0000	1100
S. 70° W.	1300	0000	444·63	0000	1221·60
S. 85° W.	1400	0000	122·02	0000	1394·66
N. 5° W.	2605	2595·07	0000	0000	227·05
		4595·07	566·65	0000	3943·31

Difference northing and southing $(4595{\cdot}07 - 566{\cdot}65) = 4028{\cdot}42$; then $\frac{3943{\cdot}31}{4028{\cdot}42} = {\cdot}97882$ = natural tangent R N G = 44° 23′ = course

Fig. 8.



of N M = N. 44° 23′ west, and angle S M D = 39·23, or 44° 23′ — 5°.

To calculate M N make the difference of latitude 4028·42 = cosine 44° 23′, and the required distance N M = radius. Then we have by natural cosines $\frac{4028 \cdot 42}{\text{cosine } 44^\circ\ 23'} = \frac{4028 \cdot 42}{\cdot 71468} = 5636 \cdot 7 =$ M N.

Or by logarithms:

As cosine R N M = 44° 23′ . . .	9·854109
Is to R.	10·000000
So is difference of latitude 4028·42 .	3·605134
To N M = 5636·7	3·751025

The triangles A N I and B M I being similar, we have by logarithms (Davies' Legendre, book II., prop. X)—that is, by "composition and division:"

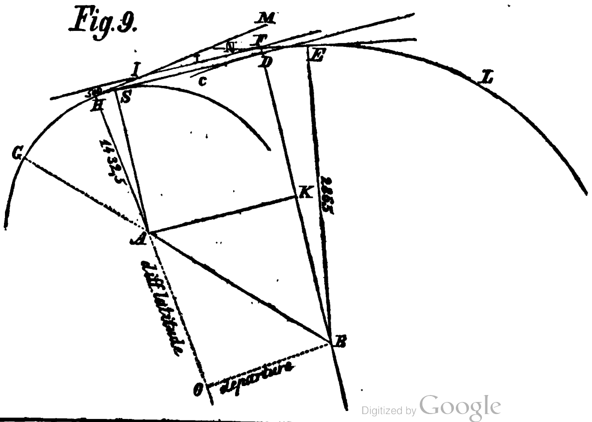
As N M = 5636·7	3·751025
Is to R.	10·000000
So is sum of radii 4605 = (2000 + 2605) .	3·663230
To cosine A N I = B M I = 35° 13′ . .	9·912205

Having now determined the angle R N I = 44° 23′, and the angle A N I = 35° 13′, the angle R N A becomes = to their difference = 9° 10′.

Therefore continue the curve from R towards A, 9° 10′ of curvature, and we have the tangent point A required. Again, we have S M I = 39° 23′, and the angle B M I = 35° 13′, consequently curve from S to B 4° 10′ of curvature, and we have the tangent point B required.

Now to find the length of tangent A B, multiply the sum of the radii 4605 by the natural tangent of 35° 13′, and we have the length required.

CASE 2D. FIG. 9.



Suppose the two curves to be connected by a common tangent, nstead of running in opposite directions as in Case 1st, curve the same way, as G H S and C D E L. It is required to find the position of the tangent S D.

Assume the points H and E; from H lay off tangent H I; from E lay off tangent E F; join F and I by a straight line, if convenient, or by a traverse, if there be obstructions. Let A H be an artificial meridian, and, as in Case 1st, calculate the distance A B, also its course = angle H A G; this will give also the angle E B A.

Suppose radius A H = 1432·5, tangent H I = 500 feet, angle M I F = 6°, I F = 1000 feet, N F T = 8°, E F = 600 feet, and radius E B = 2865 feet. We will then have the following traverse, by which to find the course and distance of A B:

Course.	Distance.	Northing.	Southing.	Easting.	Westing.
North . .	1432·5	1432·50			
East . . .	500			500	
S. 84° E. .	1000		104·50	984·60	
S. 76° E. .	600		145·20	582·20	
S. 14° W.	2865		2780·07		692·72
	Total	1432·50	3029·77	2066·80	692·72

Difference of latitude = 1597·27; departure = 1374·08.

$\frac{\text{Departure}}{\text{diff. lat.}} = \frac{1374{\cdot}08}{1597{\cdot}27} = {\cdot}86026 =$ natural tangent $40° 42' =$ course A B = angle H A G.

$\frac{\text{Diff. lat.}}{\text{cosine course}} = \frac{1597{\cdot}27}{\text{cosine } 40° 42'} = \frac{1597{\cdot}27}{{\cdot}75813} = 2106{\cdot}86 =$ distance A B.

Then $\frac{\text{diff. radii}}{\text{A B}} = \frac{1432{\cdot}50}{2106{\cdot}86} = {\cdot}67992 =$ natural cosine $47° 10' =$ D B A = S A G.

Now $47° 10' - 40° 42' = 6° 28' =$ H A S. Then curve from H $6° 28' = 162$ feet nearly to S. Now A B makes with B E an angle of $40° 42' + 8° + 6° = 54° 42'$ Hence we must curve from E to D $54° 42' - 47° 10' = 7° 32'$ curvature $= 377$ feet distance. The points S and D will be the termini of the required tangent.

Then difference of radii × natural tangent (D B E = 47° 10') = 1432·5 × 1·07864 = 1545 15 = A K = S D = length of tangent. Now when the two curves are so situated as to be seen the one from the other, assume two points as near as you can judge to the true termini of common tangent. Cause about a dozen small

straight stakes or pins to be set up endway about twenty feet apart from one of the assumed points or curves. Then set the instrument at the other, and see how tangent from instrument strikes the row of stakes. Note the difference, and move the instrument until tangent therefrom strikes as tangent to the row of stakes. Make a point where it does. Set the instrument over said point, and in like manner see how tangent from instrument strikes the other curve. Thus we dispense with all the previous calculation.

PROPOSITION X. FIG. 10.

Having located two curves connected by a tangent, as in Case 2d, Prop. IX., it is required to throw out the tangent, and introduce instead a curve with given radius.

Let the radius A S = 1432·5 feet, B D = 1637 feet, and their common tangent S D = 220 feet. It is required to find on the two

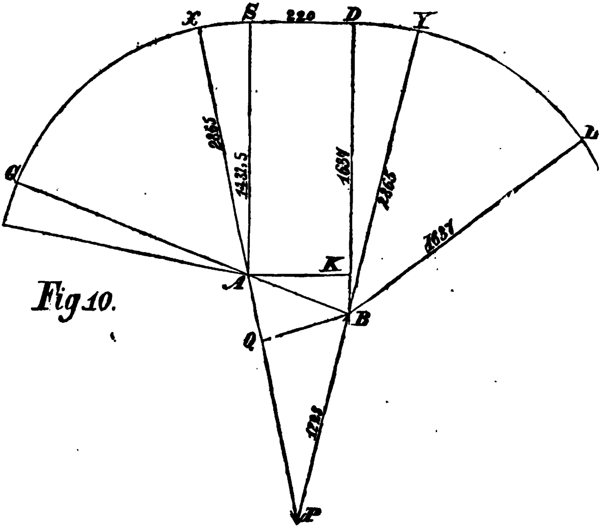


Fig. 10.

curves two tangent points, X and Y, from which, if the required radius (say 2865 feet) be drawn, it will pass through the points A and B, intersecting in the centre P, equi-distant from X and Y.

Now in the triangle B A K we have given, difference of radii

K = 1637 — 1432·5 = 204·5; also, A K = S D = 220, to find the gle K A B, its complement K B A = S A G,* and the distance A B.

$$\text{en}\ \frac{BK}{AK} = \frac{204{\cdot}5}{220} = {\cdot}92954 = \text{natural tangent of } 42^\circ\ 54\tfrac{1}{2}' = KAB.$$

erefore its complement K B A = S A G = 47° 5½′. Now B K × ant K B A = 204·5 × 1·468801 = 300·37 = A B; call it 300 t. Again, in the triangle B A P we have A B = 300, A P = 55 — 1432·5 = 1432·5, B P = 2865 — 1637 = 1228. To find the gles A B P, B P A, and B A P, make A P = 1432·5 feet the base, l let Q be the foot of the perpendicular from B. Then by trigonetry we have:

A P : B P + B A :: B P — B A : P Q — Q A, or

432·5 : 1228 + 300 :: 1228 — 300 : 989·8 = P Q — Q A. Then

$$\frac{32{\cdot}5 + 989{\cdot}8}{2} = PQ = 1211{\cdot}15, \text{ and } \frac{1432{\cdot}5 - 989{\cdot}8}{2} = QA = 221{\cdot}35.$$

$$\text{Then } \frac{AQ}{AB} = \frac{221{\cdot}35}{300} = {\cdot}73783 = \text{nat. cos. of } BAP = 42^\circ\ 27'.$$

$$\text{Again, } \frac{PQ}{PA} = \frac{1211{\cdot}15}{1228} = {\cdot}98628 = \text{nat. cos. of } BPA = 9^\circ\ 30'.$$

Now Y B P being a straight line, the angle Y B A = 42° 27′ + 30′ = 51° 57′ and X A P being a straight line, the angle X A G = A P = 42° 27′. Now the angle S A G being 47° 5½′ the angle X will equal 47° 5½′ — 42° 27′ = 4° 38½′, and Y B D = 51° 57′ 47° 5½′ = 4° 51½′.

We therefore move back from S 4° 38½′ of curvature, or 116 feet X; also from D 4° 51½′ of curvature, or 139 feet to Y; we then ve the points X and Y, which are to be connected with a 2° rve of 2865 feet radius.

PROPOSITION XI. Fig. 11.

wing located a compound curve terminating in a given tangent, it is required to change the p. c. c., also the length of the last radius, so as to pass through the same terminating point with a given difference in the direction of the tangent.

Let the given curve H A be a 2° of 2865 feet radius compounded A B, a 2° 30′ curve 2292 feet radius, 800 feet in length, andconning 20° of curvature; it is required to move the p. c. c. forward m A towards B, curving therefrom with a shorter radius than 92 feet, passing through the fixed point B on to tangent with 30′ additional curvature.

The following method, though not perfectly accurate, will be

* Because the three angles in the triangle K A B = 180°. Also the sum of the gles on one side the line B G = 180°. Subtracting from 180° the angle A and the *ht* angle at K, we have left the angle at B. Subtracting from 180° the angle as before) and the *right* angle S A K, we have the angle S A G; hence the angle B A = the angle S A G.

found sufficiently so for most practical purposes. Had the 2° curve H A been continued 800 feet farther, to a point C, the variation B C would be equal 28 feet.* Now by compounding to a 2° 30′ curve I turn off with the instrument for the chord A B 2°

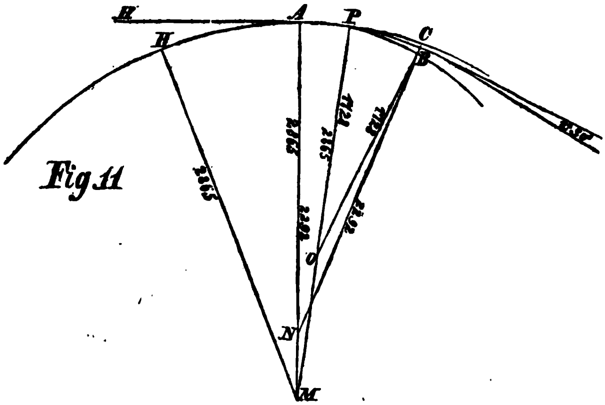


Fig. 11

more than I would for the chord A C; for $\frac{20° - 16°}{2} = 2°$; but if the instrument set at the required point P, with a backsight on A, and a foresight on B, I turn off $\frac{20° + 2° \, 30'}{2} = 11° \, 15'$, that is 3° 15′ instrumental deflection over and above that required for a continuous 2° curve to C; the curve P B will therefore be shorter than A B in the ratio of 3° 15′ to 2°; hence the proportion:

$$3\tfrac{1}{4} : 2 :: 800 : 492 = \text{length of curve P B.}$$

A P then will equal 800 — 492 = 308 feet of 2° curve; but 308 feet of a 2° curve gives 6° 10′ of curvature; hence P B contains 22° 30′ — 6° 10′ = 16° 20′ of curvature in 492 feet distance; then we have $\frac{16{\cdot}333\ldots}{4{\cdot}92} = 3{\cdot}3198° = 3° \, 19'$, or 1728 feet radius for the curve P B. It will be sufficiently accurate, however, to continue the 2° curve 310 feet to P, and then run 490 feet of a 3° 20′ curve.

Were H A a tangent by making A P the same length and rate of curvature as above, the curve P B would be the same also.

* 2° × 1.75 × 8 = 28.

PROPOSITION XII.

Having located a compound curve terminating in a tangent, it is required to change the point of compound curvature so that the curve will terminate in a tangent parallel to a given tangent at any required distance perpendicular thereto.

RULE. Divide the required distance between parallel tangents by the difference of radii of the two last branches of curve. From the cosine of total amount of curvature in last branch subtract this quotient; the remainder will be the natural cosine of amount of curvature required for last radius.

Given a curve 600 feet long, 2865 feet radius, compounded with a curve of 1910 feet radius 400 feet long, then tangent; required to fix point of compounding, to give parallel tangent 30 feet outside or inside of tangent given:

$$\frac{30}{955} = \cdot 03141$$

400 feet curvature = 12°   cosine 12° = ·97815
less ·03141

cosine of curvature required 18° 47′ — ·94674

18° 47′ — 12° = 6° 47′ curvature to be used in moving p. c. c. back to throw a tangent 30 feet inside.

If we move tangent inside 6° 47′ ÷ 2° = 339 feet. Length of 2° curve = 600 — 339 = 261. Length of 3° curve = 400 + 226 = 626 ft.

The entire length of curve by alteration becomes 261 + 626 = 887 instead of 1000 feet as before, admitting of more tangent at the end.

This last rule is applicable when the movement of the p. c. c. is retrograde or from the terminating tangent, thereby increasing the amount of curvature in last curve, and diminishing that of the preceding curve.

When it is required to move the point c. c. forward, diminishing the amount of curvature in last curve, *add* the quotient of the required distance divided by difference of radii, to the cosine of given amount of curvature; and the sum will be the cosine of the amount of curvature required in the last curve. Find the distance as before, and move the point forward the difference of curvature, *always* reckoning said difference according to the rate of curvature *back* of p. c. c.

PROPOSITION XIII. FIG. 12.

Having located a curve between two tangent points, it is proposed to lengthen the radii at the two termini, and shorten the radius in the middle.

Let the proposed curve be one of 1146 feet radius = 5°, 800 feet in length, and containing 40° of curvature. It is proposed to introduce at each end 100 feet of a 2° 30′ curve = 2292 feet radius. Required the other radius.

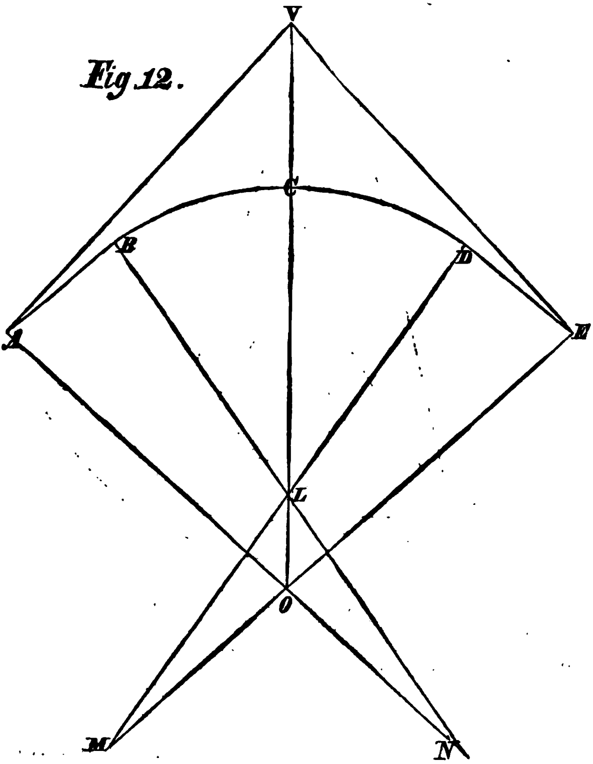
From the t. p. to the centre is 400 feet, or 20° of curvature.

Introducing 100 feet of a 2° 30′ 2292 feet R., there will be 2° 30′ of 2292 feet radius + 17° 30′ of a shorter radius.

By logarithms:

As sine 17° 30′	9·478142
Is to 2° 30′	8·639680
So is diff. radii = 1146 feet = O M .	3·059185
To difference between given radius and required = 167 = O L . .	2·220723

Fig. 12.



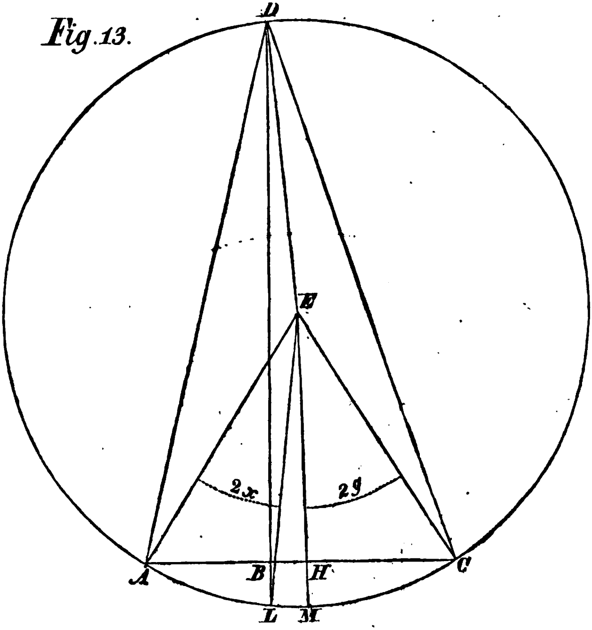
Given radius being 1146, radius required will be 1146 — 167 = 979 = L C = a 5° 51′ curve.

PROPOSITION XIV. LEMMA. FIG. 13.

To divide a given angle into two parts, so that the tangents of the angles will be in a given ratio.

Let the required ratio be as three to five, and the given angle A D C = 30°; let the straight line A B C be = 8. Make A C a chord of 60°, or twice 30°. Describe the circle A C D passing

Fig. 13.



through A and C. At B, a distance of three from A, erect the perpendicular B D, produce it to L, then A D B and B D C will be the angles required. For B D is common to two right angled triangles, and hence the tangents of the vertical angles are as A B to B C.

To calculate the required vertical angles let A D B = x, B D C = y, then A E L = $2x$, and L E C = $2y$ = central angle. Then A E C = $2(x + y) = 60°$.

Erect a perpendicular from E upon H, then will A E H = H E C = $x + y$. Then L E M (H being produced to M) equals $x + y - 2x = y - x$; then E L = E C = R_1, and L M = B H. Then

$$H C : (L M = B H) :: \text{sine } (x + y) : \text{sine } (y - x).$$

But H C is half of A C, and B H is half B C — A B, therefore (B C + A B) : (B C — A B) :: sine $(x + y)$: sine $(y - x)$, that is, as the sum of the numbers expressing the ratio is to difference, so is sine of the given angle to sine of the difference required.

By logarithms;

As 3 + 5 = 8	0·903090
Is to 5 — 3 = 2	0·301030
So is sine 30°	9·698970
To sine $y - x = 7° 10' 38''$ . . .	9·096910

$$y + x = 30°, \; y - x = 7° 10' 38'',$$

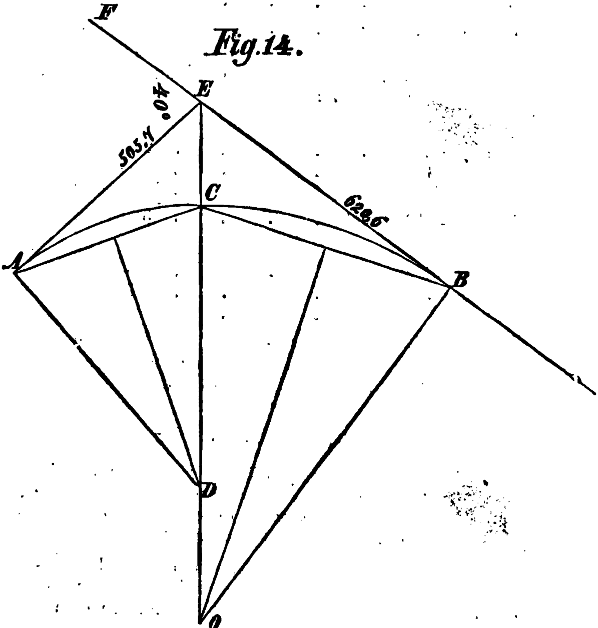
therefore $2y = 37° 10' 38''$, $y = 18° 35' 19''$, $x = 11° 24' 41''$.

PROPOSITION XV. FIG. 14.

From two fixed points, having produced tangents uniting in a vertex at unequal distances from them, it is required to locate a compound curve.

Suppose the tangents produced to E, and let A E = 505·7 feet,

Fig. 14.



E B = 620·6 feet, the angle F E A = 40°. Required the radii of a c. c. to join A and B, and also the point of compound curvature.

We observe the external secant E C is common to both curves. Now by construction of the tables we have: external secant a = tangent a × tangent $\frac{1}{2}a$, radius being unity. The angles E B C and E A C are measured by half their arcs C B and C A.

Call these angles x and y respectively. Then $x + y = \frac{40°}{2} =$ 20°; then 620·6 × tangent y = 505·7 × tangent x, or 620·6 : 505·7 = tangent y : tangent x. Then by previous proposition

$$620{\cdot}6 + 505{\cdot}7 : 620{\cdot}6 - 505{\cdot}7 :: \text{sine } (x + y = 20°) : \text{sine } (x - y)$$
$$\text{or, } 1126{\cdot}3 : 114{\cdot}9 :: \text{sine } 20° : y - x.$$

Neither of the radii being given, we will assume the condition, that the p. c. C shall be in line with the vertex E and the centres O and D. We have by logarithms:

As 1126·3	3·051654
Is to 114·9	2·060320
So is sine 20°	9·534052
To sine = 2°	8·542718

Now $x + y = 20$. Then $x = 9° + 2° = 11°$; consequently C O B = 18°, and A D C = 22°. Now we have the length of tangent and curvature given, to find the radius.

By logarithms:

As tangent 18°	9·511776
Is to 620·6	2·792812
So is R.	10·000000
To O B = 1910 feet	3·281036

To find A D:

As tangent 22°	9·606410
Is to 505·7	2·703895
So is R.	10·000000
To A D = 1251·6 = 4° 34′ . . .	3·097485

1910 (external secant 18° = ·051462) = 1251·6 × (external secant 22° = ·078535) = C E = 98·2 feet.

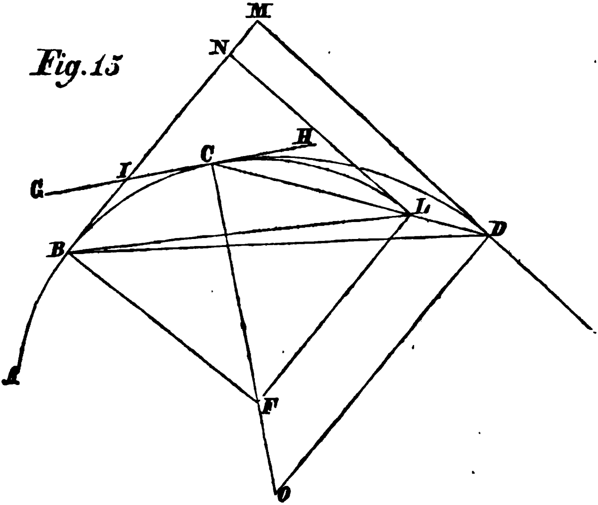
PROPOSITION XVI. FIG. 15.

Let B be a point in a curve whose radius B F is given, and let D be another fixed tangent point. It is required to find point of c. c., the curve A B being produced, from which to start a curve to terminate in tangent at D, also the radius of last curve.

Given the angles M D B, M B D, distance B D, and radius B F. Imagine the simple curve B C L to be run with a given radius B F till L N becomes parallel to D M. Now by the nature of a curve,

upon whatever point on the curve the transit be placed, the difference between backsight on B and foresight on I, is always the

Fig. 15



same, namely, $\frac{B + D}{2}$. Now at the true point of c. curvature C, the difference between backsight on B and foresight on D is also equal to $\frac{B + D}{2}$, therefore the transit reading the same on D as on L, C L D must be in the same straight line.

Hence whenever the nature of the ground will admit of it, erect a flagstaff at D, curve round from B towards L until taking a backsight the foresight necessary to fall upon L should strike the flagstaff at D. The transit will then be at the point of c. curvature sought.

Then measure C D, and make this proportion: sine H C L*: $\frac{1}{2}$ C D :: R : x = O D.

Suppose H C L = 8°, and the distance C D = 600 feet. Then by substituting in the above proportion, we have by logarithms:

As sine 8° = H C L	9·143555
Is to $\frac{1}{2}$ C D = 300	2·477121
So is R.	10·000000
To x = O D = 2855·6	3·333566

* Because H C L = $\frac{1}{2}$ C O D.

When the ground will not admit of this method, ascertain by measurement or calculation the distance from B to D.

$2\,(B\,F) \times \frac{B + D}{2} = B\,L$. Now angle $L\,B\,D = \frac{B - D}{2}$.* The triangle will then have the two sides B D and B L, and the included angle B, to find the angle $L\,D\,B = C\,D\,B$.†

Now in the triangle B C D we have the angle B C D = to the supplement of $\frac{B + D}{2}$, also the angle at D, consequently the angle at B.

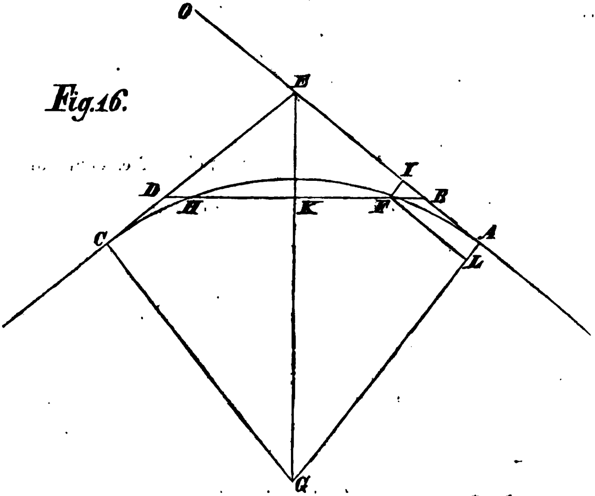
These angles, together with base B D, determine the chord C D; from which, with the angle H C L, calculate R as before. H C L becomes known from the fact, that C B D gives C B M = G C B, C B M being B — C B D. This taken from $\frac{B + D}{2}$ will give the angle H C L required.

PROPOSITION XVII. FIG. 16.

Between two tangents to locate a curve passing through a given point.

Suppose A B and C D to be the tangents permanently fixed with

Fig. 16.



* Because N B L (isosceles) = ½ exterior angles at N and M = $\frac{B + D}{2}$, B being = N B D, and $B - \frac{B + D}{2} = \frac{B - D}{2} = L\,B\,D$.

† $L\,D\,B = \left(C\,L\,B = \frac{B + D}{4}\right) - L\,B\,D$ which will make all the angles known.

reference to some agreement between individuals; and let F be the given point at which it is necessary to keep a given distance from some building or other object. Suppose A B and C D produced to meet in E. The angle O E D, and consequently its half E B D, are known. The distance I E is also known.

Let the angle O E D = 60°, let I F = 17·5 feet. It is required to find the point B, so that the angle F B I shall = 30°.

By natural sines:

$$\frac{17 \cdot 5}{\text{sine } 30°} = 35 = \text{F B} = \text{H D}.$$

Now $\sqrt{(35 + 17 \cdot 5) \times (35 - 17 \cdot 5)}$* $= \sqrt{52 \cdot 5 \times 17 \cdot 5} = 30 \cdot 3 =$ I B.

Suppose I E measures 462 feet. Then B E will equal 462 + 30·3 = 492·3.

By similar triangles, F B : B E :: B I : B K, or

35 : 492·3 :: 30·3 : 426·2 = B K = D K.

Then B D = 852·4 and B H = 852·4 — 35 = 817·4.

Now we have by geometry $\sqrt{\text{B H} \times \text{B F}} =$ B A, or $\sqrt{817 \cdot 3 \times 35}$ = 169·1 = B A.

Hence A B + B E = A E, or 169·1 + 492·3 = 661·4.

To find radius:

$$\frac{\text{A E}}{\text{tangent } 30°} = \frac{661 \cdot 4}{0 \cdot 57755} = 1145 \cdot 5 = \text{R}.$$

Now suppose it is inexpedient to produce the tangents to a vertex, the angle O E D being known, find the point B as before, and turn off E B D = ½ O E D, measure B D, and calculate by trigonometry the side E D = B E, and also B A as before.

Again, suppose the angle at E is not known, neither is it practicable to measure a direct line between the two tangents, calculate by traverse the true course and distance between any two convenient points on the tangents by Proposition IX., from which calculate the position of E.

Without ascertaining the distance to E, the radius A G can be calculated thus:

$\frac{\text{A F}^2}{2 \text{ I F}} =$ A G, or let A F = 200, then $\frac{200^2}{17 \cdot 5 \times 2} = \frac{40000}{35} = 1146 =$ AG.

Therefore commence at A, and run 800 feet of a 5° curve to C.

PROPOSITION XVIII. FIG. 17.

Given the length of a common tangent D G = a, and the angles of intersection n and m, to determine the common radius C E = C F = radius of a reversed curve to unite the tangents H D and B L.

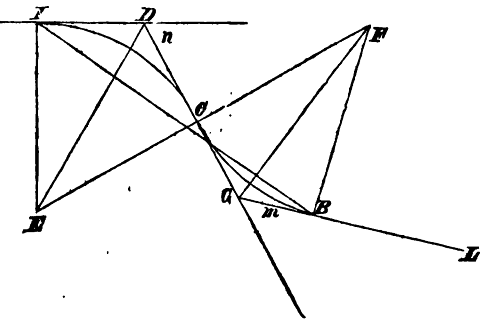
Now D C = R × tangent ½ *n*, and

C G = R × tangent ½ *m*; we have therefore

D G tangent = 800 ft., *n* = 16° and *m* = 12°.

* The sum of two quantities multiplied by their difference is equal to the difference of their squares.

Fig. 17.



$$R = \frac{a}{\tan.\frac{1}{2}n + \tan.\frac{1}{2}m} = \frac{800}{\tan.8^\circ + \tan.6^\circ} = \frac{800}{\cdot 14054 + \cdot 10510} = 3256{\cdot}7$$

$$3256{\cdot}7 \times \cdot 14054 = DC = DA = 457{\cdot}69$$
$$3256{\cdot}7 \times \cdot 10510 = CG = GB = \underline{342{\cdot}27}$$
$$799{\cdot}96$$

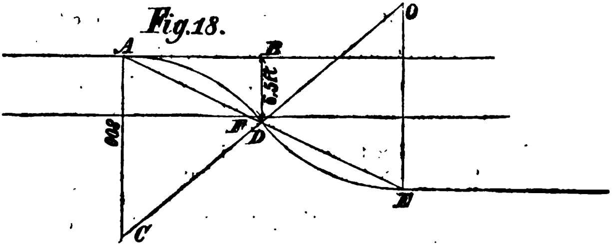
Suppose it to be required to introduce 200 feet of tangent between the curves, that portion of the tangent D G taken by the two curves will be 600 feet. Then we have:

$$800 : 600 :: 3256{\cdot}7 : 2442{\cdot}5 \text{ radius}:$$
$$\left.\begin{array}{l} 800 : 600 :: 457{\cdot}69 : 343{\cdot}27 \\ 800 : 600 :: 342{\cdot}27 : \underline{256{\cdot}70} \end{array}\right\} = \text{new tangent.}$$
$$599{\cdot}97 \quad = \text{“} \quad \text{“}$$

ON REVERSED CURVES, TURNOUTS, ETC. FIG. 18.

A F = 98 feet, A D = 102 feet, and D E = 102 feet.
Let G = gauge of track, and R = radius of turnout, x = distance

Fig. 18.



on chord from A, the origin of curve, to F, the point of frog; then will

$$x = \sqrt{2 R . G}.$$

Now suppose R = 800 feet, and G = 6 feet, then will

$$x = \sqrt{2 \times 800 \times 6} = \sqrt{9600} = 98 \text{ feet nearly.}$$

Or let x = distance on main track to a point opposite of the frog. Then will

$$x = \sqrt{G(2R - G)} \text{ or } \sqrt{6(2 \times 800) - 6)} = \sqrt{6 \times 1594} = \sqrt{9564} = 97{\cdot}79 \text{ feet.}$$

Hence the following rule is sufficiently correct for all practical purposes:

Multiply twice the radius by the gauge of track, extract the square root of the product, and we have the distance from origin of curve to point of frog.

Formula for angle of frog: G ÷ R = versed sine of curvature to frog = angle of frog. Ex. $\frac{6}{800} = {\cdot}0075 = 7° \ 2'$.

Make the movable end of the switch rail such a distance from the origin of the curve, that the departure of a curve of that radius for that distance will be equal to the opening of that rail at the movable end, say $5\frac{1}{2}$ inches.

With an 800 feet radius, the distance from origin of curve to opening of switch rail will be = 27 feet, for $\frac{27 \times 27}{1600} = \frac{11}{24} = 5\frac{1}{2}$ inches nearly.

It will appear therefore that the opening of a 20 feet rail, with an 800 feet radius curve commencing at the other end, will be only 3 inches, for $\frac{20 \times 20}{1600} = 3$ inches.

If we consider the movable rail as a movable tangent, and the origin of the curve as the opening of the rail, the angle of frog and length of curve will be obtained by Proposition XII.

EXAMPLE.

A 20 feet rail, with $5\frac{1}{2}$ inches opening, makes an angle with the main track = 1° 18′, then on 6 feet gauge the distance from opening to other side = 5 feet $6\frac{1}{2}$ inches = 5·54 feet. Then by Proposition XII. we have:

$$\text{cosine } 1° \ 8' = {\cdot}99974$$

$$\frac{554}{800} = {\cdot}00692$$

$${\cdot}99282 = \text{cosine } 6° \ 52' = \text{angle of frog.}$$

And $6°\,52' - 1°\,18' = 5°\,34'$ = amount of curvature between opening of rail and point of frog.

By the first method, when the distance between tracks = 13 feet we have $\sqrt{13 \times 800} = 102$ feet nearly for distance from origin of curve to point of reversion.

But if the point of reversion be made at the point of frog, the distance between nearest rails of tracks being 7 feet, we have 6 : 7 :: 800 : 933·3 = radius of curve with which to leave frog, and 6 : 7 :: 98 : 114·3 = distance from frog to end of turnout.

Or making the movable rail tangent, and its opening $5\frac{1}{2}$ inches, angle of opening being 1° 18′, the point of reversion being made at frog, to find the angle of frog, we have:

$$\text{cosine } 1°\,18' = \cdot 99974$$

$$\frac{6\cdot 54}{933\cdot 3} = \cdot 00700$$

$$\overline{\cdot 99274} = \text{cosine } 6°\,55' \text{ nearly the same as before.}$$

TURNOUTS ON CURVES. FIG. 19.

Suppose the turnout is on a curve running in the same direction, say a 2°, with a radius of 2865 feet. Now an 800 feet radius gives a 7° 10′ curve, and 7° 10′ − 2° = 5° 10′ = relative departure from main track. But the radius of a 5° 10′ = 1109 feet; then

$\sqrt{2 \times 1109 \times 6} = x = 115{\cdot}3$ — distance from origin of curve to point of frog.

Therefore to make a turnout from a 2° curve and running the same way would require 115 feet.

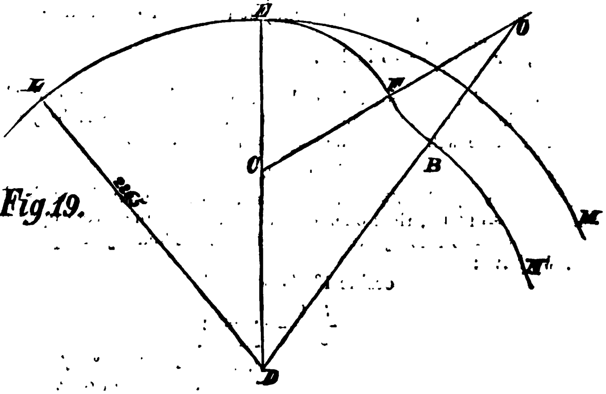


Fig. 19.

If it were required to keep the distance the same as on a straight line, it would be necessary to make the 7° 10′ curve a 9° 10′ curve of 625 feet radius.

If the 2° curve run in the opposite direction of the turnout, and the radius was 800 feet, then the convergence will be 7° 10′ + 2° = 9° 10′ curve, and the radius of a 9° 10′ curve being 625 feet, we have:

$$x = \sqrt{2 \times 625 \times 6} = \sqrt{7500} = 86{\cdot}6$$ — distance from origin of curve to point of frog.

When the main track is a curve, and it is required to get on to a side track running parallel thereto.

Note—In treating of turnouts. When the main and side track are curves, the movable rail is considered a part of the curve used for turnout, according to method 1st.

Let E M be the main track on a curve of 2865 feet radius. It is proposed with a turnout from E, with a curve of 800 feet radius, to fall upon the side track B N, distant 13 feet from the main track, and running parallel thereto. Now 2865 feet radius denotes a 2° curve, and 800 feet radius is a 7° 10′ curve. Therefore the divergence of the curve E F from the curve E M is equal to (7° 10′ — 2°) = 5° 10′ curve; and the radius of a 5° 10′ curve being 1109 feet, the divergence of the curve E F from the curve E M is equal to that of a curve of 1109 feet radius.

By similar reasoning, the convergence of the curve F B towards being parallel with E M is 9° 10′ per hundred feet, which may be expressed by a radius of 625 feet from tangent. Then we have $1109 + 625 = 1734 : 1109 :: 13 : 8{\cdot}31$ = distance of point of reversion from main track. Now since $x = \sqrt{2\,R\,.\,G}$, we have by substituting $\sqrt{2 \times 1109 \times 8{\cdot}31} = 135{\cdot}7$ = distance from origin of curve to point of reversion, radius used being 800 feet. The radius of relative curvature being expressed in the formula, we have the proportion $1109 : 625 :: 135{\cdot}7 : 76{\cdot}56$ distance from reversion to 2d track.

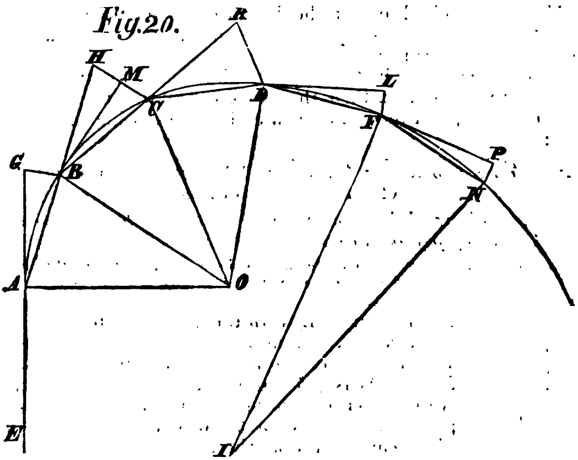
Suppose it be required to put the side track on the opposite side, then we have $1734 : 625 :: 13 : 4{\cdot}68$ = distance of point of reversion from side track. Then we have the formula $\sqrt{2 \times 625 \times 4{\cdot}68} = 76{\cdot}48$ distance from origin of curve to point of reversion. Then $625 : 1109 :: 76{\cdot}48 : 135{\cdot}7$ = distance from point of reversion to side track.

ON RUNNING CURVES BY OFFSETS, OR WITHOUT THE USE OF AN INSTRUMENT FOR MEASURING ANGLES.

FIG. 20.

From a tangent E A let it be required to run a curve A B C D, having for its radius O C. To do this we have only to find H C and its half M C = G B.

Fig. 20.



Suppose the chords A B, B C, C D are equal in length, being 100 et each. The chords, and consequently the arcs, being equal, the igle H B C is twice the angle G A B. But G A B is measured by alf the arc A B = B C, consequently the angle H B C is measured y the whole arc B C. But the angle B O C is also measured by ie arc B C, consequently the angles H B C and B O C are equal. ow triangle B O C is isosceles, and B H being equal to B C triangle . B C is isosceles also; consequently the two triangles are similar, id we have the proportion:

$$H\,C : B\,C :: B\,C : B\,O, \text{ consequently } H\,C = \frac{B\,C^2}{B\,O}, \text{ or } H\,C = \frac{10000}{R}.$$

Therefore $M\,C = G\,B = \frac{A\,B^2}{2\,R}$; hence the following rule:

The square of the uniform length of chord divided by radius will ve the linear deflection from chord produced to curve, or half of iis will give the deflection from tangent produced to curve.

EXAMPLES.

Suppose $A\,O = 2500$ feet, then $\frac{10000}{2500} = H\,C = 4$ feet, and G B : 2 feet.

Suppose A O = 2865 feet, the radius of a 2° curve, then we have

$$H\,C = \frac{10000}{2865} = 3{\cdot}49 \text{ or } 3{\cdot}5 \text{ feet nearly; and } G\,B = \tfrac{1}{2} \text{ of } 3{\cdot}5 = 1{\cdot}75.$$

Since the angle G A B = 1° the deflection for 1° per hundred feet is 1·75, or $0°\ 1' = \frac{1{\cdot}75}{60} = {\cdot}029$, and one minute for one foot = ·00029, as by tables of natural sines.

Case 2d.

Suppose we run the curve around to a point which we will call station 10, or 1000 feet from beginning. The point Q, which is less than 100 feet distant from station 10, say 50 feet, being at station 10 + 50.

Suppose this a 2° curve compounded at station 10 + 50 to a 3° curve of 1910 feet radius. Now the instrument setting on station 10 with a backsight on station 9, the instrumental deflection to 10 + 50, 150 feet, will be 1° 30′. Now since 1° per 100 feet is 1·75, that of 1° 30′ will be 2·62 feet. But the last chord being but 50 feet, or half of a hundred, the deflection will be half of 2·62 = 1·31; hence we have the following rule:

Multiply together half the curvature in degrees = instrumental deflection between the backsight and point required, the length of the last chord and 1·75, and the product is the distance from chord produced to point required.

Case 3d.

Suppose the curve from 10 + 50 to station 11 is a 3° curve of 1910 feet radius. Now the deflection from chord to tangent, from station 10 to station 10 + 50, is 0° 30′, and the deflection from tangent to chord between 10 + 50 and 11 is 0° 45′, therefore the entire deflection = 30′ + 45′ = 1° 15′. Now 1° 15′ in a hundred = 1·75 × 1¼ = 2·18, and for 50 feet will be = 1·09 feet.

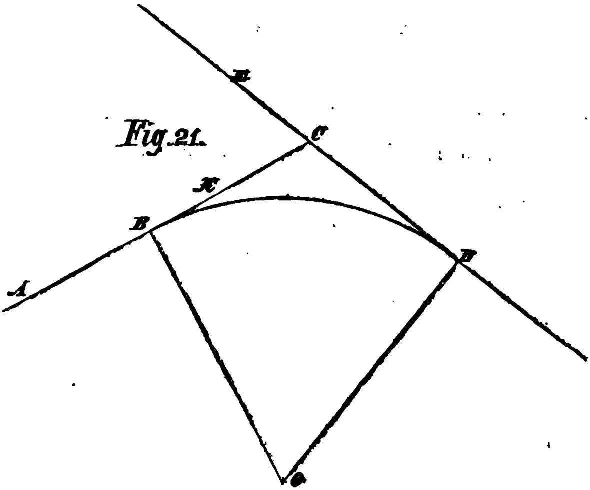
Find station 12 by Case 2d, thus 2¼° (= instrumental deflection for 150 feet) × 1·75 = 3·93 = deflection from chord produced to station 12 on curve.

Continue the curve around as at first, observing to measure from curve to tangent the same deflection as from tangent to curve, or half the usual chord deflection; the tangent point being supposed a full station. If not a full station, ascertain the tangent point by Case 2d, and the next full station on tangent by Case 3d.

Having produced two tangents to an intersection at C, it is required to connect them with a curve of given length. FIG. 21.

When the angle made by tangents is not greater than 15° the distance from vertex to the two ends of the curve will not differ materially from half the length of the curve.

Fig.21.



Suppose the tangent D C produced 100 feet to E, measure C X = 100 feet, measure E X. Now suppose it is 21 feet.

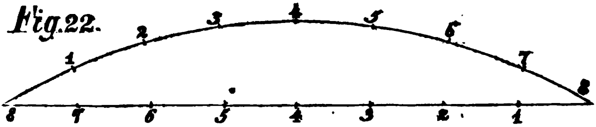
Now the deflection of 1° for 100 feet is 1·75, and $\frac{21}{1 \cdot 75} = 12°$ curvature.

Suppose it is required to divide the curve into 6 stations. Then $\frac{21}{6} = 3 \cdot 5$, the deflection for 2° in 100 feet. Hence it is a 2° curve.

Or 12° divided by 6 stations gives a 2° curve also. The deflection being = 1·75 from tangent to curve.

Between two fixed points to supply the intermediate points by ordinates from the chord. FIG. 22.

Fig.22.



By what has been previously demonstrated, the middle ordinate 4 to 4 will be expressed by $\frac{4 \times 4}{2 R}$. At 3 the deflection from tan-

gent run each way from 4 to curve is $\frac{1 \times 1}{2R}$ at 2 it is $\frac{2 \times 2}{2R}$.

Hence the ordinate 4 to 4 $= \frac{4 \times 4}{2R}$. Or 2 R being a common denominator, its relative value may be expressed by 4×4. At points 3 and 5 on chord the distance will be $(4 \times 4) - (1 \times 1) = 3 \times 5 = 15$. At 6 and $2 = (4 \times 4) - (2 \times 2) = 2 \times 6 = 12$. At 7 and $1 = (4 \times 4) - (3 \times 3) = 1 \times 7 = 7$.

The ordinates are as follows:

$$1 \times 7 = 7$$
$$2 \times 6 = 12$$
$$3 \times 5 = 15$$
$$4 \times 4 = 16.$$

Then we observe that the sum of the two factors is equal, namely the length of chord. Hence the following rule:

Multiply together the two segments of the chord or distance, divide by twice the radius, and the result is the distance from chord to curve.

Suppose for example the radius = 5000 feet, then at points 1 and 7 we have $\frac{100 \times 700}{10000} = \frac{70000}{10000} = 7$ feet = offset at station 1 from end.

For 2 and 6 $\frac{200 \times 600}{10000} = 12 = $ 2d offset.

For 3 and 5 $\frac{300 \times 500}{10000} = 15 = $ 3d offset,

and the entire length being 8 stations $\frac{400 \times 400}{10000} = 16 = $ greatest or middle ordinate.

Had it been a 1° curve of 5730 feet radius, the ordinates would have been:

$$1 \times 7 \times \tfrac{7}{8}^{*} = 6{\cdot}12$$
$$2 \times 6 \times \tfrac{7}{8} = 10{\cdot}50$$
$$3 \times 5 \times \tfrac{7}{8} = 13{\cdot}12$$
$$4 \times 4 \times \tfrac{7}{8} = 14{\cdot}00 = \text{middle ordinate;}$$

and so in proportion to any other rate of curvature in degrees.

Hence when the rate of curvature is in degrees and no minutes, we have the following rule:

Multiply together the distances in stations each side of the point, and the rate of curvature, deduct from this product $\frac{1}{8}$ of itself, the remainder will be the ordinate required.

* The departure in 100 ft. of a 1° curve from tangent being $= {\cdot}75 = \frac{7}{8}$ of a foot.

CASE 2D.

Suppose that between the points 0 and 8 there occurs a point of c. c., for instance at 3 or 5, the curves compound from a 5000 feet radius to a 4000 feet radius.

By 1st method $\frac{300 \times 300}{8000} = 11{\cdot}25$ = distance from end of chord to tangent run from p. c. c., and $\frac{500 \times 500}{10000} = 25$ = distance from other end to said tangent.

Measure from ends of chords respectively 11·25 and 25 feet; on this line, at a distance 300 feet from 11·25 offset, and 500 feet from 25 feet offset, would be the point of compound curvature sought.

Or imagine either curve produced to a point opposite the end of the other; calculate by Proposition XI., and measure the distance between the two curves, then on the new chord find the p. c. c. as by simple curves. Thus:

$$\frac{300 \times 300}{8000} - \frac{300 \times 300}{10000} = 2{\cdot}25.$$

Measure 2·25 from the old chord, and you have the direction of the new. Having found the p. c. c. calculate the offsets from each chord separately.

The above rule for ordinates, although not perfectly accurate, considering the divisor always $= 2$ R, while it is variable, is sufficiently near for centres to grade by, when the chord subtends not more than 20° curvature.

This rule will also apply to placing centre points between stations. Thus:

On a chord of 100 feet, radius 1000 feet, let it be required to locate a point 30 feet from one end and 70 feet from the other.

Then we have $\frac{30 \times 70}{2000} = 1{\cdot}05.$

FOR SPRINGING RAILS.

Let L = length of rail and R = length of radius. Then:

$$\frac{\left(\frac{L}{2}\right)^2}{2R} = \frac{L^2}{8R} = \text{spring in feet.}$$

$$\frac{L^2 \times 1\frac{1}{2}}{R} = \text{spring in inches.}$$

$$\frac{L^2 \times 12}{R.} = \text{spring in eighths of an inch.}$$

$$\left(\frac{24\,L^2}{R}\right) = \text{spring in sixteenths of an inch.}$$

EXAMPLE.

Let the rail be 20 feet long, and the radius 1200 feet. Then

$$\frac{24 \times 20^2}{1200} = \frac{9600}{1200} = \tfrac{8}{16}.$$

Hence the rule:

24 times the square of the length of rail in feet divided by length of radius in feet, will give the spring in middle in sixteenths of an inch.

To find the length of chord for any rate of curvature (less than 8°) not specified in the Table of Chords (p 414.)

EXAMPLE.

Let it be required to find the length of chord corresponding to 800 feet of curve for a 7° 10′ curve.

7° curve gives	.	769·01
7° 15′ curve gives		766·79
Difference	. .	2·22

Then 15 : 10 :: 2·22 : 1·48, and 769·01 — 1·48 = 767·53;

or 15 : 5 :: 2·22 : ·74, and 766·79 + 0·74 = 767·53.

The result, as obtained by the table of sines, is 767·54, only $\frac{1}{100}$ of a foot difference.

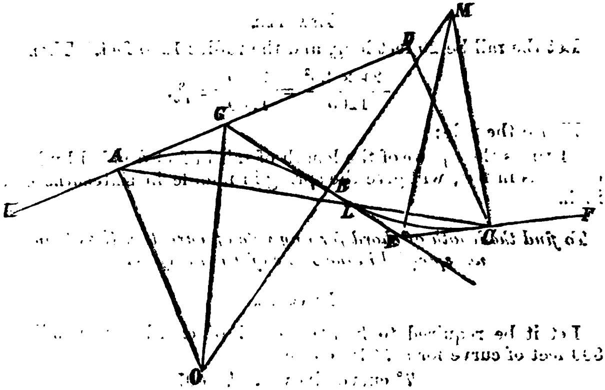
That is, sine 28° 40′ × radius 800 × 2 = 767·54.

Suppose now it be required to find the length of chord corresponding to 950 feet of a 6° curve.

900 feet gives length of chord	.	867·45
1000 " " "	.	955·37
Sum	.	1822·82
Mean 950	.	911·41

Now sine 28° 30′ × radius 955·37 × 2 = length of chord = 911·71 being only $\frac{3}{10}$ of a foot difference, so that this table will be sufficient for ordinary purposes. For common rates of curvature for a less distance, say 650 feet, the variations from the true length would be scarcely perceptible.

PROBLEM.—*Let A and C be two fixed tangent points, the positions of whose tangents are determined by the angles D A C = m = 18°, B C E = n = 6°, and the perpendicular distance D C = p = 463½ ft.* Required the amount of curvature in the arc A B, its reversion B C, and the length of the common radius O B = M B by which the arcs A B and B C are described.*



Let m = nat. vers. sine D A C, and n = nat. vers. sine B C E.
Let x = nat. vers. sine (A O B − m) = (B M C − n).
Or curvature A B = $m + x$, and curvature B C = $n + x$.†

To find x we have, $x = \dfrac{m + n}{2} = \dfrac{\text{v. s. } 18° + \text{v. s. } 6°}{2} =$

$$\frac{0{\cdot}048944 + 0{\cdot}005478}{2} = 0{\cdot}027211 = \text{nat. vers. sine } 13°\ 23'\ 48''.$$

Therefore arc A B = 18° + 13° 23′ 48″ = 31° 23′ 48″ and B C = 6° + 13° 23′ 48″ = 19° 23′ 48″. Then by principles from which Proposition XII. is derived, to find O B = R,

we have $\dfrac{\text{perpd. dist. D C} = p}{\text{twice nat. vers. sine AB} - \text{nat. vers. sine } (m - n)} = \text{R}$, Or

$$\frac{p = 463{\cdot}5}{\text{nat. v. s. } 31°\ 23'\ 48'' \times 2 - \text{nat. v. s. } 12°} = \frac{463{\cdot}5}{0{\cdot}146420 \times 2 - 0{\cdot}021852} =$$

$$\frac{463{\cdot}5}{0{\cdot}270988} = 1710{\cdot}4 = \text{O B} = \text{radius of a } 3°\ 21' \text{ curve.}$$

Then $\dfrac{31°\ 24'}{3°\ 21'}$ gives 937 ft. = arc A B, and $\dfrac{19°\ 24'}{3°\ 21'}$ gives 579 ft = arc BC.

* If D C cannot be measured, measure A C and calculate D C. Thus if A C = 1500 ft. we have 1500 × sine 18° = 1500 × 0·30902 = 463·53.

† D G E being equal to A O B, A O B − m = A L G = C L E. Therefore x = nat. vers. sine A L G = 13° 23′ 48″.

(THE CURVATURE IS SUBTENDED BY A CHORD OF 100 FEET.)

0 DEGREE.			1 DEGREE.			2 DEGREES.		
M.	Radius Infinite.	Logarithm Infinite.	M.	Radius.	Logarithm.	M.	Radius	Logarithms.
			0	5730	3·758128	0	2865	3·457115
1	343775	5·536274	1	5636	3·750949	1	2841	3·453511
2	171887	5·235244	2	5545	3·743888	2	2818	3·449937
3	114592	5·059153	3	5457	3·736940	3	2795	3·446391
4	85944	4·934214	4	5372	3·730106	4	2772	3·442876
5	68755	4·837304	5	5289	3·723367	5	2750	3·439387
6	57296	4·758123	6	5209	3·716737	6	2729	3·435928
7	49111	4·691176	7	5131	3·710206	7	2707	3·432493
8	42972	4·633184	8	5056	3·703772	8	2686	3·429089
9	38197	4·582031	9	4982	3·697432	9	2665	3·425708
10	34377	4·536274	10	4911	3·691183	10	2645	3·422356
11	31252	4·494881	11	4842	3·685023	11	2624	3·419028
12	28648	4·457093	12	4775	3·678947	12	2605	3·415727
13	26444	4·422331	13	4709	3·672958	13	2585	3·412448
14	24555	4·390146	14	4646	3·667057	14	2566	3·409197
15	22918	4·360183	15	4584	3·661220	15	2547	3·405967
16	21486	4·332154	16	4523	3·655469	16	2528	3·402763
17	20222	4·305825	17	4465	3·649792	17	2509	3·399581
18	19099	4·281002	18	4407	3·644189	18	2491	3·396424
19	18093	4·257520	19	4352	3·638656	19	2473	3·393288
20	17189	4·235245	20	4297	3·633194	20	2456	3·390176
21	16370	4·214055	21	4244	3·627800	21	2438	3·387085
22	15626	4·193852	22	4192	3·622470	22	2421	3·384016
23	14947	4·174546	23	4142	3·617196	23	2404	3·380968
24	14324	4·156064	24	4093	3·612005	24	2387	3·377943
25	13751	4·138334	25	4045	3·606866	25	2371	3·374937
26	13222	4·121802	26	3997	3·601787	26	2355	3·371954
27	12732	4·104910	27	3952	3·596766	27	2339	3·368989
28	12278	4·089117	28	3907	3·591803	28	2323	3·366046
29	11854	4·073876	29	3863	3·586896	29	2307	3·363121
30	11459	4·059154	30	3820	3·582044	30	2292	3·360217
31	11090	4·044912	31	3778	3·577246	31	2277	3·357331
32	10743	4·031125	32	3737	3·572499	32	2261	3·354466
33	10417	4·017760	33	3697	3·567804	33	2247	3·351618
34	10111	4·004797	34	3657	3·563160	34	2232	3·348789
35	9822	3·992206	35	3619	3·558564	35	2218	3·345978
36	9549	3·979973	36	3581	3·554011	36	2204	3·343187
37	9291	3·968072	37	3544	3·549516	37	2190	3·340411
38	9047	3·956493	38	3508	3·545063	38	2176	3·337655
39	8815	3·945209	39	3473	3·540654	39	2162	3·334915
40	8594	3·934216	40	3438	3·536289	40	2149	3·332193
41	8385	3·923490	41	3404	3·531968	41	2135	3·329487
42	8185	3·913029	42	3370	3·527690	42	2122	3·326799
43	7995	3·902806	43	3338	3·523452	43	2109	3·324126
44	7813	3·892824	44	3306	3·519257	44	2096	3·321471
45	7639	3·883063	45	3274	3·515100	45	2083	3·318831
46	7473	3·873519	46	3243	3·510985	46	2071	3·316208
47	7314	3·864179	47	3213	3·506907	47	2059	3·313600
48	7162	3·855036	48	3183	3·502868	48	2046	3·311008
49	7016	3·846081	49	3154	3·498866	49	2034	3·308430
50	6876	3·837308	50	3125	3·494900	50	2022	3·305869
51	6741	3·828708	51	3097	3·490970	51	2010	3·303323
52	6611	3·820275	52	3069	3·487075	52	1999	3·300797
53	6486	3·812002	53	3042	3·483205	53	1987	3·298274
54	6366	3·803885	54	3016	3·479389	54	1976	3·295771
55	6250	3·795915	55	2989	3·475596	55	1965	3·293283
56	6139	3·788091	56	2964	3·471836	56	1953	3·290809
57	6031	3·780403	57	2938	3·468108	57	1942	2·288349
58	5927	3·772851	58	2913	3·464413	58	1931	3·285902
59	5827	3·765426	59	2889	3·460748	59	1921	3·283470
60	5730	3·758128	60	2865	3·457115	60	1910	3·281051

3 DEGREES.			4 DEGREES.			5 DEGREES.		
M.	Radius.	Logarithm.	M.	Radius	Logarithm.	M.	Radius.	Logarithm
0	1910	3·281051	0	1433	3·156151	0	1146	3·059290
1	1900	3·278646	1	1427	3·154544	1	1142	3·057845
2	1889	3·276258	2	1421	3·152548	2	1139	3·056407
3	1879	3·273875	3	1415	3·150758	3	1135	3·055010
4	1869	3·271508	4	1409	3·148975	4	1131	3·053542
5	1858	3·269155	5	1403	3·147100	5	1127	3·052115
6	1848	3·266814	6	1398	3·145431	6	1124	3·050696
7	1839	3·264486	7	1392	3·143670	7	1120	3·049279
8	1829	3·262170	8	1386	3·141916	8	1116	3·047808
9	1819	3·259867	9	1381	3·140170	9	1113	3·046461
10	1810	3·257576	10	1375	3·138430	10	1109	3·045059
11	1800	3·255297	11	1370	3·136697	11	1106	3·043662
12	1791	3·253029	12	1364	3·134977	12	1102	3·042268
13	1781	3·250771	13	1359	3·133251	13	1099	3·040879
14	1772	3·248530	14	1354	3·131539	14	1095	3·039495
15	1763	3·246297	15	1348	3·129833	15	1092	3·038114
16	1754	3·244077	16	1343	3·128134	16	1088	3·036740
17	1745	3·241867	17	1338	3·126441	17	1085	3·035368
18	1736	3·239669	18	1333	3·124756	18	1081	3·034002
19	1728	3·237481	19	1328	3·123075	19	1078	3·032636
20	1719	3·235305	20	1322	3·121404	20	1075	3·031281
21	1710	3·233140	21	1317	3·119737	21	1071	3·029927
22	1702	3·230985	22	1312	3·118078	22	1068	3·028577
23	1694	3·228841	23	1307	3·116428	23	1065	3·027230
24	1686	3·226707	24	1302	3·114778	24	1061	3·025890
25	1677	3·224584	25	1298	3·113134	25	1058	3·024552
26	1669	3·222479	26	1293	3·111401	26	1055	3·023219
27	1661	3·220369	27	1288	3·109871	27	1052	3·021889
28	1653	3·218277	28	1283	3·108249	28	1048	3·020565
29	1645	3·216128	29	1278	3·106632	29	1045	3·019243
30	1637	3·214122	30	1273	3·105022	30	1042	3·017927
31	1630	3·212060	31	1269	3·103418	31	1039	3·016614
32	1622	3·210007	32	1264	3·101818	32	1036	3·015305
33	1614	3·207963	33	1260	3·100225	33	1033	3·013999
34	1607	3·205930	34	1255	3·098638	34	1030	3·012698
35	1599	3·203906	35	1250	3·097056	35	1027	3·011400
36	1592	3·201892	36	1246	3·095481	36	1024	3·010107
37	1584	3·199891	37	1241	3·093910	37	1021	3·008817
38	1577	3·197890	38	1237	3·092374	38	1017	3·007532
39	1570	3·195903	39	1232	3·090788	39	1014	3·006249
40	1563	3·193925	40	1228	3·089236	40	1011	3·004972
41	1556	3·191957	41	1224	3·087689	41	1008	3·003698
42	1549	3·189996	42	1219	3·086147	42	1006	3·002427
43	1542	3·188045	43	1215	3·084610	43	1003	3·001159
44	1535	3·186103	44	1211	3·083079	44	1000	2·999897
45	1528	3·184168	45	1207	3·081553	45	996·9	2·998636
46	1521	3·182244	46	1202	3·080033	46	994·1	2·997381
47	1515	3·180327	47	1198	3·078518	47	991·1	2·996128
48	1508	3·178419	48	1194	3·077002	48	988·3	2·994880
49	1501	3·176519	49	1190	3·075503	49	985·4	2·993634
50	1495	3·174627	50	1186	3·074005	50	982·6	2·992393
51	1489	3·172742	51	1182	3·072511	51	979·8	2·991156
52	1482	3·170868	52	1178	3·071022	52	977·1	2·989921
53	1476	3·169001	53	1174	3·069537	53	974·3	2·988690
54	1469	3·167142	54	1170	3·068059	54	971·5	2·987463
55	1463	3·165290	55	1166	3·066584	55	968·7	2·986199
56	1457	3·163447	56	1162	3·065116	56	966·1	2·985018
57	1451	3·161612	57	1158	3·063648	57	963·4	2·983801
58	1445	3·159784	58	1154	3·062194	58	960·7	2·982587
59	1439	3·157963	59	1150	3·060738	59	958·0	2·981377
60	1433	3·156151	60	1146	3·059290	60	955·4	2·980170

6 DEGREES.			7 DEGREES.			8 DEGREES.		
M.	Radius.	Logarithm.	M.	Radius.	Logarithm.	M.	Radius.	Logarithm.
0	955·4	2·980170	0	819·0	2·913295	0	716·8	2·855385
1	952·7	2·978967	1	817·1	2·912266	1	715·3	2·854483
2	950·1	2·977766	2	815·1	2·911234	2	713·8	2·853583
3	947·5	2·976569	3	813·2	2·910208	3	712·3	2·852684
4	944·9	2·975375	4	811·3	2·909183	4	710·9	2·851787
5	942·3	2·974186	5	809·4	2·908161	5	709·4	2·850891
6	939·7	2·972997	6	807·5	2·907142	6	707·9	2·849999
7	937·2	2·971814	7	805·6	2·906124	7	706·5	2·849107
8	934·6	2·970633	8	803·7	2·905111	8	705·0	2·848219
9	932·1	2·969456	9	801·9	2·904097	9	703·6	2·847329
10	929·6	2·968282	10	800·0	2·903090	10	702·2	2·846445
11	927·1	2·967111	11	798·1	2·902082	11	700·7	2·845562
12	924·6	2·965943	12	796·3	2·901076	12	699·3	2·844679
13	922·1	2·964778	13	794·5	2·900073	13	697·9	2·843799
14	919·6	2·963616	14	792·6	2·899073	14	696·5	2·842921
15	917·2	2·962458	15	790·8	2·898075	15	695·1	2·842044
16	914·8	2·961303	16	789·0	2·897078	16	693·7	2·841169
17	912·3	2·960150	17	787·2	2·896085	17	692·3	2·840296
18	909·9	2·959001	18	785·4	2·895094	18	690·9	2·839424
19	907·5	2·957854	19	783·6	2·894103	19	689·5	2·838554
20	905·1	2·956711	20	781·8	2·893118	20	688·2	2·837687
21	902·8	2·955572	21	780·1	2·892134	21	686·8	2·836821
22	900·4	2·954434	22	778·3	2·891151	22	685·4	2·835956
23	898·0	2·953300	23	776·6	2·890171	23	684·1	2·835093
24	895·7	2·952168	24	774·8	2·889193	24	682·7	2·834232
25	893·4	2·951040	25	773·1	2·888218	25	681·4	2·833373
26	891·1	2·949915	26	771·3	2·887244	26	680·0	2·832515
27	888·8	2·948792	27	769·6	2·886272	27	678·7	2·831659
28	886·5	2·947673	28	767·9	2·885303	28	677·4	2·830805
29	884·2	2·946555	29	766·2	2·884336	29	676·0	2·829953
30	882·0	2·945452	30	764·4	2·883371	30	674·7	2·829102
31	879·7	2·944330	31	762·8	2·882409	31	673·4	2·828253
32	877·5	2·943223	32	761·1	2·881445	32	672·1	2·827405
33	875·2	2·942116	33	759·4	2·880490	33	670·7	2·826560
34	873·0	2·941015	34	757·8	2·879534	34	669·4	2·825715
35	870·8	2·939914	35	756·1	2·878580	35	668·1	2·824873
36	868·6	2·938819	36	754·4	2·877627	36	666·9	2·824032
37	866·4	2·937722	37	752·8	2·876678	37	665·6	2·823192
38	864·2	2·936633	38	751·2	2·875730	38	664·3	2·822355
39	862·1	2·935543	39	749·5	2·874783	39	663·0	2·821519
40	859·9	2·934459	40	747·9	2·873840	40	661·7	2·820685
41	857·7	2·933377	41	746·3	2·872900	41	660·5	2·819852
42	855·6	2·932295	42	744·7	2·871959	42	659·2	2·819021
43	853·5	2·931218	43	743·1	2·871022	43	657·9	2·818191
44	851·4	2·930142	44	741·5	2·870086	44	656·7	2·817363
45	849·3	2·929070	45	739·9	2·869153	45	655·4	2·816537
46	847·2	2·928000	46	738·3	2·868221	46	654·2	2·815712
47	845·1	2·926933	47	736·7	2·867291	47	653·0	2·814888
48	843·1	2·925867	48	735·1	2·866363	48	651·7	2·814068
49	841·0	2·924806	49	733·6	2·865438	49	650·5	2·813246
50	839·0	2·923747	50	732·0	2·864514	50	649·3	2·812428
51	836·9	2·922691	51	730·5	2·863593	51	648·1	2·811611
52	834·9	2·921637	52	728·9	2·862673	52	646·8	2·810796
53	832·9	2·920585	53	727·4	2·861756	53	645·6	2·809982
54	830·9	2·919536	54	725·8	2·860840	54	644·4	2·809169
55	828·9	2·918489	55	724·3	2·859926	55	643·2	2·808358
56	826·9	2·917446	56	722·8	2·859014	56	642·0	2·807594
57	824·9	2·916403	57	721·3	2·858104	57	640·8	2·806741
58	822·9	2·915365	58	719·8	2·857196	58	639·6	2·805935
59	821·0	2·914327	59	718·3	2·856289	59	638·5	2·805130
60	819·0	2·913295	60	716·8	2·855385	60	637·3	2·804327

9 DEGREES.			10 DEGREES.			11 DEGREES.		
M.	Radius	Logarithm.	M.	Radius.	Logarithm.	M.	Radius.	Logarithm.
0	637·3	2·804327	0	573·7	2·758674	0	521·7	2·717397
1	636·1	2·803526	1	572·7	2·757953	1	520·9	2·716742
2	634·9	2·802724	2	571·8	2·757232	2	520·1	2·716087
3	633·8	2·801926	3	570·8	2·756514	3	519·3	2·715434
4	632·6	2·801128	4	569·9	2·755796	4	518·5	2·714781
5	631·4	2·800332	5	569·0	2·755079	5	517·8	2·714130
6	630·3	2·799538	6	568·0	2·754364	6	517·0	2·713479
7	629·1	2·798745	7	567·1	2·753650	7	516·2	2·712830
8	628·0	2·797953	8	566·2	2·752937	8	515·4	2·712181
9	626·8	2·797163	9	565·2	2·752225	9	514·7	2·711533
10	625·7	2·796374	10	564·3	2·751514	10	513·9	2·710887
11	624·6	2·795587	11	563·4	2·750804	11	513·1	2·710241
12	623·5	2·794801	12	562·5	2·750096	12	512·4	2·709596
13	622·3	2·794017	13	561·6	2·749389	13	511·6	2·708953
14	621·2	2·793234	14	560·6	2·748683	14	510·9	2·708310
15	620·1	2·792452	15	559·7	2·747978	15	510·1	2·707668
16	619·0	2·791673	16	558·8	2·747274	16	509·3	2·707027
17	617·9	2·790894	17	557·9	2·746572	17	508·6	2·706387
18	616·8	2·790117	18	557·0	2·745870	18	507·9	2·705748
19	615·7	2·789340	19	556·1	2·745170	19	507·1	2·705110
20	614·6	2·788566	20	555·2	2·744471	20	506·4	2·704473
21	613·5	2·787794	21	554·3	2·743773	21	505·6	2·703837
22	612·4	2·787021	22	553·4	2·743076	22	504·9	2·703202
23	611·3	2·786252	23	552·6	2·742380	23	504·1	2·702568
24	610·2	2·785482	24	551·7	2·741686	24	503·4	2·701934
25	609·1	2·784715	25	550·8	2·740993	25	502·7	2·701302
26	608·1	2·783948	26	549·9	2·740300	26	501·9	2·700671
27	607·0	2·783183	27	549·0	2·739609	27	501·2	2·700040
28	605·9	2·782420	28	548·2	2·738918	28	500·5	2·699410
29	604·8	2·781657	29	547·3	2·738229	29	499·8	2·698782
30	603·8	2·780897	30	546·4	2·737541	30	499·0	2·698154
31	602·8	2·780138	31	545·6	2·736854	31	498·3	2·697527
32	601·7	2·779379	32	544·7	2·736169	32	497·6	2·696901
33	600·7	2·778622	33	543·8	2·735484	33	496·9	2·696276
34	599·6	2·777863	34	543·0	2·734800	34	496·2	2·695652
35	598·6	2·777113	35	542·1	2·734118	35	495·5	2·695029
36	597·5	2·776360	36	541·3	2·733436	36	494·8	2·694407
37	596·5	2·775608	37	540·4	2·732756	37	494·1	2·693785
38	595·5	2·774858	38	539·6	2·732077	38	493·4	2·693165
39	594·4	2·774108	39	538·8	2·731398	39	492·7	2·692545
40	593·4	2·773361	40	537·9	2·730721	40	492·0	2·691926
41	592·4	2·772616	41	537·1	2·730045	41	491·3	2·691308
42	591·4	2·771870	42	536·3	2·729370	42	490·6	2·690692
43	590·4	2·771124	43	535·4	2·728696	43	489·9	2·690076
44	589·4	2·770383	44	534·6	2·728023	44	489·2	2·689460
45	588·4	2·769642	45	533·8	2·727351	45	488·5	2·688846
46	587·4	2·768902	46	532·9	2·726684	46	487·8	2·688233
47	586·4	2·768163	47	532·1	2·726010	47	487·1	2·687620
48	585·4	2·767426	48	531·3	2·725342	48	486·4	2·687008
49	584·4	2·766689	49	530·5	2·724674	49	485·7	2·686398
50	583·4	2·765955	50	529·7	2·724008	50	485·0	2·685788
51	582·4	2·765223	51	528·9	2·723342	51	484·4	2·685179
52	581·4	2·764489	52	528·0	2·722677	52	483·7	2·684570
53	580·4	2·763758	53	527·2	2·722014	53	483·0	2·683963
54	579·5	2·763028	54	526·4	2·721351	54	482·3	2·683357
55	578·5	2·762299	55	525·6	2·720690	55	481·7	2·682751
56	577·5	2·761572	56	524·8	2·720019	56	481·0	2·682146
57	576·6	2·760845	57	524·0	2·719370	57	480·3	2·681542
58	575·6	2·760120	58	523·2	2·718711	58	479·7	2·680939
59	574·6	2·759398	59	522·5	2·718054	59	479·0	2·680337
60	573·7	2·758674	60	521·7	2·717397	60	478·3	2·679735

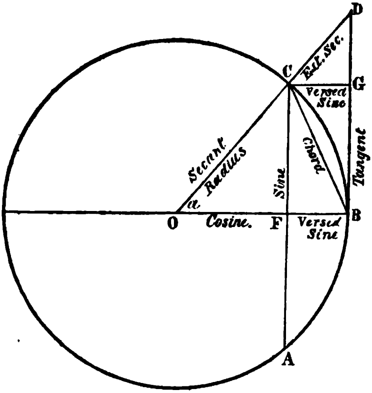
12 DEGREES.			13 DEGREES.			14 DEGREES.		
M.	Radius.	Logarithm.	M.	Radius.	Logarithm.	M.	Radius.	Logarithm.
0	478·3	2·679735	0	441·7	2·645111	0	410·3	2·613075
1	477·7	2·679135	1	441·1	2·644557	1	409·8	2·612561
2	477·0	2·678535	2	440·5	2·644004	2	409·3	2·612048
3	476·3	2·677936	3	440·0	2·643451	3	408·8	2·611535
4	475·6	2·677338	4	439·4	2·642900	4	408·3	2·611023
5	475·0	2·676741	5	438·9	2·642348	5	407·9	2·610511
6	474·4	2·676145	6	438·3	2·641798	6	407·4	2·610000
7	473·8	2·675549	7	437·8	2·641248	7	406·9	2·609490
8	473·1	2·674954	8	437·2	2·640699	8	406·4	2·608980
9	472·5	2·674360	9	436·7	2·640150	9	406·0	2·608471
10	471·8	2·673767	10	436·1	2·639603	10	405·5	2·607962
11	471·2	2·673175	11	435·6	2·639056	11	405·0	2·607454
12	470·5	2·672584	12	435·0	2·638510	12	404·5	2·606946
13	469·9	2·671993	13	434·5	2·637964	13	404·0	2·606439
14	469·2	2·671403	14	433·9	2·637419	14	403·6	2·605933
15	468·6	2·670814	15	433·4	2·636875	15	403·1	2·605428
16	468·0	2·670226	16	432·8	2·636332	16	402·6	2·604923
17	467·3	2·669638	17	432·3	2·635789	17	402·2	2·604418
18	466·7	2·669052	18	431·8	2·635247	18	401·7	2·603914
19	466·1	2·668466	19	431·2	2·634705	19	401·2	2·603411
20	465·5	2·667881	20	430·7	2·634164	20	400·8	2·602908
21	464·8	2·667297	21	430·2	2·633624	21	400·3	2·602406
22	464·2	2·666713	22	429·6	2·633085	22	399·9	2·601905
23	463·6	2·666131	23	429·1	2·632546	23	399·4	2·601404
24	463·0	2·665549	24	428·6	2·632008	24	398·9	2·600904
25	462·3	2·664968	25	428·0	2·631471	25	398·5	2·600404
26	461·7	2·664387	26	427·5	2·630934	26	398·0	2·599905
27	461·1	2·663808	27	427·0	2·630398	27	397·6	2·599406
28	460·5	2·663229	28	426·4	2·629863	28	397·1	2·598908
29	459·9	2·662651	29	425·9	2·629328	29	396·7	2·598411
30	459·2	2·662074	30	425·4	2·628794	30	396·2	2·597914
31	458·6	2·661498	31	424·9	2·628261	31	395·7	2·597418
32	458·0	2·660922	32	424·4	2·627728	32	395·3	2·596922
33	457·4	2·660347	33	423·8	2·627196	33	394·8	2·596427
34	456·8	2·659773	34	423·3	2·626665	34	394·4	2·595933
35	456·2	2·659200	35	422·8	2·626134	35	393·9	2·595439
36	455·6	2·658628	36	422·3	2·625604	36	393·5	2·594946
37	455·0	2·658056	37	421·8	2·625074	37	393·0	2·594453
38	454·4	2·657485	38	421·3	2·624546	38	392·6	2·593961
39	453·8	2·656915	39	420·7	2·624018	39	392·2	2·593469
40	453·2	2·656345	40	420·2	2·623490	40	391·7	2·592978
41	452·7	2·655776	41	419·7	2·622963	41	391·2	2·592487
42	452·1	2·655208	42	419·2	2·622437	42	390·8	2·591997
43	451·5	2·654641	43	418·7	2·621912	43	390·4	2·591508
44	450·9	2·654075	44	418·2	2·621387	44	390·0	2·591019
45	450·3	2·653509	45	417·7	2·620863	45	389·5	2·590531
46	449·7	2·652944	46	417·2	2·620339	46	389·1	2·590043
47	449·1	2·652380	47	416·7	2·619816	47	388·6	2·589556
48	448·6	2·651816	48	416·2	2·619294	48	388·2	2·589069
49	448·0	2·651254	49	415·7	2·618772	49	387·8	2·588583
50	447·4	2·650691	50	415·2	2·618251	50	387·3	2·588097
51	446·8	2·650130	51	414·7	2·617731	51	386·9	2·587612
52	446·2	2·649570	52	414·2	2·617211	52	386·5	2·587128
53	445·7	2·649010	53	413·7	2·616692	53	386·0	2·586644
54	445·1	2·648451	54	413·2	2·616173	54	385·6	2·586161
55	444·5	2·647892	55	412·7	2·615655	55	385·2	2·585678
56	444·0	2·647335	56	412·2	2·615138	56	384·8	2·585196
57	443·4	2·646778	57	411·7	2·614622	57	384·3	2·584714
58	442·8	2·646222	58	411·2	2·614106	58	383·9	2·584233
59	442·2	2·645666	59	410·8	2·613590	59	383·5	2·583752
60	441·7	2·645111	60	410·3	2·613075	60	383·1	2·583272

TABLE

Of Chords corresponding to every 100 *feet on curve from* 200 *to* 1000 *feet, calculated to every* 15 *minutes' rate of curvature, from* 15 *minutes to* 8 *degrees, radius of* 1° *being* 5730 *feet.*

Rate of curvature.	200 feet.	300 feet.	400 feet.	500 feet.	600 feet.	700 feet.	800 feet.	900 feet.	1000 ft.
15′	200·00	300·00	400·00	499·99	599·98	699·97	799·96	899·94	999·92
30′	[illegible]	299·99	399·98	499·96	599·93	699·89	799·84	899·77	999·69
45′	[illegible]	299·98	399·95	499·91	599·84	699·76	799·64	899·49	999·30
1°	199·99	299·97	399·92	499·85	599·73	699·57	799·36	899·09	998·75
1° 15′	199·99	299·95	399·88	499·76	599·58	699·33	799·00	898·57	998·05
1° 30′	199·98	299·93	399·83	499·66	599·40	699·04	798·56	897·95	997·18
1° 45′	199·98	299·91	399·77	499·53	599·18	698·69	798·04	897·20	996·15
2°	199·97	299·88	399·70	499·39	598·94	698·30	797·44	896·35	994·98
2° 15′	199·96	299·85	399·61	499·23	598·65	697·84	796·76	895·38	993·65
2° 30′	199·95	299·81	399·52	499·05	598·34	697·34	796·01	894·30	992·17
2° 45′	199·94	299·77	399·42	498·85	597·99	696·78	795·17	893·10	990·52
3°	199·93	299·73	399·32	498·63	597·61	696·17	794·25	891·80	988·73
3° 15′	199·92	299·68	399·19	498·39	597·19	695·50	793·26	890·38	986·78
3° 30′	199·91	299·63	399·07	498·14	596·74	694·79	792·18	888·85	984·68
3° 45′	199·89	299·57	398·93	497·86	596·26	694·02	791·03	887·21	982·42
4°	199·88	299·51	398·78	497·57	595·74	693·20	789·80	885·45	979·99
4° 15′	199·86	299·45	398·63	497·25	595·20	692·32	788·49	883·58	977·46
4° 30′	199·85	299·38	398·46	496·92	594·62	691·40	787·11	881·61	974·75
4° 45′	199·83	299·31	398·28	496·57	594·00	690·42	785·64	879·52	971·89
5°	199·81	299·24	398·10	496·20	593·36	689·39	784·10	877·32	968·87
5° 15′	199·79	299·16	397·90	495·81	592·68	688·30	782·48	875·02	965·72
5° 30′	199·77	299·08	397·70	495·40	591·97	687·17	780·79	872·61	962·42
5° 45′	199·75	299·00	397·49	494·98	591·22	685·98	779·01	870·08	958·96
6°	199·73	298·90	397·26	494·53	590·45	684·75	777·16	867·45	955·37
6° 15′	199·70	298·81	397·03	494·07	589·64	683·46	775·24	864·72	951·63
6° 30′	199·68	298·72	396·80	493·60	588·81	682·13	773·26	861·90	947·75
6° 45′	199·65	298·61	396·54	493·09	587·93	680·73	771·16	858·93	943·71
7°	199·63	298·51	396·28	492·57	587·02	679·29	769·01	855·87	939·54
7° 15′	199·60	298·40	396·01	492·03	586·08	677·79	766·79	852·72	935·23
7° 30′	199·57	298·29	395·73	491·47	585·11	676·25	764·49	849·45	930·78
7° 45′	199·54	298·17	495·44	490·90	584·12	674·66	762·12	846·09	926·20
8°	199·51	298·05	395·14	490·31	583·08	673·01	759·67	842·62	921·47

TABLES OF NATURAL AND LOGARITHMIC VERSED SINES, AND EXTERNAL SECANTS.



On the Construction of the Tables of Versed Sines and External Secants.

In the above figure it is required to find the value of versed sine F B = C G, of arc B C = A B angle a, and of external secant C D in terms of sine C F and tangent B D.

The chord B C = 2 sine $\frac{1}{2}$ B C, and angle F C B is measured by $\frac{1}{2}$ arc A B = $\frac{1}{2}$ arc B C.

Therefore making chord B C radius, B F will be the sine of angle F C B, and we have:

$$\text{Versed sine BF} = 2 \times \overline{\text{sine FCB}}^2 = 2 \times (\text{sine } \tfrac{1}{2} a)^2.$$

That is, twice the square of sine of half given arc = versed sine. Making C F radius. B F becomes tangent, and we have, versed sine B F = C F × tangent F C B, or sine a × tangent $\frac{1}{2}$ a.

Now by similar triangles v. s. a : ex. sec. a :: cos. a : radius;
and v. s. a : ex. sec. a :: sine a : tangent a;

$$\text{or, ex. sec. } a = \left.\frac{\text{v. s. } a \times \text{radius}}{\text{cosine } a}\right\} = \text{tan. } a \times \text{tangent } \tfrac{1}{2} a.$$

Then log. v. s. a = log. sine a + log. tan. $\frac{1}{2}$ a — (10 = log. of R.),
and log ex. sec. a = log. v. s. a + 10 — log. cos. a.;
or, log. ex sec. a = log. tan. a + log. tan. $\frac{1}{2}$ a — 10.

EXAMPLE.

Log. sine 40° = 9·808067	Log. tan. 40° = 9·923813
Log. tan. 20° = 9·561066	Log. tan. 20° = 9·561066
Log. v. s. 40° = 9·369133	Ex. sec. 40° = 9·484879

0 DEGREE.			1 DEGREE.			2 DEGREES.		
Min.	Nat. No	Logarithm	Min.	Nat. No.	Logarithm.	Min.	Nat. No	Logarithm.
0	0·000000	0·000000	0	0·000152	6·182714	0	0·000609	6·784740
1	·000000	2·621422	1	·000157	·197071	1	·000619	·791948
2	·000000	3·228482	2	·000162	·211194	2	·000630	·799097
3	·000000	·580664	3	·000168	·225091	3	·000640	·806187
4	·000000	·830542	4	·000173	·238770	4	·000650	·813219
5	·000001	4·024362	5	·000179	·252227	5	·000661	·820194
6	·000002	·182724	6	·000184	·265496	6	·000672	·827114
7	·000002	·316618	7	·000190	·278557	7	·000682	·833980
8	·000003	·432602	8	·000196	·291426	8	·000693	·840792
9	·000003	·534936	9	·000201	·304106	9	·000704	·847551
10	·000004	·626422	10	·000207	·316604	10	·000715	·854257
11	·000005	·709206	11	·000213	·328925	11	·000726	·860912
12	·000006	·784784	12	·000219	·341072	12	·000737	·867516
13	·000007	·854306	13	·000225	·353051	13	·000748	·874070
14	·000008	·918678	14	·000232	·364868	14	·000759	·880577
15	·000010	·978602	15	·000238	·376528	15	·000771	·887033
16	·000011	5·034662	16	·000244	·388032	16	·000782	·893444
17	·000012	·087316	17	·000251	·399387	17	·000794	·899806
18	·000014	·136966	18	·000257	·410592	18	·000805	·906123
19	·000015	·183924	19	·000264	·421657	19	·000817	·912393
20	·000017	·228480	20	·000271	·432582	20	·000829	·918618
21	·000018	·270856	21	·000278	·443372	21	·000841	·924800
22	·000020	·311266	22	·000284	·454030	22	·000853	·930937
23	·000022	·349877	23	·000291	·464538	23	·000865	·937032
24	·000024	·386842	24	·000298	·474960	24	·000877	·943084
25	·000026	·422302	25	·000306	·485238	25	·000889	·949093
26	·000029	·456366	26	·000313	·495396	26	·000902	·955062
27	·000031	·489140	27	·000320	·505436	27	·000914	·960991
28	·000033	·520736	28	·000328	·515364	28	·000926	·966879
29	·000035	·551216	29	·000335	·525179	29	·000939	·972726
30	·000038	·580662	30	·000343	·534882	30	·000952	·978536
31	·000040	·609143	31	·000350	·544480	31	·000964	·984305
32	·000043	·636720	32	·000358	·553972	32	·000977	·990038
33	·000046	·663449	33	·000366	·563362	33	·000990	·995733
34	·000049	·689376	34	·000374	·572651	34	·001003	7·001391
35	·000052	·714558	35	·000382	·581841	35	·001016	·007013
36	·000055	·739024	36	·000390	·590937	36	·001029	·012597
37	·000058	·762821	37	·000398	·599936	37	·001043	·018147
38	·000061	·785984	38	·000406	·608845	38	·001056	·023660
39	·000064	·808549	39	·000415	·617662	39	·001069	·029139
40	·000068	·830538	40	·000423	·626393	40	·001083	·034584
41	·000071	·851985	41	·000431	·635084	41	·001097	·039995
42	·000075	·872916	42	·000440	·643591	42	·001110	·045372
43	·000078	·893353	43	·000449	·652064	43	·001124	·050717
44	·000082	·913322	44	·000458	·660456	44	·001138	·056027
45	·000086	·932845	45	·000466	·668768	45	·001152	·061307
46	·000090	·951932	46	·000475	·676999	46	·001166	·066554
47	·000093	·970611	47	·000484	·685156	47	·001180	·071771
48	·000097	·988893	48	·000493	·693234	48	·001194	·076955
49	·000102	6·006770	49	·000503	·701240	49	·001208	·082119
50	·000106	·024354	50	·000512	·709171	50	·001222	·087232
51	·000110	·041559	51	·000521	·717032	51	·001237	·092325
52	·000114	·058420	52	·000531	·724820	52	·001251	·097388
53	·000119	·074965	53	·000540	·732540	53	·001266	·102423
54	·000123	·091200	54	·000550	·740192	54	·001281	·107428
55	·000128	·107146	55	·000559	·747778	55	·001295	·112405
56	·000133	·122788	56	·000569	·755297	56	·001310	·117353
57	·000137	·138167	57	·000579	·762752	57	·001325	·122272
58	·000142	·153268	58	·000589	·770144	58	·001340	·127165
59	·000147	·168116	59	·000599	·777472	59	·001355	·132031
60	·000152	·182714	60	·000609	·784740	60	·001370	·136868

0 DEGREE.			1 DEGREE.			2 DEGREES.		
Min	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min	Nat. No.	Logarithm.
0	0·000000	·000000	0	0·000152	6·182780	0	0·000610	6·785005
1	·000000	2·626422	1	·000157	·197189	1	·000620	·792217
2	·000000	3·228482	2	·000163	·211265	2	·000630	·799371
3	·000000	·580664	3	·000168	·225164	3	·000640	·806465
4	·000000	·830542	4	·000173	·238845	4	·000651	·813502
5	·000000	4·024362	5	·000179	·252305	5	·000661	·820481
6	·000001	·182725	6	·000184	·265576	6	·000672	·827406
7	·000002	·316619	7	·000190	·278639	7	·000683	·834276
8	·000003	·432603	8	·000196	·291511	8	·000694	·841093
9	·000003	·534907	9	·000201	·304193	9	·000704	·847857
10	·000004	·626424	10	·000207	·316694	10	·000715	·854568
11	·000005	·709209	11	·000213	·329018	11	·000726	·861227
12	·000006	·784787	12	·000219	·341167	12	·000738	·867836
13	·000007	·854309	13	·000225	·353149	13	·000749	·874395
14	·000008	·918682	14	·000232	·364969	14	·000760	·880907
15	·000010	·978606	15	·000238	·376631	15	·000772	·887368
16	·000011	5·034667	16	·000244	·388138	16	·000783	·893784
17	·000012	·087321	17	·000251	·399486	17	·000795	·900157
18	·000014	·136974	18	·000257	·410704	18	·000806	·906473
19	·000015	·183933	19	·000264	·421772	19	·000818	·912748
20	·000017	·228487	20	·000271	·432700	20	·000830	·918978
21	·000018	·270864	21	·000278	·443493	21	·000842	·925165
22	·000020	·311275	22	·000285	·454154	22	·000854	·931308
23	·000022	·349882	23	·000292	·464685	23	·000866	·937408
24	·000024	·386858	24	·000299	·475090	24	·000878	·943465
25	·000026	·422314	25	·000306	·485371	25	·000890	·949479
26	·000029	·456378	26	·000313	·495532	26	·000903	·955454
27	·000031	·489153	27	·000320	·505577	27	·000915	·961388
28	·000033	·520750	28	·000328	·515506	28	·000927	·967282
29	·000036	·551280	29	·000335	·525325	29	·000940	·973134
30	·000038	·580679	30	·000343	·535031	30	·000953	·978950
31	·000040	·619151	31	·000350	·544632	31	·000965	·984724
32	·000043	·636789	32	·000358	·554128	32	·000978	·990463
33	·000046	·663469	33	·000366	·563521	33	·000991	·996163
34	·000049	·689397	34	·000374	·572813	34	·001014	7·001827
35	·000052	·714581	35	·000382	·582007	35	·001017	·007455
36	·000055	·739048	36	·000390	·591106	36	·001031	·013044
37	·000058	·762846	37	·000398	·600109	37	·001044	·018600
38	·000061	·786018	38	·000406	·609022	38	·001057	·024119
39	·000064	·808577	39	·000415	·617842	39	·001071	·029604
40	·000068	·830567	40	·000423	·626577	40	·001084	·035055
41	·000071	·852016	41	·000432	·635222	41	·001098	·040471
42	·000075	·872948	42	·000440	·643782	42	·001111	·045854
43	·000078	·893387	43	·000449	·652259	43	·001125	·051205
44	·000082	·913358	44	·000458	·660655	44	·001139	·056521
45	·000086	·932882	45	·000466	·668771	45	·001153	·061807
46	·000090	·951971	46	·000476	·677206	46	·001167	·067061
47	·000094	·970652	47	·000485	·685366	47	·001181	·072284
48	·000097	·988940	48	·000494	·693448	48	·001195	·077474
49	·000102	6·006814	49	·000503	·701458	49	·001210	·082644
50	·000106	·024400	50	·000512	·709398	50	·001224	·087763
51	·000110	·041607	51	·000522	·717258	51	·001238	·092862
52	·000114	·058470	52	·000531	·725051	52	·001253	·097932
53	·000119	·075017	53	·000540	·732775	53	·001268	·102973
54	·000123	·091254	54	·000550	·740431	54	·001282	·107985
55	·000128	·107202	55	·000560	·748021	55	·001297	·112968
56	·000133	·122846	56	·000570	·755544	56	·001312	·117922
57	·000137	·128227	57	·000579	·763004	57	·001327	·122848
58	·000142	·153330	58	·000589	·770400	58	·001342	·127747
59	·000147	·168180	59	·000599	·777732	59	·001357	·132620
60	·000152	·182780	60	·000610	·785005	60	·001372	·137464

3 DEGREES.			4 DEGREES.			5 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat No	Logarithm	Min.	Nat. No.	Logarithm
0	0·001370	7·136868	0	0·002436	7·386669	0	0·003805	7·580389
1	·001386	·141679	1	·002456	·390278	1	·003831	·583272
2	·001401	·146464	2	·002477	·393824	2	·003856	·586156
3	·001417	·151225	3	·002497	·397455	3	·003882	·589026
4	·001432	·155955	4	·002518	·401019	4	·003907	·591886
5	·001448	·160661	5	·002538	·404572	5	·003933	·594735
6	·001463	·165342	6	·002559	·408108	6	·003959	·597578
7	·001479	·169999	7	·002580	·411629	7	·003985	·600410
8	·001495	·174629	8	·002601	·415137	8	·004010	·603234
9	·001511	·179236	9	·002622	·418632	9	·004037	·606048
10	·001527	·183819	10	·002643	·422111	10	·004063	·608852
11	·001543	·188377	11	·002664	·425577	11	·004089	·611647
12	·001559	·192912	12	·002685	·429029	12	·004116	·614434
13	·001576	·197422	13	·002707	·432468	13	·004142	·617210
14	·001592	·201910	14	·002728	·435892	14	·004169	·619980
15	·001608	·206376	15	·002750	·439303	15	·004195	·622740
16	·001625	·210817	16	·002771	·442702	16	·004222	·625490
17	·001641	·215237	17	·002793	·446087	17	·004248	·628234
18	·001658	·219632	18	·002815	·449458	18	·004275	·630966
19	·001675	·224018	19	·002837	·452817	19	·004302	·633692
20	·001692	·228360	20	·002859	·456162	20	·004329	·636409
21	·001709	·232692	21	·002881	·459494	21	·004356	·639117
22	·001726	·237000	22	·002903	·462815	22	·004382	·641816
23	·001743	·241289	23	·002925	·466121	23	·004410	·644506
24	·001760	·245555	24	·002947	·469417	24	·004438	·647170
25	·001777	·249802	25	·002970	·472699	25	·004466	·649864
26	·001795	·254026	26	·002992	·475969	26	·004493	·652532
27	·001813	·258232	27	·003015	·479227	27	·004521	·655190
28	·001830	·262416	28	·003037	·482472	28	·004548	·657840
29	·001847	·266582	29	·003060	·485705	29	·004576	·660482
30	·001865	·270725	30	·003083	·488926	30	·004604	·663116
31	·001883	·274852	31	·003106	·492137	31	·004632	·665743
32	·001901	·278957	32	·003129	·495334	32	·004660	·668360
33	·001919	·283043	33	·003152	·498523	33	·004688	·670972
34	·001937	·287109	34	·003175	·501694	34	·004716	·673574
35	·001955	·291156	35	·003198	·504857	35	·004744	·676168
36	·001973	·295187	36	·003221	·508008	36	·004773	·678759
37	·001992	·299198	37	·003244	·511147	37	·004801	·681334
38	·002010	·303190	38	·003268	·514275	38	·004830	·683906
39	·002028	·307162	39	·003291	·517391	39	·004858	·686470
40	·002047	·311119	40	·003315	·520498	40	·004887	·689026
41	·002066	·315056	41	·003339	·523593	41	·004916	·691574
42	·002085	·318977	42	·003362	·526677	42	·004944	·694116
43	·002103	·322879	43	·003386	·529750	43	·004973	·696649
44	·002122	·326764	44	·003410	·532812	44	·005002	·699176
45	·002141	·330634	45	·003434	·535863	45	·005031	·701696
46	·002160	·334483	46	·003459	·538904	46	·005061	·704208
47	·002179	·338316	47	·003483	·541933	47	·005090	·706713
48	·002198	·342133	48	·003507	·544953	48	·005119	·709210
49	·002218	·345932	49	·003531	·547961	49	·005149	·711700
50	·002237	·349716	50	·003556	·550961	50	·005178	·714184
51	·002257	·353482	51	·003581	·553948	51	·005208	·716658
52	·002276	·357233	52	·003605	·556927	52	·005238	·719128
53	·002296	·360967	53	·003630	·559895	53	·005267	·721589
54	·002316	·364687	54	·003655	·562852	54	·005297	·724044
55	·002336	·368390	55	·003680	·565800	55	·005327	·726492
56	·002355	·372076	56	·003705	·568737	56	·005357	·728934
57	·002375	·375746	57	·003730	·571665	57	·005387	·731367
58	·002396	·379403	58	·003755	·574582	58	·005417	·733796
59	·002416	·383043	59	·003780	·577492	59	·005448	·736217
60	·002436	·386669	60	·003805	·580389	60	·005478	·738630

3 DEGREES.			4 DEGREES.			5 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·001372	7·137464	0	0·002442	7·387728	0	0·003820	7·582045
1	·001388	·142281	1	·002462	·391346	1	·003845	·584946
2	·001403	·147073	2	·002483	·394951	2	·003871	·587834
3	·001419	·151841	3	·002503	·398541	3	·003897	·590715
4	·001434	·156577	4	·002524	·402114	4	·003923	·593586
5	·001450	·161290	5	·002545	·405676	5	·003949	·596446
6	·001465	·165978	6	·002566	·409221	6	·003975	·599301
7	·001481	·170642	7	·002587	·412751	7	·004001	·602144
8	·001497	·175279	8	·002608	·416268	8	·004027	·604979
9	·001513	·179893	9	·002629	·419772	9	·004053	·607805
10	·001529	·184483	10	·002650	·423261	10	·004080	·610620
11	·001545	·189048	11	·002671	·426736	11	·004107	·613427
12	·001562	·193590	12	·002693	·430197	12	·004133	·616225
13	·001578	·198107	13	·002714	·433645	13	·004159	·619018
14	·001594	·202602	14	·002736	·437079	14	·004186	·621794
15	·001611	·207075	15	·002757	·440499	15	·004213	·624566
16	·001628	·211523	16	·002779	·443907	16	·004240	·627327
17	·001644	·215951	17	·002801	·447302	17	·004267	·630183
18	·001661	·220353	18	·002823	·450682	18	·004294	·632827
19	·001678	·224736	19	·002845	·454051	19	·004321	·635564
20	·001695	·229095	20	·002867	·457405	20	·004348	·638293
21	·001712	·233435	21	·002889	·460747	21	·004375	·641013
22	·001729	·237750	22	·002911	·464077	22	·004403	·643724
23	·001746	·242047	23	·002934	·467393	23	·004430	·646426
24	·001763	·246320	24	·002956	·470699	24	·004458	·649102
25	·001781	·250575	25	·002978	·473991	25	·004485	·651808
26	·001798	·254806	26	·003001	·477210	26	·004513	·654488
27	·001816	·259020	27	·003024	·480538	27	·004541	·657158
28	·001833	·263211	28	·003046	·483793	28	·004569	·659820
29	·001851	·267385	29	·003069	·487036	29	·004597	·662474
30	·001869	·271536	30	·003092	·490267	30	·004625	·665120
31	·001887	·275671	31	·003115	·493488	31	·004653	·667759
32	·001905	·279783	32	·003138	·496694	32	·004681	·670388
33	·001923	·283877	33	·003161	·499894	33	·004710	·673013
34	·001941	·287951	34	·003185	·503075	34	·004738	·675627
35	·001959	·292006	35	·003208	·506248	35	·004767	·678233
36	·001977	·293045	36	·003232	·509409	36	·004796	·680837
37	001996	·300062	37	·003255	·512558	37	·004824	·683424
38	·002014	·304064	38	·003279	·515697	38	·004853	·686009
39	·002032	·308044	39	·003302	·518823	39	·004882	·688585
40	·002051	·312009	40	·003326	·521940	40	·004911	·691154
41	·002070	·315954	41	·003350	·525045	41	·004940	·693714
42	·002089	·319883	42	·003374	·528140	42	·004969	·696269
43	·002108	·323793	43	·003398	·531223	43	·004998	·698814
44	·002127	·327688	44	·003422	·534296	44	·005028	·701354
45	·002146	·331565	45	·003446	·537357	45	·005057	·703887
46	·002165	·335422	46	·003471	·540409	46	·005086	·706411
47	·002184	·339263	47	·003495	·543448	47	·005116	·708929
48	·002203	·343089	48	·003519	·546479	48	·005146	·711439
49	·002223	·346896	49	·003544	·549497	49	·005175	·713942
50	·002242	·350689	50	·003569	·552508	50	·005205	·716439
51	·002262	·354463	51	·003593	·555506	51	·005235	·718926
52	·002281	·358223	52	·003618	·558496	52	·005265	·721409
53	·002301	·361965	53	·003643	·561474	53	·005295	·723883
54	·002321	·365694	54	·003668	·564442	54	·005325	·726351
55	002341	·369406	55	·003693	·537401	55	·005356	·728812
56	002361	·373100	56	·003718	·570349	56	·005386	·731267
57	·002381	·376779	57	·003744	·573288	57	·005416	·733713
58	·002401	·380445	58	·003769	·576216	58	·005447	·736155
59	·002422	·384093	59	·003794	·579137	59	·005478	·738589
60	002442	·387728	60	·003820	·582045	60	·005508	·741016

6 DEGREES.			7 DEGREES.			8 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·005478	7·738630	0	0·007454	7·872380	0	0·009732	7·988199
1	·005509	·741088	1	·007489	·874444	1	·009772	·990003
2	·005539	·743488	2	·007525	·876502	2	·009808	·991804
3	·005570	·745831	3	·007561	·878555	3	·009854	·993601
4	·005600	·748220	4	·007596	·880603	4	·009894	·995396
5	·005631	·750601	5	·007632	·882647	5	·009935	·997185
6	·005662	·752974	6	·007668	·884686	6	·009976	·998972
7	·005693	·755342	7	·007704	·886719	7	·010017	8·000754
8	·005724	·757704	8	·007740	·888749	8	·010058	·002532
9	·005755	·760057	9	·007776	·890773	9	·010099	·004307
10	·005786	·762406	10	·007813	·892793	10	·010141	·006079
11	·005818	·764749	11	·007849	·894808	11	·010181	·007847
12	·005849	·767084	12	·007885	·896818	12	·010223	·009611
13	·005880	·769413	13	·007922	·898824	13	·010265	·011371
14	·005912	·771738	14	·007958	·900825	14	·010307	·013128
15	·005944	·774055	15	·007995	·902821	15	·010348	·014883
16	·005975	·776364	16	·008032	·904813	16	·010390	·016632
17	·006007	·778671	17	·008069	·906800	17	·010432	·018379
18	·006039	·780968	18	·008106	·908783	18	·010474	·020121
19	·006071	·783261	19	·008143	·910761	19	·010516	·021861
20	·006103	·785547	20	·008180	·912734	20	·010558	·023597
21	·006135	·787829	21	·008217	·914704	21	·010600	·025329
22	·006167	·790102	22	·008254	·916670	22	·010643	·027058
23	·006200	·792369	23	·008291	·918628	23	·010685	·028783
24	·006232	·794633	24	·008329	·920584	24	·010728	·030505
25	·006265	·796891	25	·008366	·922536	25	·010770	·032223
26	·006297	·799140	26	·008404	·924483	26	·010813	·033939
27	·006330	·801385	27	·008442	·926425	27	·010856	·035651
28	·006362	·803624	28	·008479	·928363	28	·010898	·037359
29	·006395	·805859	29	·008517	·930297	29	·010941	·039064
30	·006428	·808086	30	·008555	·932227	30	·010984	·040766
31	·006461	·810307	31	·008593	·934152	31	·011027	·042465
32	·006494	·812524	32	·008631	·936074	32	·011070	·044159
33	·006527	·814734	33	·008669	·937990	33	·011113	·045850
34	·006560	·816939	34	·008708	·939903	34	·011157	·047539
35	·006594	·819139	35	·008746	·941811	35	·011200	·049225
36	·006627	·821332	36	·008784	·943715	36	·011243	·050906
37	·006661	·823521	37	·008823	·945615	37	·011287	·052584
38	·006694	·825704	38	·008862	·947511	38	·011331	·054260
39	·006728	·827881	39	·008900	·949403	39	·011374	·055931
40	·006762	·830052	40	·008939	·951290	40	·011418	·057601
41	·006795	·832218	41	·008978	·953173	41	·011462	·059266
42	·006829	·834379	42	·009017	·955052	42	·011506	·060928
43	·006863	·836535	43	·009056	·956927	43	·011550	·062588
44	·006897	·838685	44	·009095	·958799	44	·011594	·064243
45	·006932	·840830	45	·009134	·960666	45	·011638	·065896
46	·006966	·842969	46	·009173	·962529	46	·011682	·067546
47	·007000	·845115	47	·009213	·964388	47	·011727	·069192
48	·007034	·847232	48	·009252	·966243	48	·011772	·070836
49	·007069	·849356	49	·009292	·968094	49	·011816	·072476
50	·007104	·851475	50	·009331	·969942	50	·011860	·074113
51	·007138	·853589	51	·009371	·971784	51	·011905	·075747
52	·007173	·855697	52	·009411	·973624	52	·011950	·077378
53	·007208	·857800	53	·009450	·975459	53	·011995	·079007
54	·007243	·859898	54	·009491	·977291	54	·012040	·080631
55	·007278	·861991	55	·009531	·979118	55	·012085	·082253
56	·007313	·864079	56	·009552	·980942	56	·912130	·083872
57	·007348	·866162	57	·009611	·982762	57	·012175	·085488
58	·007383	·868240	58	·009651	·984578	58	·012220	·087100
59	·007418	·870312	59	·009691	·986390	59	·012266	·088710
60	·007454	·872380	60	·009732	·988199	60	·012311	·090316

6 DEGREES.			7 DEGREES.			8 DEGREES.		
Min.	Nat. No	Logarithm.	Min	Nat. No.	Logarithm.	Min	Nat. No	Logarithm.
0	0·005508	7·741016	0	0·007510	7·875628	0	0·009828	7·992447
1	·005539	·743437	1	·007546	·877708	1	·009873	·994268
2	·005570	·745850	2	·007581	·879782	2	·009910	·996087
3	·005601	·748257	3	·007618	·881851	3	·009952	·997902
4	·005632	·750659	4	·007654	·883915	4	·009993	·999715
5	·005663	·753054	5	·007691	·885974	5	·010035	8·001521
6	·005694	·755440	6	·007727	·888029	6	·010077	·003326
7	·005726	·757822	7	·007764	·890078	7	·010119	·005126
8	·005757	·760197	8	·007801	·892124	8	·010160	·006921
9	·005788	·762564	9	·007837	·894163	9	·010203	·008716
10	·005820	·764926	10	·007874	·896199	10	·010245	·010505
11	·005852	·767288	11	·007911	·898230	11	·010287	·012292
12	·005883	·769632	12	·007948	·900256	12	·010329	·014074
13	·005915	·771974	13	·007985	·902278	13	·010372	·015852
14	·005947	·774313	14	·008022	·904295	14	·010414	·017627
15	·005979	·776644	15	·008059	·906307	15	·010457	·019401
16	·006011	·778967	16	·008097	·908315	16	·010499	·021148
17	·006043	·781288	17	·008134	·910318	17	·010542	·022933
18	·006076	·783599	18	·008172	·912318	18	·010585	·024694
19	·006108	·785906	19	·008209	·914312	19	·010628	·026452
20	·006141	·788206	20	·008247	·916301	20	·010671	·028207
21	·006173	·790502	21	·008285	·918287	21	·010714	·029957
22	·006206	·792789	22	·008323	·920270	22	·010757	·031705
23	·006238	·795070	23	·008361	·922244	23	·010800	·033449
24	·006271	·797348	24	·008399	·924216	24	·010844	·035189
25	·006304	·799620	25	·008437	·926185	25	·010887	·036926
26	·006337	·801883	26	·008475	·928148	26	·010931	·038661
27	·006370	·804143	27	·008513	·930107	27	·010975	·040391
28	·006403	·806396	28	·008552	·932061	28	·011018	·042118
29	·006436	·808645	29	·008590	·934012	29	·011062	·043842
30	·006470	·810887	30	·008629	·935958	30	·011106	·045563
31	·006503	·813122	31	·008668	·937900	31	·011150	·047281
32	·006537	·815354	32	·008706	·939839	32	·011194	·048994
33	·006570	·817578	33	·008745	·941771	33	·011238	·050704
34	·006604	·819798	34	·008784	·943701	34	·011282	·052412
35	·006638	·822012	35	·008823	·945626	35	·011326	·054117
36	·006671	·824220	36	·008862	·947547	36	·011371	·055817
37	·006705	·826423	37	·008901	·949464	37	·011416	·057514
38	·006739	·828621	38	·008941	·951377	38	·011461	·059209
39	·006773	·830813	39	·008980	·953286	39	·011506	·060899
40	·006808	·832998	40	·009020	·955190	40	·011550	·062588
41	·006842	·835179	41	·009059	·957090	41	·011595	·064273
42	·006876	·837355	42	·009099	·958986	42	·011640	·065954
43	·006911	·839526	43	·009139	·960878	43	·011685	·067633
44	·006945	·841691	44	·009178	·962767	44	·011730	·069308
45	·006980	·843851	45	·009218	·964651	45	·011776	·070980
46	·007015	·846005	46	·009258	·966531	46	·011821	·072650
47	·007049	·848155	47	·009298	·968408	47	·011866	·074315
48	·007084	·850298	48	·009339	·970280	48	·011912	·075979
49	·007119	·852437	49	·009379	·972148	49	·011957	·077638
50	·007154	·854571	50	·009419	·974013	50	·012003	·079295
51	·007189	·856700	51	·009460	·975873	51	·012049	·080949
52	·007225	·858823	52	·009500	·977730	52	·012095	·082599
53	·007260	·860942	53	·009541	·979583	53	·012140	·084248
54	·007295	·863055	54	·009581	·981432	54	·012187	·085892
55	·007331	·865163	55	·009622	·983277	55	·012232	·087534
56	·007367	·867267	56	·009663	·985119	56	·012279	·089172
57	·007402	·869365	57	·009704	·986956	57	·012325	·090808
58	·007438	·871458	58	·009745	·988790	58	·012372	·092440
59	·007474	·873546	59	·009786	·990619	59	·012418	·094060
60	·007510	·875628	60	·009828	·992446	60	·012465	·095696

9 DEGREES.			10 DEGREES.			11 DEGREES.		
Min	Nat No	Logarithm	Min.	Nat. No.	Logarithm.	Min.	Nat No	Logarithm.
0	0·012311	8·090816	0	0·015192	8·181622	0	0·018373	8·264176
1	·012357	·091920	1	·015242	·183065	1	·018428	·265486
2	·012403	·093521	2	·015293	·184505	2	·018484	·266796
3	·012448	·095119	3	·015343	·185943	3	·018541	·268133
4	·012494	·096714	4	·015395	·187378	4	·018596	·269407
5	·012540	·098306	5	·015446	·188811	5	·018651	·270711
6	·012586	·099894	6	·015497	·190242	6	·018707	·272012
7	·012632	·101480	7	·015548	·191671	7	·018762	·273298
8	·012678	·103064	8	·015599	·193097	8	·018819	·274608
9	·012724	·104644	9	·015650	·194520	9	·018876	·275904
10	·012770	·106221	10	·015701	·195942	10	·018932	·277197
11	·012817	·107796	11	·015752	·197361	11	·018988	·278487
12	·012864	·109367	12	·015804	·198778	12	·019045	·279777
13	·012910	·110936	13	·015856	·200192	13	·019101	·281065
14	·012957	·112501	14	·015908	·201604	14	·019158	·282350
15	·013003	·114063	15	·015959	·203014	15	·019215	·283634
16	·013050	·115625	16	·016011	·204421	16	·019272	·284915
17	·013097	·117182	17	·016063	·205826	17	·019328	·286194
18	·013144	·118737	18	·016115	·207229	18	·019385	·287474
19	·013191	·120288	19	·016167	·208639	19	·019442	·288749
20	·013238	·121838	20	·016219	·210028	20	·019499	·290023
21	·013286	·123384	21	·016271	·211424	21	·019557	·291296
22	·013333	·124927	22	·016323	·212827	22	·019614	·292566
23	·013380	·126468	23	·016376	·214219	23	·019671	·293835
24	·013428	·128006	24	·016428	·215598	24	·019729	·295101
25	·013475	·129542	25	·016481	·216986	25	·019786	·296366
26	·013523	·131074	26	·016533	·218371	26	·019844	·297629
27	·013570	·132603	27	·016586	·219753	27	·019902	·298889
28	·013618	·134131	28	·016639	·221134	28	·019959	·300149
29	·013666	·135635	29	·016692	·222502	29	·020017	·301406
30	·013714	·137176	30	·016745	·223887	30	·020075	·302661
31	·013762	·138695	31	·016798	·225261	31	·020133	·303916
32	·013810	·140212	32	·016851	·226633	32	·020191	·305167
33	·013859	·141726	33	·016904	·228002	33	·020250	·306417
34	·013907	·143236	34	·016958	·229370	34	·020308	·307666
35	·013955	·144745	35	·017011	·230734	35	·020366	·308912
36	·014003	·146251	36	·017065	·232097	36	·020425	·310156
37	·014052	·147754	37	·017118	·233458	37	·020483	·311399
38	·014101	·149255	38	·017171	·234817	38	·020541	·312680
39	·014149	·150752	39	·017225	·236173	39	·020600	·313880
40	·014198	·152248	40	·017279	·237528	40	·020659	·315117
41	·014247	·153741	41	·017333	·238880	41	·020718	·316352
42	·014296	·155231	42	·017387	·240230	42	·020777	·317587
43	·014345	·156719	43	·017441	·241578	43	·020836	·318818
44	·014394	·158203	44	·017496	·242924	44	·020895	·320049
45	·014443	·159686	45	·017550	·244267	45	·020954	·321278
46	·014493	·161165	46	·017604	·245609	46	·021014	·322505
47	·014542	·162643	47	·017658	·246949	47	·021073	·323730
48	·014592	·164118	48	·017712	·248286	48	·021133	·324953
49	·014641	·165589	49	·017767	·249621	49	·021192	·326174
50	·014691	·167060	50	·017822	·250955	50	·021252	·327395
51	·014741	·168527	51	·017877	·252286	51	·021311	·328613
52	·014791	·169992	52	·017931	·253615	52	·021371	·329829
53	·014841	·171454	53	·017986	·254942	53	·021431	·331044
54	·014891	·172914	54	·018041	·256267	54	·021491	·332256
55	·014941	·174372	55	·018096	·257591	55	·021551	·333469
56	·014991	·175827	56	·018151	·258911	56	·021611	·334678
57	·015041	·177279	57	·018206	·260230	57	·021671	·335885
58	·015091	·178789	58	·018262	·261548	58	·021732	·337092
59	·015141	·180177	59	·018317	·262862	59	·021792	·338296
60	·015192	·181622	60	·018373	·264176	60	·021852	·339499

9 DEGREES.			10 DEGREES.			11 DEGREES.		
Min	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm	Min	Nat. No.	Logarithm
0	0·012465	8·095696	0	0·015426	8·188271	0	0·018717	8·272229
1	·012512	·097320	1	·015478	·189732	1	·018774	·273564
2	·012559	·098941	2	·015530	·191193	2	·018832	·274899
3	·012605	·100559	3	·015583	·192653	3	·018891	·276260
4	·012652	·102174	4	·015636	·194116	4	·018948	·277559
5	·012699	·103787	5	·015688	·195571	5	·019006	·278888
6	·012746	·105395	6	·015740	·197025	6	·019064	·280213
7	·012794	·107001	7	·015793	·198476	7	·019122	·281527
8	·012841	·108605	8	·015846	·199925	8	·019180	·282859
9	·012889	·110206	9	·015899	·201371	9	·019239	·284180
10	·012936	·111804	10	·015952	·202815	10	·019297	·285494
11	·012984	·113399	11	·016005	·204257	11	·019356	·286813
12	·013031	·114990	12	·016058	·205697	12	·019415	·288128
13	·013079	·116579	13	·016111	·207133	13	·019473	·289441
14	·013127	·118165	14	·016164	·208568	14	·019532	·290751
15	·013175	·119749	15	·016218	·210001	15	·019591	·292060
16	·013223	·121330	16	·016271	·211431	16	·019650	·293366
17	·013271	·122908	17	·016325	·212859	17	·019709	·294670
18	·013319	·124483	18	·016379	·214285	18	·019769	·295976
19	·013367	·126055	19	·016433	·215718	19	·019828	·297276
20	·013416	·127626	20	·016486	·217130	20	·019887	·298575
21	·013464	·129193	21	·016540	·218549	21	·019947	·299874
22	·013513	·130756	22	·016594	·219965	22	·020006	·301169
23	·013561	·132318	23	·016649	·221380	23	·020066	·302463
24	·013610	·133877	24	·016703	·222792	24	·020126	·303755
25	·013659	·135434	25	·016757	·224203	25	·020186	·305045
26	·013708	·136987	26	·016811	·225612	26	·020246	·306334
27	·013757	·138537	27	·016866	·227017	27	·020306	·307619
28	·013806	·140086	28	·016920	·228421	28	·020366	·308905
29	·013856	·141631	29	·016975	·229922	29	·020426	·310188
30	·013905	·143174	30	·017030	·231221	30	·020486	·311469
31	·013954	·144714	31	·017085	·232618	31	·020547	·312749
32	·014004	·146252	32	·017140	·234014	32	·020608	·314026
33	·014054	·147787	33	·017195	·235406	33	·020668	·315302
34	·014103	·149318	34	·017250	·236798	34	·020729	·316576
35	·014153	·150849	35	·017305	·238185	35	·020790	·317848
36	·014203	·152376	36	·017360	·239572	36	·020851	·319118
37	·014253	·153900	37	·017416	·240957	37	·020912	·320387
38	·014303	·155423	38	·017472	·242339	38	·020973	·321644
39	·014353	·156941	39	·017527	·243719	39	·021034	·322920
40	·014403	·158458	40	·017583	·245098	40	·021095	·324183
41	·014453	·159973	41	·017638	·246474	41	·021156	·325444
42	·014503	·161485	42	·017695	·247848	42	·021218	·326705
43	·014554	·162994	43	·017751	·249219	43	·021280	·327963
44	·014605	·164500	44	·017807	·250589	44	·021341	·329220
45	·014655	·166004	45	·017863	·251956	45	·021403	·330475
46	·014706	·167505	46	·017919	·253322	46	·021465	·331728
47	·014757	·169005	47	·017975	·254686	47	·021527	·332979
48	·014808	·170502	48	·018032	·256047	48	·021589	·334229
49	·014859	·171995	49	·018089	·257407	49	·021651	·335477
50	·014909	·173488	50	·018145	·258765	50	·021713	·336724
51	·014961	·174977	51	·018202	·260120	51	·021776	·337969
52	·015013	·176464	52	·018258	·261473	52	·021838	·339211
53	·015064	·177948	53	·018315	·262825	53	·021900	·340453
54	·015115	·179430	54	·018372	·264174	54	·021963	·341691
55	·015167	·180910	55	·018430	·265522	55	·022026	·342931
56	·015218	·182387	56	·018487	·266867	56	·022089	·344167
57	·015270	·183861	57	·018544	·268210	57	·022151	·345400
58	·015322	·185343	58	·018601	·269552	58	·022214	·346634
59	·015374	·186808	59	·018659	·270891	59	·022277	·347865
60	·015426	·188271	60	·018717	·272229	60	·022341	·349095

12 DEGREES.			13 DEGREES.			14 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·021852	8·339499	0	0·025630	8·408747	0	0·029704	8·472819
1	·021913	·340700	1	·025695	·409856	1	·029775	·473843
2	·021974	·341900	2	·025761	·410962	2	·029845	·474874
3	·022034	·343097	3	·025827	·412067	3	·029916	·475899
4	·022095	·344293	4	·025892	·413171	4	·029986	·476925
5	·022156	·345488	5	·025958	·414273	5	·030057	·477948
6	·022217	·346681	6	·026024	·415374	6	·030127	·478970
7	·022278	·347877	7	·026090	·416474	7	·030199	·479991
8	·022338	·349062	8	·026156	·417573	8	·030270	·481011
9	·022400	·350249	9	·026222	·418669	9	·030341	·482029
10	·022461	·351435	10	·026288	·419764	10	·030412	·483046
11	·022523	·352620	11	·026355	·420858	11	·030483	·484062
12	·022584	·353802	12	·026421	·421951	12	·030555	·485078
13	·022646	·354984	13	·026488	·423042	13	·030626	·486091
14	·022707	·356163	14	·026554	·424131	14	·030697	·487103
15	·022769	·357342	15	·026621	·425219	15	·030769	·488115
16	·022831	·358518	16	·026687	·426309	16	·030841	·489125
17	·022892	·359693	17	·026754	·427393	17	·030912	·490133
18	·022954	·360867	18	·026821	·428477	18	·030984	·491141
19	·023016	·362039	19	·026888	·429560	19	·031056	·492148
20	·023079	·363208	20	·026955	·430641	20	·031128	·493153
21	·023141	·364376	21	·027022	·431722	21	·031200	·494157
22	·023203	·365543	22	·027089	·432800	22	·031272	·495160
23	·023266	·366719	23	·027157	·433877	23	·031345	·496162
24	·023328	·367872	24	·027224	·434954	24	·031417	·497162
25	·023390	·369035	25	·027292	·436029	25	·031489	·498162
26	·023453	·370195	26	·027359	·437102	26	·031562	·499160
27	·023515	·371354	27	·027427	·438174	27	·031634	·500158
28	·023578	·372511	28	·027494	·439244	28	·031707	·501153
29	·023641	·373667	29	·027562	·440314	29	·031780	·502148
30	·023704	·374822	30	·027630	·441382	30	·031852	·503142
31	·023767	·375974	31	·027698	·442449	31	·031925	·504134
32	·023830	·377125	32	·027766	·443514	32	·031998	·505125
33	·023893	·378275	33	·027834	·444578	33	·032071	·506115
34	·023956	·379423	34	·027902	·445641	34	·032144	·507105
35	·024020	·380569	35	·027971	·446702	35	·032218	·508093
36	·024083	·381715	36	·028039	·447762	36	·032291	·509079
37	·024147	·382858	37	·028107	·448821	37	·032364	·510065
38	·024210	·384001	38	·028176	·449878	38	·032438	·511049
39	·024273	·385141	39	·028245	·450935	39	·032511	·512032
40	·024338	·386279	40	·028313	·451990	40	·032585	·513014
41	·024402	·387417	41	·028382	·453043	41	·032659	·513996
42	·024465	·388553	42	·028451	·454096	42	·032732	·514976
43	·024529	·389687	43	·028520	·455147	43	·032806	·515955
44	·024594	·390821	44	·028589	·456196	44	·032880	·516933
45	·024658	·391952	45	·028658	·457244	45	·032954	·517909
46	·024722	·393082	46	·028727	·458291	46	·033028	·518884
47	·024786	·394210	47	·028796	·459338	47	·033102	·519858
48	·024851	·395338	48	·028866	·460382	48	·033177	·520832
49	·024915	·396463	49	·028935	·461425	49	·033251	·521804
50	·024980	·397587	50	·029005	·462468	50	·033325	·522775
51	·025044	·398710	51	·029074	·463508	51	·033400	·523745
52	·025109	·399831	52	·029144	·464547	52	·033475	·524714
53	·025174	·400951	53	·029214	·465586	53	·033549	·525681
54	·025239	·402069	54	·029283	·466623	54	·033624	·526648
55	·025304	·403185	55	·029353	·467659	55	·033699	·527614
56	·025369	·404300	56	·029423	·468693	56	·033774	·528578
57	·025434	·405414	57	·029493	·469726	57	·033849	·529541
58	·025499	·406527	58	·029564	·470759	58	·033924	·530504
59	·025564	·407637	59	·029634	·471789	59	·033999	·531465
60	·025630	·408747	60	·029704	·472819	60	·034074	·532425

	12 DEGREES.			13 DEGREES.			14 DEGREES.	
Min	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·022341	8·349095	0	0·026304	8·420023	0	0·030614	8·485915
1	·022404	·350322	1	·026373	·421161	1	·030688	·486975
2	·022467	·351540	2	·026442	·422296	2	·030763	·488033
3	·022531	·352778	3	·026511	·423431	3	·030838	·489090
4	·022594	·353996	4	·026581	·424564	4	·030913	·490147
5	·022658	·355213	5	·026650	·425695	5	·030988	·491202
6	·022722	·356438	6	·026720	·426826	6	·031064	·492256
7	·022786	·357662	7	·026789	·427955	7	·031139	·493308
8	·022849	·358874	8	·026859	·429084	8	·031215	·494360
9	·022913	·360088	9	·026928	·430209	9	·031290	·495410
10	·022977	·361301	10	·026998	·431334	10	·031366	·496459
11	·023042	·362513	11	·027068	·432457	11	·031442	·497507
12	·023106	·363723	12	·027138	·433580	12	·031518	·498555
13	·023170	·364932	13	·027208	·434700	13	·031594	·499600
14	·023235	·366138	14	·027278	·435819	14	·031670	·500644
15	·023299	·367345	15	·027349	·436937	15	·031746	·501688
16	·023364	·368548	16	·027419	·438055	16	·031822	·502730
17	·023429	·369751	17	·027490	·439170	17	·031899	·503770
18	·023994	·370952	18	·027560	·440284	18	·031975	·504810
19	·023559	·372152	19	·027631	·441397	19	·032052	·505849
20	·023624	·373348	20	·027702	·442508	20	·032128	·506887
21	·023689	·374544	21	·027773	·443619	21	·032205	·507923
22	·023754	·375739	22	·027844	·444727	22	·032282	·508958
23	·023820	·376942	23	·027915	·445834	23	·032359	·509993
24	·023885	·378123	24	·027986	·446941	24	·032436	·511025
25	·023950	·379314	25	·028057	·448046	25	·032513	·512058
26	·024016	·380502	26	·028129	·449149	26	·032590	·513088
27	·024082	·381689	27	·028200	·450252	27	·032668	·514119
28	·024148	·382874	28	·028272	·451352	28	·032745	·515146
29	·024214	·384058	29	·028343	·452452	29	·032823	·516174
30	·024280	·385240	30	·028415	·453551	30	·032900	·517200
31	·024346	·386421	31	·028487	·454648	31	·032978	·518225
32	·024412	·387600	32	·028559	·455743	32	·033056	·519249
33	·024478	·388778	33	·028631	·456838	33	·033134	·520272
34	·024544	·389954	34	·028703	·457931	34	·033212	·521294
35	·024611	·391128	35	·028775	·459023	35	·033290	·522315
36	·024678	·392302	36	·028248	·460113	36	·033368	·523334
37	·024744	·393474	37	·028920	·461203	37	·033447	·524353
38	·024811	·394645	38	·028993	·462290	38	·033525	·525370
39	·024878	·395813	39	·029065	·463378	39	·033604	·526386
40	·024945	·396979	40	·029138	·464464	40	·033682	·527401
41	·025012	·398146	41	·029211	·465547	41	·033761	·528416
42	·025079	·399310	42	·029284	·466631	42	·033840	·529429
43	·025146	·400473	43	·029357	·467713	43	·033919	·530441
44	·025214	·401635	44	·029430	·468793	44	·033998	·531453
45	·025281	·402795	45	·029503	·469872	45	·034077	·532463
46	·025348	·403954	46	·029577	·470950	46	·034156	·533470
47	·025416	·405110	47	·029650	·472028	47	·034236	·534478
48	·025484	·406267	48	·029724	·473103	48	·034315	·535485
49	·025552	·407421	49	·029797	·474177	49	·034395	·536490
50	·025620	·408573	50	·029871	·475251	50	·034474	·537495
51	·025688	·409725	51	·029945	·476322	51	·034554	·538498
52	·025756	·410875	52	·030019	·477392	52	·034634	·539501
53	·025824	·412024	53	·030093	·478462	53	·034714	·540501
54	·025892	·413171	54	·030167	·479531	54	·034794	·541502
55	·025961	·414316	55	·030241	·480598	55	·034874	·542501
56	·026029	·415460	56	·030315	·481663	56	·034954	·543499
57	·026098	·416603	57	·030390	·482728	57	·035035	·544496
58	·026166	·417745	58	·030464	·483792	58	·035115	·545493
59	·026235	·418884	59	·030539	·484853	59	·035195	·546487
60	·026304	·420023	60	·030614	·485915	60	·035276	·547481

15 DEGREES.			16 DEGREES.			17 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·034074	8·532425	0	0·038738	8·588140	0	0·043695	8·640434
1	·034150	·533384	1	·038818	·589038	1	·043780	·641279
2	·034225	·534342	2	·038899	·589936	2	·043865	·642123
3	·034300	·535299	3	·038979	·590833	3	·043951	·642966
4	·034376	·536255	4	·039060	·591728	4	·044036	·643809
5	·034452	·537210	5	·039140	·592623	5	·044121	·644650
6	·034527	·538163	6	·039221	·593517	6	·044207	·645491
7	·034603	·539116	7	·039301	·594409	7	·044292	·646330
8	·034679	·540068	8	·039382	·595301	8	·044378	·647169
9	·034755	·541018	9	·039463	·596193	9	·044464	·648008
10	·034831	·541968	10	·039544	·597083	10	·044550	·648845
11	·034907	·542916	11	·039625	·597971	11	·044636	·649682
12	·034983	·543863	12	·039706	·598859	12	·044722	·650517
13	·035060	·544809	13	·039787	·599747	13	·044808	·651352
14	·035136	·545755	14	·039869	·600633	14	·044894	·652187
15	·035213	·546699	15	·039950	·601518	15	·044980	·653020
16	·035289	·547642	16	·040032	·602403	16	·045066	·653852
17	·035366	·548584	17	·040113	·603286	17	·045153	·654683
18	·035443	·549525	18	·040195	·604169	18	·045239	·655514
19	·035520	·550466	19	·040276	·605051	19	·045326	·656345
20	·035596	·551405	20	·040358	·605931	20	·045412	·657174
21	·035673	·552342	21	·040440	·606811	21	·045499	·658002
22	·035750	·553279	22	·040522	·607690	22	·045586	·658830
23	·035827	·554215	23	·040604	·608568	23	·045673	·659657
24	·035905	·555150	24	·040686	·609445	24	·045760	·660485
25	·035982	·556084	25	·040768	·610322	25	·045847	·661308
26	·036059	·557017	26	·040850	·611196	26	·045934	·662132
27	·036137	·557948	27	·040933	·612071	27	·046021	·662956
28	·036214	·558880	28	·041015	·612945	28	·046109	·663779
29	·036292	·559809	29	·041098	·613817	29	·046196	·664601
30	·036370	·560738	30	·041180	·614689	30	·046283	·665422
31	·036447	·561665	31	·041263	·615560	31	·046370	·666242
32	·036525	·562592	32	·041346	·616430	32	·046458	·667061
33	·036603	·563518	33	·041428	·617299	33	·046546	·667881
34	·036681	·564443	34	·041511	·618167	34	·046633	·668698
35	·036759	·565366	35	·041594	·619035	35	·046721	·669516
36	·036837	·566289	36	·041678	·619901	36	·046809	·670333
37	·036916	·567211	37	·041761	·620766	37	·046897	·671148
38	·036994	·568132	38	·041844	·621631	38	·046985	·671963
39	·037072	·569052	39	·041927	·622495	39	·047073	·672776
40	·037151	·569971	40	·042010	·623357	40	·047162	·673590
41	·037230	·570888	41	·042094	·624220	41	·047250	·674403
42	·037308	·571805	42	·042178	·625081	42	·047339	·675215
43	·037387	·572721	43	·042261	·625941	43	·047427	·676025
44	·037466	·573635	44	·042345	·626800	44	·047516	·676836
45	·037545	·574550	45	·042429	·627659	45	·047604	·677646
46	·037624	·575462	46	·042513	·628517	46	·047693	·678454
47	·037703	·576374	47	·042597	·629373	47	·047782	·679262
48	·037782	·577285	48	·042681	·630230	48	·047871	·680069
49	·037861	·578194	49	·042765	·631085	49	·047960	·680875
50	·037940	·579103	50	·042849	·631939	50	·048049	·681681
51	·038020	·580012	51	·042933	·632792	51	·048138	·682485
52	·038100	·580919	52	·043017	·633645	52	·048227	·683290
53	·038179	·581825	53	·043102	·634496	53	·048316	·684093
54	·038259	·582730	54	·043186	·635347	54	·048406	·684896
55	·038338	·583634	55	·043271	·636197	55	·048495	·685698
56	·038418	·584537	56	·043356	·637046	56	·048584	·686498
57	·038498	·585439	57	·043441	·637895	57	·048674	·687298
58	·038578	·586341	58	·043525	·638742	58	·048764	·688098
59	·038658	·587241	59	·043610	·639588	59	·048854	·688896
60	·038738	·588140	60	·043695	·640434	60	·048944	·689694

15 DEGREES.			16 DEGREES.			17 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·035276	8·547481	0	0·040299	8·605299	0	0·045692	8·659838
1	·035357	·548473	1	·040386	·606238	1	·045785	·660721
2	·035438	·549466	2	·040473	·607167	2	·045878	·661604
3	·035519	·550457	3	·040560	·608100	3	·045971	·662485
4	·035600	·551447	4	·040647	·609032	4	·046065	·663867
5	·035681	·552436	5	·040735	·609963	5	·046158	·664247
6	·035762	·553423	6	·040822	·610893	6	·046252	·665127
7	·035843	·554400	7	·040909	·611822	7	·046345	·666005
8	·035925	·555396	8	·040997	·612750	8	·046439	·666883
9	·036006	·556381	9	·041085	·613679	9	·046533	·667761
10	·036088	·557364	10	·041172	·614605	10	·046627	·668637
11	·036170	·558347	11	·041260	·615530	11	·046721	·669513
12	·036252	·559328	12	·041348	·616455	12	·046815	·670387
13	·036334	·560309	13	·041437	·617380	13	·046910	·671261
14	·036416	·561289	14	·041524	·618302	14	·047004	·672135
15	·036498	·562267	15	·041613	·619224	15	·047099	·673008
16	·036580	·563245	16	·041701	·620146	16	·047193	·673879
17	·036662	·564221	17	·041789	·621066	17	·047288	·674749
18	·036745	·565197	18	·041878	·621982	18	·047383	·675619
19	·036828	·566172	19	·041967	·622905	19	·047478	·676490
20	·036910	·567146	20	·042055	·623822	20	·047573	·677358
21	·036993	·568118	21	·042144	·624739	21	·047668	·678226
22	·037076	·569089	22	·042233	·625655	22	·047763	·679093
23	·037159	·570060	23	·042322	·626570	23	·047859	·679960
24	·037242	·571030	24	·042412	·627484	24	·047954	·680827
25	·037325	·571999	25	·042501	·628398	25	·048050	·681690
26	·037408	·572967	26	·042590	·629310	26	·048145	·682553
27	·037492	·573937	27	·042680	·630222	27	·048241	·683417
28	·037575	·574899	28	·042770	·631133	28	·048337	·684280
29	·037658	·575863	29	·042859	·632043	29	·048433	·685142
30	·037742	·576827	30	·042949	·632952	30	·048529	·686002
31	·037826	·577790	31	·043039	·633861	31	·048625	·686862
32	·037910	·578752	32	·043129	·634768	32	·048722	·687721
33	·037994	·579713	33	·043219	·635674	33	·048818	·688581
34	·038078	·580673	34	·043309	·636580	34	·048915	·689438
35	·038162	·581631	35	·043400	·637486	35	·049011	·690296
36	·038246	·582589	36	·043490	·638389	36	·049108	·691153
37	·038331	·583547	37	·043580	·639292	37	·049205	·692008
38	·038415	·584503	38	·043671	·640195	38	·049302	·692863
39	·038500	·585458	39	·043762	·641096	39	·049399	·693717
40	·038585	·586412	40	·043853	·641997	40	·049296	·694571
41	·038669	·587365	41	·043943	·642890	41	·049593	·695424
42	·038754	·588318	42	·044035	·643796	42	·049691	·696276
43	·038839	·589269	43	·044126	·644694	43	·049788	·697127
44	·038924	·590219	44	·044217	·645591	44	·049886	·697978
45	·039009	·591169	45	·044309	·646488	45	·049983	·698829
46	·039095	·592117	46	·044400	·647384	46	·050081	·699677
47	·039181	·593075	47	·044491	·648272	47	·050179	·700526
48	·039266	·594012	48	·044583	·649173	48	·050277	·701373
49	·039351	·594958	49	·044676	·650076	49	·050376	·702220
50	·039437	·595902	50	·044767	·650958	50	·040474	·703066
51	·039523	·596846	51	·044859	·651850	51	·050572	·703911
52	·039608	·597789	52	·044951	·652741	52	·050671	·704757
53	·039695	·598731	53	·045043	·653630	53	·050769	·705600
54	·039781	·599672	54	·045136	·654520	54	·050868	·706444
55	·039867	·600612	55	·045228	·655408	55	·050967	·707287
56	·039953	·601551	56	·045321	·656296	56	·051066	·708128
57	·040040	·602489	57	·045413	·657183	57	·051165	·708969
58	·040126	·603427	58	·045506	·658069	58	·051264	·709810
59	·040213	·604263	59	·045599	·658953	59	·051363	·710649
60	·040299	·605299	60	·045692	·659838	60	·051462	·711489

18 DEGREES.			19 DEGREES.			20 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·048944	8·689694	0	0·054481	8·736248	0	0·060308	8·780371
1	·049033	·690492	1	·054576	·737003	1	·060407	·781087
2	·049123	·691288	2	·054671	·737758	2	·060507	·781802
3	·049213	·692084	3	·054766	·738510	3	·060606	·782517
4	·049304	·692880	4	·054861	·739263	4	·060706	·783232
5	·049394	·693674	5	·054956	·740014	5	·060806	·783945
6	·049484	·694467	6	·055051	·740766	6	·060906	·784659
7	·049575	·695260	7	·055146	·741517	7	·061006	·785371
8	·049665	·696052	8	·055242	·742266	8	·061106	·786082
9	·049756	·696843	9	·055337	·743015	9	·061206	·786794
10	·049846	·697633	10	·055432	·743764	10	·061306	·787505
11	·049937	·698423	11	·055527	·744512	11	·061407	·788215
12	·050028	·699213	12	·055623	·745259	12	·061507	·788924
13	·050119	·700001	13	·055719	·746005	13	·061608	·789634
14	·050210	·700788	14	·055815	·746752	14	·061708	·790341
15	·050301	·701576	15	·055911	·747497	15	·061809	·791049
16	·050392	·702361	16	·056007	·748241	16	·061909	·791756
17	·050483	·703147	17	·056103	·748945	17	·062010	·792463
18	·050574	·703931	18	·056199	·749728	18	·062111	·793169
19	·050666	·704716	19	·056295	·750472	19	·062212	·793874
20	·050757	·705498	20	·056391	·751213	20	·062313	·794579
21	·050849	·706282	21	·056488	·751955	21	·062414	·795283
22	·050941	·707063	22	·056584	·752696	22	·062515	·795987
23	·051032	·707845	23	·056681	·753436	23	·062617	·796690
24	·051124	·708624	24	·056777	·754175	24	·062718	·797393
25	·051216	·709404	25	·056874	·754913	25	·062820	·798094
26	·051308	·710183	26	·056971	·755652	26	·062921	·798795
27	·051400	·710961	27	·057068	·756390	27	·063023	·799496
28	·051492	·711739	28	·057164	·757126	28	·063124	·800196
29	·051584	·712516	29	·057261	·757862	29	·063226	·800895
30	·051676	·713291	30	·057358	·758597	30	·063328	·801594
31	·051769	·714067	31	·057456	·759333	31	·063430	·802293
32	·051861	·714842	32	·057553	·760067	32	·063532	·802990
33	·051953	·715615	33	·057650	·760800	33	·063634	·803688
34	·052046	·716389	34	·057748	·761534	34	·063736	·804384
35	·052139	·717162	35	·057845	·762266	35	·063838	·805080
36	·052232	·717933	36	·057942	·762992	36	·063940	·805775
37	·052324	·718704	37	·058040	763728	37	·064043	·806470
38	·052417	·719474	38	·058138	·764459	38	·064145	·807165
39	·052510	·720244	39	·058236	·765189	39	·064248	·807858
40	·052603	·721013	40	·058334	·765918	40	·064350	·808551
41	·052697	·721782	41	·058432	·766647	41	·064453	·809244
42	·052790	·722549	42	·058530	·767375	42	·064556	·809937
43	·052883	·723316	43	·058628	·768101	43	·064659	·810628
44	·052976	·724082	44	·058726	·768829	44	·064762	·811319
45	·053070	·724848	45	·058824	·769555	45	·064865	·812009
46	·053163	·725613	46	·058922	·770279	46	·064968	·812693
47	·053257	·726377	47	·059021	·771005	47	·065071	·813387
48	·053351	·727140	48	·059119	771720	48	·065174	·814076
49	·053444	·727903	49	·059218	772453	49	·065278	·814765
50	·053538	·728666	50	·059316	773175	50	·065381	·815452
51	·053632	·729427	51	·059415	·773897	51	·065485	·816139
52	·053726	·730188	52	·059514	·774619	52	·065588	·816825
53	·053820	·730947	53	·059613	·775340	53	·065692	·817511
54	·053915	·731706	54	·059712	·776060	54	·065796	·818196
55	·054009	·732465	55	·059811	776780	55	·065899	·818881
56	·054103	·733224	56	·059910	777500	56	·066003	·819565
57	·054198	·733981	57	·060009	·778218	57	·066107	·820249
58	·054292	·734737	58	·060109	·778936	58	·066211	·820931
59	·054387	·735493	59	·060208	·779653	59	·066315	·821614
60	·054481	·736248	60	·060308	·780371	60	·066420	·822296

18 DEGREES.			19 DEGREES.			20 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·051462	8·711489	0	0·057621	8·760578	0	0·064178	8·807385
1	·051562	·712327	1	·057727	·761376	1	·064290	·808147
2	·051661	·713164	2	·057833	·762174	2	·064403	·808908
3	·051761	·714001	3	·057939	·762971	3	·064511	·809669
4	·051861	·714838	4	·058045	·763767	4	·064629	·810430
5	·051960	·715673	5	·058152	·764562	5	·064743	·811190
6	·052060	·716508	6	·058258	·765358	6	·064856	·811950
7	·052161	·717342	7	·058365	·766152	7	·064969	·812708
8	·052261	·718175	8	·058472	·766945	8	·065083	·813465
9	·052361	·719008	9	·058579	·767738	9	·065197	·814224
10	·052461	·719839	10	·058686	·768531	10	·065310	·814981
11	·052562	·720671	11	·058793	·769323	11	·065424	·815737
12	·052663	·721502	12	·058900	·770114	12	·065538	·816493
13	·052763	·722332	13	·059007	·770904	13	·065652	·817249
14	·052864	·723160	14	·059115	·771695	14	·065766	·818003
15	·052965	·723990	15	·059222	·772484	15	·065881	·818758
16	·053066	·724817	16	·059330	·773272	16	·065995	·819511
17	·053167	·725644	17	·059438	·774060	17	·066110	·820265
18	·053268	·726470	18	·059545	·774848	18	·066224	·821018
19	·053370	·727297	19	·059654	·775636	19	·066339	·821769
20	·053471	·728122	20	·059762	·776421	20	·066454	·822521
21	·053573	·728947	21	·059870	·777207	21	·066569	·823272
22	·053675	·729770	22	·059978	·777993	22	·066684	·824023
23	·053777	·730594	23	·060087	·778777	23	·066800	·824773
24	·053879	·731415	24	·060195	·779561	24	·066915	·825523
25	·053981	·732237	25	·060304	·780343	25	·067030	·826271
26	·054083	·733058	26	·060412	·781127	26	·067146	·827019
27	·054185	·733878	27	·060521	·781909	27	·067262	·827767
28	·054287	·734698	28	·060630	·782690	28	·067377	·828514
29	·054390	·735517	29	·060740	·783471	29	·067493	·829260
30	·054492	·736335	30	·060849	·784252	30	·067609	·830007
31	·054595	·737153	31	·060958	·785031	31	·067726	·830753
32	·054698	·737970	32	·061068	·785810	32	·067842	·831497
33	·054801	·738785	33	·061177	·786588	33	·067958	·832242
34	·054904	·739602	34	·061287	·787367	34	·068075	·832986
35	·055007	·740417	35	·061397	·788144	35	·068191	·833729
36	·055110	·741231	36	·061506	·788915	36	·068308	·834472
37	·055213	·742044	37	·061616	·789696	37	·068425	·835214
38	·055317	·742857	38	·061726	·790472	38	·068542	·835957
39	·055420	·743670	39	·061837	·791247	39	·068659	·836697
40	·055524	·744482	40	·061947	·792021	40	·068775	·837439
41	·055628	·745293	41	·062058	·792795	41	·068893	·838178
42	·055732	·746103	42	·062168	·793568	42	·069011	·838919
43	·055836	·746912	43	·062279	·794340	43	·069129	·839658
44	·055940	·747721	44	·062390	·795113	44	·069247	·840397
45	·056044	·748530	45	·062501	·795884	45	·069364	·841135
46	·056148	·749338	46	·062612	·796654	46	·069482	·841871
47	·056253	·750145	47	·062723	·797424	47	·069600	·842608
48	·056357	·750951	48	·062834	·798197	48	·069718	·843345
49	·056462	·751757	49	·062945	·798964	49	·069836	·844082
50	·056567	·752563	50	·063057	·799731	50	·069955	·844817
51	·056672	·753367	51	·063168	·800499	51	·070073	·845553
52	·056777	·754171	52	·063280	·801267	52	·070192	·846287
53	·056882	·754973	53	·063392	·802033	53	·070311	·847021
54	·056987	·755776	54	·063504	·802799	54	·070430	·847754
55	·057092	·756578	55	·063616	·803565	55	·070549	·848487
56	·057198	·757380	56	·063728	·804331	56	·070668	·849220
57	·057304	·758181	57	·063840	·805094	57	·070787	·849952
58	·057409	·758980	58	·063953	·805858	58	·070906	·850682
59	·057515	·759779	59	·064065	·806621	59	·071025	·851414
60	·057621	·760578	60	·064178	·807385	60	·071145	·852144

21 DEGREES.		
Min.	Nat. No.	Logarithm.
0	0·066420	8·822296
1	·066524	·822977
2	·066628	·823658
3	·066733	·824338
4	·066837	·825018
5	·066942	·825697
6	·067047	·826376
7	·067151	·827054
8	·067256	·827731
9	·067361	·828409
10	·067466	·829085
11	·067571	·829760
12	·067676	·830436
13	·067781	·831110
14	·067887	·831785
15	·067992	·832459
16	·068097	·833131
17	·068203	·833804
18	·068309	·834476
19	·068415	·835148
20	·068520	·835819
21	·068626	·836489
22	·068732	·837159
23	·068838	·837829
24	·068944	·838497
25	·069050	·839165
26	·069157	·839833
27	·069263	·840501
28	·069369	·841167
29	·069476	·841834
30	·069582	·842499
31	·069689	·843165
32	·069796	·843829
33	·069903	·844493
34	·070010	·845157
35	·070117	·845820
36	·070224	·846483
37	·070331	·847145
38	·070438	·847805
39	·070545	·848467
40	·070653	·849127
41	·070760	·849787
42	·070867	·850446
43	·070975	·851106
44	·071083	·851764
45	·071190	·852422
46	·071298	·853079
47	·071406	·853735
48	·071514	·854391
49	·071622	·855048
50	·071730	·855703
51	·071839	·856358
52	·071947	·857012
53	·072055	·857665
54	·072164	·858319
55	·072272	·858972
56	·072381	·859624
57	·072490	·860275
58	·072598	·860927
59	·072707	·861578
60	·072816	·862227

22 DEGREES.		
Min.	Nat. No.	Logarithm.
0	0·072816	8·862227
1	·072925	·862877
2	·073034	·863526
3	·073143	·864175
4	·073253	·864823
5	·073362	·865471
6	·073471	·866118
7	·073581	·866765
8	·073690	·867411
9	·073800	·868056
10	·073910	·868701
11	·074020	·869346
12	·074130	·869991
13	·074239	·870634
14	·074349	·871277
15	·074460	·871920
16	·074570	·872562
17	·074680	·873203
18	·074790	·873845
19	·074901	·874486
20	·075011	·875126
21	·075122	·875766
22	·075232	·876405
23	·075343	·877044
24	·075454	·877682
25	·075565	·878320
26	·075676	·878957
27	·075787	·879594
28	·075898	·880230
29	·076009	·880866
30	·076121	·881502
31	·076232	·882137
32	·076343	·882770
33	·076455	·883405
34	·076566	·884038
35	·076678	·884670
36	·076790	·885303
37	·076902	·885935
38	·077014	·886567
39	·077125	·887197
40	·077237	·887828
41	·077350	·888458
42	·077562	·889088
43	·077574	·889717
44	·077687	·890346
45	·077799	·890974
46	·077912	·891602
47	·078024	·892229
48	·078137	·892856
49	·078250	·893482
50	·078363	·894108
51	·078476	·894784
52	·078589	·895358
53	·078702	·895983
54	·078815	·896607
55	·078928	·897230
56	·079041	·897853
57	·079154	·898475
58	·079268	·899097
59	·079382	·899719
60	·079495	·900340

23 DEGREES.		
Min.	Nat. No.	Logarithm
0	0·079498	8·900340
1	·079609	·900962
2	·079723	·901582
3	·079837	·902201
4	·079951	·902821
5	·080064	·903439
6	·080178	·904057
7	·080293	·904675
8	·080407	·905293
9	·080521	·905910
10	·080636	·906527
11	·080750	·907143
12	·080865	·907758
13	·080979	·908374
14	·081094	·908988
15	·081209	·909603
16	·081324	·910216
17	·081429	·910830
18	·081554	·911443
19	·081669	·912056
20	·081784	·912668
21	·081899	·913279
22	·082014	·913890
23	·082130	·914501
24	·082245	·915111
25	·082361	·915721
26	·082476	·916331
27	·082592	·916939
28	·082708	·917548
29	·082824	·918156
30	·082940	·918764
31	·083056	·919371
32	·083172	·919977
33	·083288	·920583
34	·083404	·921189
35	·083521	·921795
36	·083637	·922400
37	·083754	·923004
38	·083871	·923609
39	·083987	·924212
40	·084104	·924815
41	·084221	·925418
42	·084337	·926020
43	·084454	·926623
44	·084572	·927224
45	·084689	·927824
46	·084806	·928425
47	·084923	·929025
48	·085040	·929625
49	·085158	·930224
50	·085275	·930823
51	·085393	·931422
52	·085510	·932019
53	·085628	·932617
54	·085746	·933214
55	·085864	·933811
56	·085982	·934407
57	·086100	·935003
58	·086218	·935598
59	·086336	·936193
60	·086454	·936787

21 DEGREES.			22 DEGREES.			23 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·071145	8·852144	0	0·078535	8·895061	0	0·086360	8·936314
1	·071265	·852874	1	·078662	·895762	1	·086495	·936990
2	·071384	·853603	2	·078789	·896462	2	·086629	·937663
3	·071504	·854362	3	·078916	·897162	3	·086763	·938336
4	·071624	·855061	4	·079043	·897862	4	·086898	·939010
5	·071744	·855788	5	·079170	·898561	5	·087033	·939682
6	·071865	·856516	6	·079297	·899259	6	·087167	·940353
7	·071985	·857243	7	·079425	·899957	7	·087302	·941025
8	·072105	·857969	8	·079553	·900655	8	·087437	·941697
9	·072226	·858695	9	·079680	·901351	9	·087573	·942368
10	·072347	·859420	10	·079808	·902048	10	·087708	·943040
11	·072468	·860144	11	·079936	·902744	11	·087843	·943709
12	·072589	·860869	12	·080065	·903441	12	·087979	·944379
13	·072710	·861592	13	·080193	·904135	13	·088115	·945049
14	·072831	·862316	14	·080321	·904830	14	·088251	·945717
15	·072952	·863039	15	·080450	·905525	15	·088387	·946386
16	·073074	·863761	16	·080578	·906218	16	·088522	·947053
17	·073195	·864483	17	·080707	·906911	17	·088659	·947722
18	·073317	·865204	18	·080836	·907605	18	·088795	·948389
19	·073439	·865925	19	·080965	·908298	19	·088932	·949057
20	·073561	·866646	20	·081094	·908990	20	·089068	·949723
21	·073683	·867365	21	·081223	·909681	21	·089205	·950389
22	·073805	·868088	22	·081353	·910372	22	·089342	·951054
23	·073927	·868804	23	·081482	·911063	23	·089479	·951720
24	·074049	·869521	24	·081612	·911754	24	·089616	·952384
25	·074172	·870239	25	·081742	·912444	25	·089753	·953049
26	·074294	·870956	26	·081872	·913133	26	·089891	·953714
27	·074417	·871674	27	·082002	·913822	27	·090028	·954377
28	·074540	·872390	28	·082132	·914510	28	·090165	·955040
29	·074663	·873106	29	·082262	·915198	29	·090303	·955703
30	·074787	·873829	30	·082392	·915887	30	·090441	·956366
31	·074910	·874537	31	·082523	·916574	31	·090579	·957028
32	·075033	·875251	32	·082653	·917260	32	·090717	·957689
33	·075156	·875965	33	·082784	·917947	33	·090855	·958350
34	·075280	·876678	34	·082915	·918632	34	·090994	·959011
35	·075404	·877391	35	·083046	·919317	35	·091132	·959672
36	·075527	·878103	36	·083177	·920002	36	·091271	·960333
37	·075651	·878816	37	·083308	·920687	37	·091410	·960992
38	·075775	·879527	38	·083440	·921372	38	·091550	·961652
39	·075900	·880239	39	·083570	·922054	39	·091688	·962310
40	·076024	·880949	40	·083702	·922738	40	·091827	·962969
41	·076148	·881659	41	·083834	·923421	41	·091966	·963627
42	·076273	·882368	42	·083966	·924104	42	·092105	·964285
43	·076398	·883079	43	·084098	·924786	43	·092245	·964942
44	·076522	·883787	44	·084230	·925467	44	·092385	·965600
45	·076647	·884495	45	·084362	·926148	45	·092524	·966255
46	·076772	·885203	46	·084495	·926829	46	·092664	·966912
47	·076897	·885909	47	·084627	·927510	47	·092804	·967567
48	·077022	·886616	48	·084760	·928190	48	·092944	·968223
49	·077148	·887323	49	·084893	·928869	49	·093085	·968878
50	·077273	·888029	50	·085025	·929548	50	·093225	·969532
51	·077399	·888734	51	·085158	·930227	51	·093366	·970187
52	·077525	·889439	52	·085291	·930904	52	·093506	·970840
53	·077650	·890143	53	·085425	·931583	53	·093647	·971494
54	·077776	·890848	54	·085558	·932260	54	·093788	·972147
55	·077903	·891551	55	·085691	·932936	55	·093929	·972800
56	·078029	·892254	56	·085825	·933613	56	·094070	·973452
57	·078155	·892956	57	·085958	·934288	57	·094212	·974104
58	·078282	·893659	58	·086092	·934964	58	·094353	·974755
59	·078408	·894361	59	·086225	·935631	59	·094495	·975407
60	·078535	·895061	60	·086360	·936314	60	·094636	·976057

	24 DEGREES.			25 DEGREES.			26 DEGREES.	
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·086454	8·936787	0	0·093692	8·971703	0	0·101206	9·005206
1	·086573	·937382	1	·093815	·972273	1	·101333	·005753
2	·086695	·937975	2	·093939	·972843	2	·101461	·006300
3	·086819	·938569	3	·094061	·973411	3	·101588	·006843
4	·086929	·939162	4	·094184	·973979	4	·101716	·007392
5	·087047	·939754	5	·094308	·974547	5	·101845	·007938
6	·087166	·940346	6	·094431	·975115	6	·101973	·008485
7	·087286	·940938	7	·094555	·975683	7	·102100	·009027
8	·087404	·941529	8	·094678	·976250	8	·102229	·009572
9	·087523	·942120	9	·094802	·976816	9	·102357	·010116
10	·087642	·942711	10	·094925	·977380	10	·102485	·010660
11	·087761	·943300	11	·095049	·977948	11	·102613	·011204
12	·087880	·943889	12	·095173	·978514	12	·102743	·011746
13	·087999	·944478	13	·095297	·979078	13	·102870	·012289
14	·088119	·945067	14	·095421	·979643	14	·102999	·012832
15	·088238	·945656	15	·095545	·980207	15	·103128	·013373
16	·088358	·946243	16	·095669	·980771	16	·103256	·013915
17	·088477	·946830	17	·095793	·981334	17	·103385	·014456
18	·088597	·947418	18	·095918	·981898	18	·103514	·014998
19	·088716	·948004	19	·096042	·982460	19	·103643	·015538
20	·088836	·948590	20	·096166	·983023	20	·103772	·016078
21	·088956	·949175	21	·096291	·983585	21	·103901	·016618
22	·089076	·949761	22	·096415	·984146	22	·104030	·017157
23	·089196	·950346	23	·096540	·984707	23	·104159	·017696
24	·089316	·950931	24	·096665	·985268	24	·104288	·018235
25	·089437	·951515	25	·096790	·985829	25	·104417	·018773
26	·089557	·952098	26	·096914	·986388	26	·104547	·019311
27	·089677	·952681	27	·097040	·986948	27	·104676	·019847
28	·089798	·953265	28	·097164	·987507	28	·104806	·020387
29	·089918	·953848	29	·097290	·988066	29	·104936	·020924
30	·090039	·954429	30	·097415	·988624	30	·105066	·021460
31	·090159	·955011	31	·097540	·989182	31	·105196	·021997
32	·090280	·955592	32	·097666	·989741	32	·105326	·022533
33	·090401	·956173	33	·097791	·990298	33	·105456	·023069
34	·090522	·956753	34	·097916	·990854	34	·105586	·023603
35	·090643	·957333	35	·098042	·991411	35	·105716	·024139
36	·090764	·957913	36	·098168	·991968	36	·105846	·024673
37	·090885	·958492	37	·098293	·992524	37	·105977	·025210
38	·091006	·959071	38	·098419	·993079	38	·106107	·025742
39	·091127	·959649	39	·098545	·993634	39	·106237	·026275
40	·091249	·960228	40	·098671	·994189	40	·106367	·026808
41	·091370	·960805	41	·098797	·994743	41	·106498	·027342
42	·091492	·961382	42	·098923	·995297	42	·106629	·027874
43	·091613	·961959	43	·099049	·995851	43	·106760	·028406
44	·091735	·962535	44	·099175	·996404	44	·106890	·028933
45	·091857	·963111	45	·099302	·996957	45	·107021	·029470
46	·091979	·963687	46	·099428	·997509	46	·107152	·030000
47	·092101	·964262	47	·099555	·998061	47	·107283	·030531
48	·092223	·964836	48	·099681	·998613	48	·107414	·031061
49	·092345	·965412	49	·099808	·999164	49	·107545	·031592
50	·092467	·965985	50	·099934	·999715	50	·107677	·032121
51	·092589	·966559	51	·100061	9·000266	51	·107808	·032651
52	·092711	·967131	52	·100188	·000817	52	·107940	·033180
53	·092834	·967705	53	·100315	·001366	53	·108071	·033709
54	·092956	·968277	54	·100442	·001916	54	·108202	·034237
55	·093078	·968849	55	·100570	·002465	55	·108334	·034765
56	·093201	·969421	56	·100697	·003014	56	·108466	·035293
57	·093324	·969991	57	·100824	·003563	57	·108598	·035820
58	·093447	·970563	58	·100951	·004111	58	·108730	·036348
59	·093569	·971133	59	·101079	·004660	59	·108862	·036874
60	·093692	·971703	60	·101206	·005206	60	·108994	·037401

24 DEGREES.			25 DEGREES.			26 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·094636	8·976057	0	0·103378	9·014427	0	0·112602	9·051546
1	·094778	·976708	1	·103528	·015056	1	·112760	·052154
2	·094920	·977357	2	·103678	·015685	2	·112918	·052763
3	·095062	·978008	3	·103828	·016312	3	·113076	·053370
4	·095204	·978657	4	·103977	·016939	4	·113235	·053979
5	·095347	·979306	5	·104128	·017566	5	·113393	·054586
6	·095489	·979954	6	·104279	·018194	6	·113552	·055195
7	·095632	·980603	7	·104429	·018821	7	·113710	·055799
8	·095775	·981250	8	·104580	·019447	8	·113869	·056406
9	·095918	·981898	9	·104730	·020072	9	·114028	·057012
10	·096061	·982546	10	·104881	·020696	10	·114187	·057618
11	·096204	·983191	11	·105032	·021323	11	·114347	·058224
12	·096347	·983837	12	·105184	·021948	12	·114506	·058828
13	·096490	·984483	13	·105335	·022572	13	·114666	·059434
14	·096634	·985129	14	·105486	·023196	14	·114826	·060039
15	·096777	·985774	15	·105638	·023820	15	·114985	·060642
16	·096921	·986417	16	·105790	·024444	16	·115145	·061246
17	·097065	·987062	17	·105942	·025066	17	·115306	·061850
18	·097209	·987707	18	·106094	·025690	18	·115466	·062454
19	·097353	·988350	19	·106246	·026312	19	·115626	·063057
20	·097498	·988994	20	·106398	·026934	20	·115787	·063660
21	·097642	·989636	21	·106551	·027556	21	·115948	·064262
22	·097787	·990279	22	·106703	·028177	22	·116108	·064863
23	·097931	·990921	23	·106856	·028798	23	·116269	·065465
24	·098076	·991563	24	·107009	·029419	24	·116431	·066067
25	·098221	·992205	25	·107162	·030040	25	·116592	·066667
26	·098366	·992845	26	·107315	·030659	26	·116753	·067268
27	·098511	·993486	27	·107468	·031279	27	·116915	·067869
28	·098657	·994127	28	·107621	·031898	28	·117077	·068470
29	·098802	·994766	29	·107775	·032518	29	·117239	·069070
30	·098948	·995406	30	·107929	·033136	30	·117400	·069669
31	·099094	·996046	31	·108082	·033754	31	·117563	·070269
32	·099240	·996684	32	·108236	·034373	32	·117725	·070868
33	·099386	·997323	33	·108390	·034991	33	·117888	·071467
34	·099532	·997961	34	·108544	·035607	34	·118049	·072064
35	·099678	·998599	35	·108699	·036225	35	·118212	·072663
36	·099824	·999236	36	·108854	·036842	36	·118375	·073261
37	·099971	·999873	37	·109008	·037459	37	·118539	·073861
38	·100118	9·000510	38	·109163	·038074	38	·118702	·074456
39	·100264	·001146	39	·109318	·038690	39	·118865	·075053
40	·100411	·001783	40	·109473	·039306	40	·119028	·075649
41	·100558	·002418	41	·109628	·039920	41	·119192	·076246
42	·100706	·003053	42	·109783	·040535	42	·119355	·076842
43	·100853	·003688	43	·109939	·041150	43	·119519	·077438
44	·101000	·004322	44	·110094	·041764	44	·119683	·078033
45	·101148	·004957	45	·110250	·042378	45	·119848	·078629
46	·101296	·005591	46	·110406	·042991	46	·120012	·079222
47	·101444	·006224	47	·110561	·043604	47	·120176	·079817
48	·101591	·006857	48	·110717	·044217	48	·120340	·080412
49	·101740	·007491	49	·110874	·044829	49	·120505	·081006
50	·101885	·008122	50	·111030	·045441	50	·120670	·081599
51	·102037	·008755	51	·111187	·046053	51	·120835	·082193
52	·102185	·009385	52	·111344	·046665	52	·121000	·082786
53	·102334	·010018	53	·111500	·047276	53	·121166	·083379
54	·102482	·010649	54	·111657	·047887	54	·121331	·083971
55	·102631	·011279	55	·111814	·048497	55	·121496	·084563
56	·102780	·011910	56	·111972	·049108	56	·121662	·085155
57	·102930	·012539	57	·112129	·049718	57	·121828	·085746
58	·103079	·013170	58	·112287	·050328	58	·121994	·086338
59	·103228	·013798	59	·112445	·050938	59	·122160	·086929
60	·103378	·014427	60	·112602	·051546	60	·122327	·087520

27 DEGREES.			28 DEGREES.			29 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm.
0	0·108994	9·037401	0	0·117052	9·068380	0	0·125380	9·098229
1	·109126	·037927	1	·117189	·068887	1	·125522	·098717
2	·109258	·038452	2	·117326	·069393	2	·125663	·099206
3	·109390	·038978	3	·117462	·069899	3	·125804	·099693
4	·109522	·039502	4	·117599	·070404	4	·125945	·100191
5	·109655	·040027	5	·117736	·070910	5	·126086	·100668
6	·109787	·040551	6	·117873	·071415	6	·126228	·101155
7	·109920	·041076	7	·118010	·071919	7	·126370	·101642
8	·110053	·041600	8	·118147	·072424	8	·126512	102129
9	·110185	·042123	9	·118285	·072928	9	·126653	·102614
10	·110318	·042645	10	·118422	·073432	10	·126794	·103100
11	·110451	·043168	11	·118559	·073935	11	·126936	·103585
12	·110584	·043690	12	·118696	·074437	12	·127078	·104670
13	·110717	·044213	13	·118834	·074941	13	·127220	·104555
14	·110850	·044735	14	·118972	·075453	14	·127362	·105040
15	·110983	·045256	15	·119110	·075946	15	·127504	·105523
16	·111116	·045777	16	·119247	·076448	16	127646	·106008
17	·111249	·046297	17	·119385	·076948	17	·127789	·106491
18	·111383	·046818	18	·119523	·077450	18	·127931	·106974
19	·111516	·047338	19	·119661	·077951	19	·128073	·107457
20	·111650	·047857	20	·119799	·078452	20	·128216	·107940
21	·111783	·048377	21	·119937	·078952	21	·128358	·108423
22	·111917	·048896	22	·120075	·079452	22	·128501	·108906
23	·112051	·049415	23	·120213	·079951	23	·128643	·109387
24	·112185	·049933	24	·120351	·080451	24	·128786	·109869
25	·112319	·050451	25	·120490	·080951	25	·128929	·110350
26	·112453	·050968	26	·120628	·081449	26	·129072	·110831
27	·112587	·051487	27	·120767	·081947	27	·129215	·111312
28	·112721	·052004	28	·120905	·082445	28	·129358	·111793
29	·112855	·052520	29	·121044	·082943	29	·129501	·112273
30	·112989	·053037	30	·121183	·083441	30	·129644	·112754
31	·113124	·053553	31	·121322	·083938	31	·129788	·113233
32	·113258	·054069	32	·121461	·084435	32	·129931	·113712
33	·113393	·054584	33	·121600	·084932	33	·130074	·114192
34	·113527	·055099	34	·121739	·085428	34	·130218	·114671
35	·113662	·055614	35	·121878	·085924	35	·130362	·115149
36	·113797	·056129	36	·122017	·086420	36	·130505	·115627
37	·113931	·056642	37	·122157	·086916	37	·130649	·116105
38	·114066	·057157	38	·122296	·087411	38	·130793	·116583
39	·114201	·057670	39	·122435	·087905	39	·130936	·117060
40	·114336	·058183	40	·122575	·088401	40	·131080	·117537
41	·114471	·058696	41	·122714	·088895	41	·131224	·118015
42	·114606	·059208	42	·122854	·089388	42	·131368	·118491
43	·114742	·059721	43	·122994	·089882	43	·131513	·118968
44	·114877	·060232	44	·123134	·090376	44	·131657	·119443
45	·115013	·060745	45	·123273	·090869	45	·131801	·119919
46	·115148	·061256	46	·123413	·091361	46	·131945	·120394
47	·115283	·061766	47	·123553	·091854	47	·132090	·120870
48	·115419	·062277	48	·123693	·092346	48	·132235	·121345
49	·115555	·062788	49	·123834	·092838	49	·132379	·121819
50	·115691	·063298	50	·123974	·093329	50	·132524	·122294
51	·115827	·063807	51	·124114	·093821	51	·132669	·122768
52	·115962	·064316	52	·124255	·094312	52	·132814	·123242
53	·116098	·064826	53	·124395	·094802	53	·132958	·123715
54	·116235	·065335	54	·124535	·095293	54	·133103	·124188
55	·116371	·065843	55	·124676	·095783	55	·133248	·124661
56	·116507	·066351	56	·124817	·096272	56	·133394	·125135
57	·116643	·066859	57	·124958	·096763	57	·133539	·125607
58	·116779	·067366	58	·125098	·097251	58	·133684	·126079
59	·116916	·067874	59	·125239	·097740	59	·133830	·126551
60	·117052	·068380	60	·125380	·098229	60	·133975	·127022

27 DEGREES.			28 DEGREES.			29 DEGREES.		
Min	Nat No	Logarithm.	Min.	Nat No.	Logarithm	Min.	Nat. No	Logarithm.
0	0·122327	9·087520	0	0·132570	9·122445	0	0·143354	9·156410
1	·122493	·088111	1	·132745	·123019	1	·143538	·156968
2	·122660	·088700	2	·132920	·123593	2	·143723	·157527
3	·122826	·089290	3	·133096	·124166	3	·143908	·158084
4	·122993	·089879	4	·133272	·124738	4	·144093	·158642
5	·123160	·090469	5	·133448	·125312	5	·144278	·159199
6	·123327	·091057	6	·133624	·125884	6	·144463	·159757
7	·123495	·091647	7	·133800	·126453	7	·144649	·160314
8	·123662	·092236	8	·133976	·127028	8	·144834	·160869
9	·123829	·092823	9	·134153	·127600	9	·145020	·161427
10	·123997	·093410	10	·134330	·128171	10	·145206	·161983
11	·124165	·093998	11	·134506	·128742	11	·145391	·162539
12	·124333	·094585	12	·134683	·129312	12	·145578	·163095
13	·124501	·095173	13	·134860	·129883	13	·145764	·163650
14	·124669	·095760	14	·135037	·130453	14	·145950	·164206
15	·124838	·096346	15	·135215	·131024	15	·146137	·164760
16	·125006	·096932	16	·135392	·131594	16	·146324	·165315
17	·125175	·097579	17	·135570	·132162	17	·146511	·165869
18	·125344	·098108	18	·135748	·132732	18	·146698	·166423
19	·125513	·098688	19	·135926	·133301	19	·146885	·166977
20	·125682	·099273	20	·136104	·133870	20	·147072	·167531
21	·125851	·099858	21	·136282	·134438	21	·147260	·168085
22	·126021	·100442	22	·136460	·135006	22	·147448	·168639
23	·126191	·101027	23	·136639	·135574	23	·147636	·169191
24	·126360	·101610	24	·136818	·136142	24	·147825	·169749
25	·126530	·102194	25	·136997	·136710	25	·148012	·170296
26	·126700	·102776	26	·137176	·137277	26	·148200	·170849
27	·126871	·103361	27	·137355	·137843	27	·148389	·171401
28	·127041	·103944	28	·137534	·138409	28	·148577	·171953
29	·127211	·104525	29	·137713	·138976	29	·148766	·172505
30	·127382	·105108	30	·137893	·139542	30	·148956	·173057
31	·127553	·105690	31	·138073	·140108	31	·149145	·173608
32	·127724	·106272	32	·138253	·140674	32	·149334	·174158
33	·127895	·106853	33	·138433	·141239	33	·149524	·174710
34	·128066	·107434	34	·138613	·141804	34	·149714	·175261
35	·128237	·108014	35	·138793	·142369	35	·149903	·175810
36	·128409	·108596	36	·138974	·142934	36	·150093	·176360
37	·128581	·109175	37	·139155	·143499	37	·150283	·176910
38	·128753	·109756	38	·139336	·144063	38	·150474	·177460
39	·128925	·110335	39	·139517	·144626	39	·150664	·178008
40	·129097	·110914	40	·139698	·145191	40	·150854	·178557
41	·129269	·111493	41	·139880	·145754	41	·151045	·179107
42	·129441	·112072	42	·140061	·146316	42	·151236	·179655
43	·129614	·112651	43	·140242	·146879	43	·151427	·180205
44	·129787	·113228	44	·140424	·147442	44	·151619	·180752
45	·129960	·113808	45	·140606	·148005	45	·151810	·181300
46	·130132	·114385	46	·140788	·148566	46	·152001	·181847
47	·130305	·114962	47	·140970	·149128	47	·152193	·182395
48	·130479	·115539	48	·141153	·149690	48	·152385	·182943
49	·130652	·116117	49	·141336	·150251	49	·152577	·183489
50	·130826	·116694	50	·141518	·150812	50	·152769	·184036
51	·130999	·117269	51	·141701	·151373	51	·152962	·184583
52	·131173	·117845	52	·141884	·151934	52	·153154	·185129
53	·131347	·118422	53	·142067	·152494	53	·153347	·185675
54	·131522	·118998	54	·142250	·153054	54	·153540	·186221
55	·131696	·119573	55	·142434	·153614	55	·153733	·186766
56	·131871	·120148	56	·142618	·154173	56	·153926	·187313
57	·132046	·120723	57	·142802	·154734	57	·154120	·187858
58	·132220	·121297	58	·142986	·155292	58	·154313	·188403
59	·132395	·121872	59	·143170	·155851	59	·154507	·188947
60	·132570	·122445	60	·143354	·156410	60	·154700	·189491

30 DEGREES.			31 DEGREES.			32 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·133975	9·127022	0	0·142833	9·154827	0	0·151952	9·181706
1	·134120	·127494	1	·142983	·155283	1	·152106	·182147
2	·134266	·127964	2	·143133	·155738	2	·152261	·182587
3	·134412	·128436	3	·143282	·156192	3	·152415	·183027
4	·134558	·128906	4	·143432	·156647	4	·152569	·183466
5	·134703	·129376	5	·143583	·157102	5	·152724	·183906
6	·134849	·129846	6	·143733	·157556	6	·152878	·184345
7	·134995	·130316	7	·143883	·158010	7	·153033	·184784
8	·135141	·130786	8	·144034	·158464	8	·153188	·185223
9	·135287	·131254	9	·144184	·158917	9	·153342	·185661
10	·135433	·131724	10	·144335	·159370	10	·153497	·186100
11	·135579	·132192	11	·144485	·159823	11	·153652	·186538
12	·135725	·132660	12	·144636	·160275	12	·153807	·186975
13	·135872	·133128	13	·144787	·160728	13	·153962	·187413
14	·136018	·133596	14	·144937	·161180	14	·154117	·187850
15	·136165	·134064	15	·145088	·161632	15	·154272	·188287
16	·136311	·134531	16	·145239	·162083	16	·154427	·188724
17	·136458	·134998	17	·145390	·162535	17	·154583	·189161
18	·136605	·135465	18	·145541	·162986	18	·154738	·189598
19	·136751	·135931	19	·145692	·163436	19	·154894	·190033
20	·136898	·136397	20	·145844	·163887	20	·155049	·190469
21	·137045	·136863	21	·145995	·164337	21	·155205	·190905
22	·137192	·137329	22	·146146	·164788	22	·155360	·191340
23	·137339	·137794	23	·146298	·165238	23	·155516	·191775
24	·137486	·138259	24	·146450	·165687	24	·155672	·192210
25	·137633	·138724	25	·146601	·166137	25	·155828	·192645
26	·137781	·139189	26	·146752	·166585	26	·155984	·193080
27	·137928	·139653	27	·146904	·167034	27	·156140	·193514
28	·138076	·140117	28	·147056	·167483	28	·156296	·193948
29	·138223	·140581	29	·147208	·167931	29	·156452	·194382
30	·138371	·141045	30	·147360	·168379	30	·156609	·194815
31	·138518	·141507	31	·147512	·168827	31	·156765	·195249
32	·138666	·141971	32	·147664	·169274	32	·156921	·195682
33	·138814	·142434	33	·147817	·169722	33	·157078	·196115
34	·138962	·142896	34	·147969	·170169	34	·157234	·196547
35	·139110	·143358	35	·148121	·170615	35	·157391	·196979
36	·139258	·143820	36	·148273	·171063	36	·157548	·197412
37	·139406	·144282	37	·158426	·171509	37	·157705	·197844
38	·139554	·144743	38	·148578	·171954	38	·157861	·198275
39	·139703	·145204	39	·148730	·172400	39	·158018	·198707
40	·139851	·145665	40	·148883	·172846	40	·158175	·199138
41	·139999	·146125	41	·149036	·173292	41	·158332	·199569
42	·140147	·146585	42	·149189	·173736	42	·158490	·200000
43	·140296	·147046	43	·149342	·174181	43	·158647	·200430
44	·140445	·147506	44	·149495	·174626	44	·158804	·200860
45	·140594	·147965	45	·149648	·175070	45	·158962	·201290
46	·140742	·148424	46	·149801	·175514	46	·159119	·201720
47	·140891	·148883	47	·149954	·175958	47	·159276	·202149
48	·141040	·149342	48	·150107	·176401	48	·159433	·202579
49	·141189	·149801	49	·150261	·176845	49	·159591	·203008
50	·141338	·150259	50	·150414	·177288	50	·159749	·203437
51	·141487	·150717	51	·150568	·177731	51	·159907	·203866
52	·141636	·151175	52	·150721	·178174	52	·160065	·204295
53	·141786	·151632	53	·150875	·178616	53	·160223	·204723
54	·141935	·152089	54	·151028	·179058	54	·160380	·205151
55	·142085	·152546	55	·151182	·179500	55	·160538	·205579
56	·142234	·153003	56	·151336	·179942	56	·160697	·206006
57	·142384	·153460	57	·151490	·180384	57	·160855	·206433
58	·142533	·153916	58	·151644	·180824	58	·161013	·206860
59	·142683	·154372	59	·151798	·181265	59	·161171	·207287
60	·142833	·154827	60	·151952	·181706	60	·161330	·207714

30 DEGREES.			31 DEGREES.			32 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm
0	0·154700	9·189491	0	0·166633	9·221761	0	0·179179	9·253286
1	·154894	·190036	1	·166837	·222293	1	·179393	·253805
2	·155089	·190579	2	·167041	·222824	2	·179608	·254324
3	·155283	·191124	3	·167245	·223354	3	·179822	·254844
4	·155478	·191668	4	·167450	·223885	4	·180037	·255362
5	·155673	·192211	5	·167655	·224417	5	·180252	·255881
6	·155867	·192754	6	·167860	·224947	6	·180467	·256399
7	·156062	·193297	7	·168065	·225477	7	·180683	·256917
8	·156257	·193840	8	·168270	·226007	8	·180899	·257436
9	·156452	·194382	9	·168476	·226537	9	·181115	·257953
10	·156648	·194925	10	·168681	·227066	10	·181331	·258471
11	·156844	·195467	11	·168887	·227595	11	·181547	·258989
12	·157040	·196008	12	·169093	·228124	12	·181763	·259505
13	·157236	·196550	13	·169299	·228653	13	·181980	·260023
14	·157432	·197091	14	·169505	·229182	14	·182197	·260540
15	·157628	·197633	15	·169711	·229711	15	·182413	·261056
16	·157824	·198174	16	·169918	·230238	16	·182630	·261573
17	·158021	·198714	17	·170125	·230767	17	·182848	·262090
18	·158218	·199255	18	·170332	·231295	18	·183066	·262607
19	·158415	·199795	19	·170539	·231822	19	·183283	·263122
20	·158612	·200335	20	·170746	·232350	20	·183501	·263638
21	·158809	·200875	21	·170953	·232877	21	·183719	·264154
22	·159007	·201415	22	·171161	·233405	22	·183937	·264669
23	·159204	·201954	23	·171369	·233932	23	·184156	·265184
24	·159402	·202493	24	·171577	·234458	24	·184374	·265699
25	·159600	·203032	25	·171785	·234985	25	·184593	·266214
26	·159798	·203571	26	·171993	·235510	26	·184812	·266729
27	·159997	·204110	27	·172201	·236036	27	·185031	·267244
28	·160195	·204648	28	·172410	·236562	28	·185250	·267758
29	·160393	·205186	29	·172619	·237088	29	·185469	·268272
30	·160592	·205725	30	·172828	·237613	30	·185689	·268786
31	·160791	·206261	31	·173037	·238139	31	·185909	·269300
32	·160991	·206800	32	·173246	·238663	32	·186129	·269814
33	·161190	·207337	33	·173456	·239189	33	·186349	·270327
34	·161390	·207874	34	·173665	·239713	34	·186570	·270840
35	·161589	·208410	35	·173875	·240237	35	·186790	·271353
36	·161789	·208947	36	·174085	·240763	36	·187011	·271867
37	·161989	·209484	37	·174295	·241286	37	·187232	·272379
38	·162189	·210020	38	·174505	·241809	38	·187453	·272891
39	·162389	·210555	39	·174716	·242333	39	·187674	·273404
40	·162589	·211091	40	·174927	·242857	40	·187896	·273916
41	·162789	·211626	41	·175138	·243381	41	·188117	·274428
42	·162990	·212161	42	·175349	·243903	42	·188339	·274940
43	·163191	·212697	43	·175560	·244426	43	·188561	·275451
44	·163392	·213232	44	·175772	·244949	44	·188783	·275963
45	·163594	·213766	45	·175983	·245471	45	·189006	·276475
46	·163795	·214301	46	·176195	·245993	46	·189228	·276985
47	·163997	·214835	47	·176407	·246516	47	·189450	·277495
48	·164198	·215368	48	·176619	·247037	48	·189673	·278006
49	·164400	·215903	49	·176832	·247559	49	·189896	·278517
50	·164603	·216437	50	·177044	·248081	50	·190120	·279028
51	·164805	·216970	51	·177257	·248602	51	·190344	·279538
52	·165008	·217504	52	·177470	·249124	52	·190568	·280049
53	·165210	·218036	53	·177683	·249644	53	·190792	·280559
54	·165413	·218569	54	·177896	·250165	54	·191015	·281068
55	·165616	·219101	55	·178109	·250685	55	·191240	·281578
56	·165819	·219634	56	·178322	·251206	56	·191464	·282087
57	·166023	·220167	57	·178536	·251727	57	·191689	·282596
58	·166226	·220699	58	·178750	·252246	58	·191914	·283105
59	·166430	·221231	59	·178964	·252766	59	·192139	·283614
60	·166633	·221761	60	·179179	·253286	60	·192364	·284123

33 DEGREES.			34 DEGREES.			35 DEGREES.		
Min.	Nat. No	Logarithm.	Min	Nat. No	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·161330	9·207714	0	0·170963	9·232901	0	0·180848	9·257313
1	·161488	·208140	1	·171126	·233314	1	·181015	·257714
2	·161647	·208566	2	·171289	·233727	2	·181182	·258114
3	·161805	·208992	3	·171452	·234139	3	·181349	·258514
4	·161964	·209418	4	·171614	·234552	4	·181516	·258914
5	·162123	·209843	5	·171777	·234964	5	·181683	·259314
6	·162282	·210269	6	·171940	·235376	6	·181850	·259714
7	·162440	·210693	7	·172103	·235788	7	·182018	·260114
8	·162599	·211118	8	·172266	·236199	8	·182185	·260513
9	·162758	·211543	9	·172430	·236611	9	·182353	·260912
10	·162917	·211967	10	·172593	·237022	10	·182520	·261310
11	·163076	·212391	11	·172756	·237433	11	·182688	·261709
12	·163235	·212814	12	·172920	·237844	12	·182855	·262107
13	·163395	·213238	13	·173083	·238255	13	·183023	·262505
14	·163555	·213662	14	·173247	·238665	14	·183191	·262903
15	·163714	·214085	15	·173410	·239075	15	·183359	·263301
16	·163874	·214509	16	·173574	·239485	16	·183527	·263699
17	·164033	·214931	17	·173738	·239895	17	·183695	·264096
18	·164193	·215353	18	·173902	·240304	18	·183863	·264493
19	·164353	·215776	19	·174066	·240713	19	·184031	·264890
20	·164512	·216198	20	·174230	·241122	20	·184199	·265286
21	·164672	·216620	21	·174394	·241531	21	·184367	·265683
22	·164832	·217042	22	·174558	·241940	22	·184535	·266080
23	·164992	·217462	23	·174723	·242349	23	·184704	·266476
24	·165152	·217884	24	·174887	·242757	24	·184872	·266871
25	·165312	·218305	25	·175051	·243165	25	·185041	·267267
26	·165473	·218726	26	·175216	·243572	26	·185210	·267663
27	·165633	·219146	27	·175380	·243980	27	·185378	·268058
28	·165793	·219567	28	·175544	·244387	28	·185547	·268453
29	·165954	·219987	29	·175709	·244794	29	·185716	·268848
30	·166114	·220406	30	·175874	·245201	30	·185885	·269243
31	·166275	·221826	31	·176039	·245608	31	·186054	·269637
32	·166436	·221246	32	·176204	·246014	32	·186223	·270032
33	·166597	·221665	33	·176369	·246421	33	·186392	·270426
34	·166757	·222084	34	·176534	·246827	34	·186561	·270820
35	·166918	·222502	35	·176699	·247233	35	·186730	·271214
36	·167079	·222922	36	·176864	·247639	36	·186900	·271608
37	·167240	·223340	37	·177029	·248044	37	·187069	·272001
38	·167401	·223758	38	·177194	·248449	38	·187238	·272394
39	·167562	·224175	39	·177360	·248855	39	·187408	·272787
40	·167723	·224593	40	·177525	·249259	40	·187577	·273180
41	·167885	·225011	41	·177690	·249664	41	·187747	·273573
42	·168046	·225427	42	·177856	·250069	42	·187917	·273965
43	·168208	·225845	43	·178022	·250473	43	·188087	·274357
44	·168369	·226262	44	·178187	·250876	44	·188256	·274749
45	·168530	·226678	45	·178353	·251280	45	·188426	·275140
46	·168692	·227094	46	·178519	·251684	46	·188596	·275532
47	·168854	·227511	47	·178685	·252087	47	·188766	·275924
48	·169016	·227927	48	·178851	·252491	48	·188936	·276315
49	·169178	·228342	49	·179017	·252894	49	·189106	·276706
50	·169340	·228758	50	·179183	·253297	50	·189277	·277097
51	·169502	·229173	51	·179349	·253699	51	·189447	·277487
52	·169668	·229587	52	·179515	·254101	52	·189617	·277878
53	·169826	·230003	53	·179682	·254504	53	·189788	·278268
54	·169988	·230418	54	·179848	·354906	54	·189958	·278658
55	·170150	·230832	55	·180015	·255308	55	·190129	·279048
56	·170312	·231246	56	·180182	·255710	56	·190300	·279438
57	·170475	·231660	57	·180348	·256111	57	·190471	·279828
58	·170638	·232074	58	·180515	·256512	58	·190641	·280217
59	·170800	·232487	59	·180681	·256913	59	·190812	·280606
60	·170963	·232901	60	·180848	·257313	60	·190983	·280995

33 DEGREES.			34 DEGREES.			35 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·192364	9·284123	0	0·206218	9·314326	0	0·220775	9·343949
1	·192589	·284631	1	·206455	·314825	1	·221024	·344438
2	·192814	·285139	2	·206692	·315323	2	·221272	·344927
3	·193040	·285647	3	·206929	·315821	3	·221521	·345415
4	·193266	·286155	4	·207167	·316319	4	·221771	·345904
5	·193492	·286662	5	·207404	·316817	5	·222020	·346392
6	·193718	·287171	6	·207642	·317315	6	·222270	·346881
7	·193945	·287677	7	·207880	·317812	7	·222520	·347370
8	·194172	·288185	8	·208118	·318309	8	·222771	·347858
9	·194399	·288692	9	·208356	·318806	9	·223021	·348346
10	·194625	·289199	10	·208594	·319302	10	·223272	·348833
11	·194852	·289705	11	·208833	·319799	11	·223523	·349321
12	·195080	·290211	12	·209072	·320296	12	·223774	·349808
13	·195307	·290718	13	·209311	·320793	13	·224025	·350295
14	·195535	·291224	14	·209550	·321288	14	·224276	·350782
15	·195763	·291730	15	·209790	·321785	15	·224528	·351270
16	·195992	·292237	16	·210030	·322280	16	·225780	·351757
17	·196220	·292742	17	·210270	·322777	17	·225031	·352243
18	·196448	·293247	18	·210510	·323272	18	·225284	·352730
19	·196677	·293753	19	·210750	·323767	19	·225536	·353216
20	·196906	·294258	20	·210991	·324263	20	·225789	·353702
21	·197135	·294763	21	·211231	·324758	21	·226042	·354189
22	·197364	·295268	22	·211472	·325253	22	·226295	·354675
23	·197593	·295771	23	·211714	·325749	23	·226548	·355161
24	·197823	·296277	24	·211955	·326243	24	·226801	·355646
25	·198053	·296781	25	·212197	·326738	25	·227055	·356131
26	·198283	·297285	26	·212438	·327232	26	·227310	·356617
27	·198513	·297789	27	·212680	·327726	27	·227564	·357102
28	·198744	·298293	28	·212922	·328220	28	·227818	·357587
29	·198974	·298797	29	·213164	·328713	29	·228072	·358072
30	·199205	·299299	30	·213407	·329207	30	·228326	·358557
31	·199436	·299803	31	·213650	·329701	31	·228581	·359041
32	·199667	·300307	32	·213892	·330194	32	·228837	·359526
33	·199899	·300809	33	·214135	·330688	33	·229092	·360011
34	·200130	·301312	34	·214379	·331181	34	·229348	·360495
35	·200362	·301815	35	·214622	·331674	35	·229604	·360979
36	·200594	·302317	36	·214866	·332167	36	·229860	·361463
37	·200826	·302820	37	·215110	·332659	37	·230116	·361947
38	·201058	·303322	38	·215354	·333152	38	·230373	·362431
39	·201290	·303823	39	·215598	·333644	39	·230630	·362914
40	·201523	·304325	40	·215842	·334136	40	·230886	·363398
41	·201756	·304827	41	·216087	·334629	41	·231143	·363881
42	·201990	·305328	42	·216332	·335121	42	·231400	·364364
43	·202223	·305830	43	·216577	·335613	43	·231658	·364847
44	·202456	·306331	44	·216822	·336104	44	·231916	·365330
45	·202690	·306832	45	·217068	·336595	45	·232173	·365812
46	·202924	·307333	46	·217313	·337086	46	·232431	·366295
47	·203158	·307834	47	·217559	·337577	47	·232690	·366778
48	·203392	·308334	48	·217806	·338069	48	·232949	·367260
49	·203626	·308834	49	·218052	·338560	49	·233207	·367742
50	·203861	·309334	50	·218299	·339050	50	·233466	·368224
51	·204096	·309834	51	·218545	·339541	51	·233726	·368706
52	·204331	·310334	52	·218792	·340031	52	·233985	·369188
53	·204568	·310834	53	·219040	·340522	53	·234245	·369670
54	·204801	·311333	54	·219287	·341012	54	·234505	·370151
55	·205037	·311832	55	·219535	·341502	55	·234764	·370632
56	·205273	·312331	56	·219782	·341992	56	·235025	·371114
57	·205509	·312830	57	·220030	·342481	57	·235285	·371595
58	·205745	·313329	58	·220278	·342970	58	·235546	·372076
59	·205981	·313828	59	·220526	·343460	59	·235807	·372557
60	·206218	·314326	60	·220775	·343949	60	·236068	·373037

36 DEGREES.			37 DEGREES.			38 DEGREES.		
Min	Nat. No	Logarithm	Min.	Nat. No	Logarithm	Min.	Nat. No	Logarithm
0	0·190983	9·280905	0	0·201365	9·303983	0	0·211990	9·326314
1	·191154	·281384	1	·201540	·304361	1	·212169	·326681
2	·191325	·281772	2	·201715	·304787	2	·212348	·327047
3	·191496	·282160	3	·201890	·305115	3	·212527	·327414
4	·191667	·282548	4	·202065	·305492	4	·212706	·327780
5	·191839	·282936	5	·202240	·305868	5	·212886	·328146
6	·192010	·283324	6	·202416	·306245	6	·213065	·328512
7	·192181	·283711	7	·202591	·306621	7	·213244	·328877
8	·192353	·284099	8	·202767	·306998	8	·213424	·329243
9	·192525	·284486	9	·202943	·307374	9	·213603	·329608
10	·192796	·284873	10	·203118	·307749	10	·213783	·329974
11	·192868	·285260	11	·203294	·308125	11	·213963	·330339
12	·193040	·285947	12	·203470	·308501	12	·214143	·330704
13	·193212	·286033	13	·203646	·308876	13	·214323	·331069
14	·193384	·286419	14	·203822	·309251	14	·214503	·331433
15	·193555	·286805	15	·203993	·309625	15	·214683	·331798
16	·193727	·287191	16	·204174	·310000	16	·214863	·332162
17	·193900	·287576	17	·204350	·310375	17	·215043	·332526
18	·194072	·287962	18	·204527	·310749	18	·215224	·332890
19	·194244	·288347	19	·204703	·311124	19	·215404	·333254
20	·194416	·288732	20	·204880	·311498	20	·215585	·333618
21	·194588	·289117	21	·205056	·311871	21	·215765	·333980
22	·194761	·289502	22	·205237	·312246	22	·215945	·334344
23	·194934	·289887	23	·205409	·312619	23	·216126	·334706
24	·195106	·290271	24	·205586	·312993	24	·216307	·335070
25	·195279	·290655	25	·205762	·313366	25	·216487	·335432
26	·195452	·291039	26	·205939	·313738	26	·216668	·335795
27	·195625	·291423	27	·206115	·314111	27	·216849	·336157
28	·195797	·291806	28	·206293	·314484	28	·217030	·336520
29	·195970	·292190	29	·206470	·314856	29	·217211	·336882
30	·196143	·292573	30	·206647	·315228	30	·217392	·337244
31	·196316	·292956	31	·206824	·315601	31	·217578	·337605
32	·196490	·293339	32	·207001	·315992	32	·217754	·337966
33	·196663	·293721	33	·207178	·316344	33	·217935	·338328
34	·196836	·294104	34	·207356	·316716	34	·218117	·338689
35	·197009	·294486	35	·207533	·317087	35	·218299	·339050
36	·197182	·294868	36	·207710	·317458	36	·218480	·339411
37	·197356	·295250	37	·207880	·317829	37	·218661	·339771
38	·197530	·295632	38	·208066	·318200	38	·218843	·340132
39	·197703	·296013	39	·208243	·318571	39	·219024	·340492
40	·197877	·296395	40	·208421	·318942	40	·219206	·340852
41	·198051	·296776	41	·208599	·319311	41	·219388	·341212
42	·198225	·297157	42	·208777	·319682	42	·219570	·341573
43	·198398	·297537	43	·208954	·320051	43	·219752	·341932
44	·198572	·297918	44	·209132	·320421	44	·219934	·342292
45	·198746	·298298	45	·209309	·320791	45	·220116	·342651
46	·198920	·298679	46	·209489	·321161	46	·220298	·343010
47	·199094	·299059	47	·209667	·321530	47	·220480	·343369
48	·199269	·299439	48	·209845	·321899	48	·220662	·343728
49	·192443	·299819	49	·210023	·322267	49	·220844	·344086
50	·199617	·300198	50	·210202	·322636	50	·221027	·344445
51	·199792	·300577	51	·210380	·323005	51	·221209	·344803
52	·199966	·300956	52	·210559	·323373	52	·221311	·345161
53	·200141	·301335	53	·210738	·323742	53	·221574	·345519
54	·200815	·301714	54	·210916	·324110	54	·221757	·345877
55	·200590	·302093	55	·211095	·324477	55	·221940	·346235
56	·200665	·302471	56	·211273	·324844	56	·222122	·346592
57	·200840	·302850	57	·211452	·325212	57	·222305	·346950
58	·201015	·303228	58	·211631	·325579	58	·222488	·347307
59	·201190	·303605	59	·211810	·325947	59	·222671	·347664
60	·201365	·303983	60	·211990	·326314	60	·222854	·348021

36 DEGREES.			37 DEGREES.			38 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm
0	0·236068	9·373037	0	0·252136	9·401634	0	0·269019	9·429782
1	·236330	·373518	1	·252410	·402107	1	·269307	·430248
2	·236591	·373998	2	·252685	·402579	2	·269595	·430712
3	·236853	·374478	3	·252960	·403052	3	·269884	·431178
4	·237115	·374958	4	·253235	·403525	4	·270174	·431643
5	·237377	·375438	5	·253511	·403997	5	·270463	·432108
6	·237640	·375918	6	·253787	·404469	6	·270753	·432573
7	·237902	·376397	7	·254063	·404941	7	·271042	·433037
8	·238165	·376877	8	·254339	·405413	8	·271332	·433502
9	·238428	·377357	9	·254615	·405884	9	·271623	·433967
10	·238691	·377836	10	·254892	·406355	10	·271914	·434432
11	·238954	·378315	11	·255169	·406827	11	·272205	·434896
12	·239218	·378794	12	·255446	·407299	12	·272496	·435361
13	·239482	·379273	13	·255723	·407770	13	·272788	·435825
14	·239747	·379752	14	·256000	·408241	14	·273080	·436289
15	·240011	·380231	15	·256278	·408711	15	·273372	·436753
16	·240276	·380709	16	·256556	·409182	16	·273664	·437217
17	·240540	·381187	17	·256834	·409653	17	·273956	·437680
18	·240805	·381666	18	·257113	·410124	18	·274249	·438144
19	·241070	·382143	19	·257392	·410595	19	·274542	·438608
20	·241335	·382621	20	·257671	·411065	20	·274835	·439072
21	·241601	·383099	21	·257950	·411534	21	·275128	·439534
22	·241867	·383577	22	·258230	·412006	22	·275421	·439998
23	·242133	·384055	23	·258509	·412475	23	·275715	·440460
24	·242400	·384532	24	·258789	·412946	24	·276010	·440924
25	·242667	·385010	25	·259069	·413415	25	·276304	·441386
26	·242933	·385487	26	·259349	·413884	26	·276598	·441849
27	·243200	·385964	27	·259630	·414354	27	·276893	·442311
28	·243467	·386440	28	·259910	·414824	28	·277188	·442774
29	·243735	·386918	29	·260191	·415292	29	·277584	·443237
30	·244002	·387394	30	·260472	·415761	30	·277780	·443700
31	·244270	·387871	31	·260754	·416231	31	·278075	·444161
32	·244539	·388347	32	·261035	·416699	32	·278370	·444623
33	·244807	·388823	33	·261317	·417168	33	·278667	·445085
34	·245075	·389299	34	·261600	·417638	34	·278963	·445547
35	·245344	·389775	35	·261882	·418106	35	·279260	·446009
36	·245613	·390251	36	·262165	·418574	36	·279557	·446471
37	·245882	·390727	37	·262448	·419043	37	·279855	·446932
38	·246152	·391203	38	·262731	·419511	38	·280152	·447393
39	·246422	·391678	39	·263015	·419980	39	·280450	·447854
40	·246691	·392154	40	·263298	·420447	40	·280748	·448316
41	·246961	·392629	41	·263581	·420914	41	·281046	·448777
42	·247232	·393104	42	·263865	·421382	42	·281345	·449239
43	·247502	·393579	43	·264150	·421849	43	·281643	·449699
44	·247773	·394054	44	·264435	·422318	44	·281942	·450160
45	·248044	·394528	45	·264720	·422785	45	·282242	·450621
46	·248315	·395003	46	·265005	·423253	46	·282541	·451081
47	·248587	·395478	47	·265290	·423720	47	·282841	·451542
48	·248859	·395952	48	·265575	·424187	48	·283140	·452002
49	·249131	·396427	49	·265860	·424653	49	·283440	·452462
50	·249403	·396901	50	·266146	·425120	50	·283741	·452922
51	·249675	·397374	51	·266432	·425587	51	·284042	·453382
52	·249948	·397848	52	·266719	·426053	52	·284343	·453842
53	·250220	·398321	53	·267006	·426520	53	·284644	·454302
54	·250493	·398795	54	·267293	·426987	54	·284946	·454762
55	·250766	·399269	55	·267580	·427452	55	·285248	·455222
56	·251040	·399742	56	·267867	·427918	56	·285550	·455681
57	·251314	·400216	57	·268154	·428384	57	·285852	·456141
58	·251588	·400689	58	·268442	·428850	58	·286154	·456600
59	·251862	·401161	59	·268730	·429316	59	·286457	·457059
60	·252136	·401634	60	·269019	·429782	60	·286760	·457518

39 DEGREES.			40 DEGREES.			41 DEGREES.		
Min	Nat. No.	Logarithm.	Min	Nat. No.	Logarithm.	Min	Nat. No.	Logarithm.
0	0·222854	9·348021	0	0·233955	9·369133	0	0·245291	9·389681
1	·223037	·348377	1	·234143	·369480	1	·245481	·390018
2	·223220	·348734	2	·234330	·369827	2	·245672	·390356
3	·223404	·349090	3	·234517	·370174	3	·245864	·390694
4	·223587	·349446	4	·234704	·370520	4	·246055	·391031
5	·223770	·349802	5	·234891	·370867	5	·246246	·391368
6	·223954	·350158	6	·235079	·371213	6	·246437	·391705
7	·224137	·350514	7	·235266	·371559	7	·246629	·392042
8	·224320	·350869	8	·235454	·371905	8	·246820	·392379
9	·224504	·351224	9	·235641	·372251	9	·247010	·392715
10	·224688	·351579	10	·235829	·372597	10	·247202	·393052
11	·224871	·351934	11	·236016	·372942	11	·247394	·393388
12	·225055	·352289	12	·236204	·373287	12	·247586	·393723
13	·225239	·352644	13	·236392	·373632	13	·247777	·394060
14	·225423	·352999	14	·236580	·373978	14	·247968	·394396
15	·225607	·353353	15	·236768	·374322	15	·248160	·394732
16	·225791	·353707	16	·236956	·374667	16	·248352	·395067
17	·225976	·354062	17	·237144	·375011	17	·248544	·395403
18	·226160	·354415	18	·237332	·375355	18	·248736	·395738
19	·226344	·354769	19	·237520	·375700	19	·248928	·396074
20	·226528	·355122	20	·237708	·376044	20	·249120	·396408
21	·226713	·355476	21	·237897	·376388	21	·249312	·396743
22	·226898	·355830	22	·238085	·376731	22	·249504	·397078
23	·227082	·356183	23	·238273	·377075	23	·249696	·397412
24	·227266	·356536	24	·238462	·377418	24	·249889	·397747
25	·227451	·356888	25	·238650	·377762	25	·250082	·398082
26	·227636	·357241	26	·238839	·378105	26	·250275	·398416
27	·227821	·357593	27	·239028	·378448	27	·250466	·398749
28	·228005	·357945	28	·239216	·378790	28	·250659	·399083
29	·228190	·358297	29	·239405	·379133	29	·250852	·399417
30	·228375	·358650	30	·239594	·379476	30	·251045	·399751
31	·228560	·359001	31	·239783	·379818	31	·251237	·400083
32	·228746	·359353	32	·239972	·380160	32	·251430	·400417
33	·228931	·359704	33	·240162	·380503	33	·251623	·400750
34	·229116	·360056	34	·240351	·380845	34	·251816	·401083
35	·229301	·360407	35	·240540	·381186	35	·252009	·401415
36	·229486	·360757	36	·240729	·381528	36	·252202	·401748
37	·229672	·361108	37	·240919	·381870	37	·252395	·402081
38	·229857	·361459	38	·241108	·382211	38	·252588	·402413
39	·230043	·361809	39	·241297	·382552	39	·252782	·402745
40	·230229	·362160	40	·241486	·382893	40	·252975	·403077
41	·230415	·362510	41	·241676	·383233	41	·253168	·403409
42	·230600	·362860	42	·241866	·383574	42	·253362	·403741
43	·230786	·363210	43	·242056	·383915	43	·253556	·404073
44	·230972	·363559	44	·242245	·384255	44	·253749	·404404
45	·231158	·363909	45	·242435	·384595	45	·253942	·404735
46	·231344	·364259	46	·242625	·384935	46	·254136	·405067
47	·231530	·364608	47	·242815	·385275	47	·254330	·405398
48	·231716	·364957	48	·243005	·385615	48	·254524	·405729
49	·231903	·365306	49	·243195	·385955	49	·254718	·406060
50	·232089	·365654	50	·243385	·386294	50	·254912	·406390
51	·232275	·366003	51	·243575	·386633	51	·255106	·406721
52	·232461	·366351	52	·243766	·386973	52	·255300	·407051
53	·232649	·366699	53	·243956	·387312	53	·255494	·407381
54	·232835	·367048	54	·244146	·387650	54	·255689	·407711
55	·233022	·367396	55	·244337	·387989	55	·255883	·408041
56	·233209	·367744	56	·244528	·388328	56	·256077	·408371
57	·233396	·368091	57	·244719	·388667	57	·256272	·408701
58	·233582	·368439	58	·244909	·389004	58	·256466	·409030
59	·233769	·368786	59	·245100	·389343	59	·256660	·409359
60	·233955	·369133	60	·245291	·389681	60	·256855	·409688

39 DEGREES.			40 DEGREES.			41 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·286760	9·457518	0	0·305407	9·484879	0	0·325013	9·511901
1	·287063	·457977	1	·305726	·485332	1	·325348	·512348
2	·287366	·458436	2	·306045	·485785	2	·325684	·512796
3	·287670	·458895	3	·306364	·486238	3	·326020	·513244
4	·287974	·459353	4	·306684	·486691	4	·326355	·513691
5	·288279	·459812	5	·307004	·487144	5	·326692	·514138
6	·288583	·460270	6	·307324	·487597	6	·327029	·514585
7	·288888	·460729	7	·307644	·488049	7	·327365	·515032
8	·289193	·461187	8	·307965	·488501	8	·327702	·515480
9	·289498	·461645	9	·308286	·488954	9	·328040	·515926
10	·289803	·462102	10	·308607	·489406	10	·328378	·516374
11	·290109	·462560	11	·308928	·489858	11	·328716	·516820
12	·290415	·463018	12	·309250	·490310	12	·329055	·517268
13	·290721	·463476	13	·309572	·490761	13	·329393	·517714
14	·291028	·463934	14	·309894	·491214	14	·329731	·518160
15	·291335	·464392	15	·310217	·491665	15	·330070	·518607
16	·291641	·464849	16	·310540	·492117	16	·330410	·519053
17	·291949	·465307	17	·310863	·492568	17	·330750	·519499
18	·292257	·465764	18	·311186	·493019	18	·331090	·519945
19	·292564	·466221	19	·311510	·493471	19	·331430	·520392
20	·292872	·466678	20	·311834	·493923	20	·331770	·520837
21	·293181	·467135	21	·312158	·494374	21	·332111	·521284
22	·293490	·467593	22	·312483	·494824	22	·332452	·521730
23	·293798	·468049	23	·312807	·495276	23	·332794	·522175
24	·294107	·468506	24	·313132	·495726	24	·333136	·522621
25	·294417	·468962	25	·313457	·496178	25	·333478	·523067
26	·294727	·469419	26	·313782	·496628	26	·333820	·523513
27	·295036	·469875	27	·314108	·497079	27	·334163	·523958
28	·295346	·470331	28	·314434	·497529	28	·334506	·524403
29	·295656	·470787	29	·314760	·497980	29	·334849	·524849
30	·295967	·471244	30	·315086	·498430	30	·335193	·525295
31	·296278	·471699	31	·315413	·498880	31	·335536	·525739
32	·296589	·472155	32	·315740	·499330	32	·335880	·526185
33	·296900	·472611	33	·316068	·499781	33	·336224	·526629
34	·297212	·473067	34	·316396	·500231	34	·336569	·527074
35	·297524	·473522	35	·316724	·500681	35	·336914	·527519
36	·297836	·473977	36	·317052	·501131	36	·337260	·527964
37	·298148	·474432	37	·317381	·501581	37	·337605	·528409
38	·298461	·474888	38	·317710	·502031	38	·337950	·528853
39	·298774	·475343	39	·318039	·502480	39	·338296	·529297
40	·299088	·475798	40	·318368	·502929	40	·338643	·529742
41	·299401	·476253	41	·318697	·503378	41	·338990	·530186
42	·299715	·476708	42	·319027	·503828	42	·339337	·530631
43	·300029	·477163	43	·319356	·504277	43	·339684	·531075
44	·300343	·477617	44	·319687	·504726	44	·340031	·531519
45	·300658	·478072	45	·320018	·505175	45	·340379	·531963
46	·300973	·478527	46	·320350	·505624	46	·340728	·532408
47	·301288	·478981	47	·320681	·506073	47	·341077	·532851
48	·301603	·479435	48	·321013	·506522	48	·341425	·533295
49	·301918	·479889	49	·321345	·506971	49	·341774	·533739
50	·302234	·480344	50	·321677	·507419	50	·342123	·534182
51	·302550	·480798	51	·322009	·507867	51	·342473	·534626
52	·302866	·481251	52	·322342	·508317	52	·342823	·535070
53	·303183	·481705	53	·322675	·508765	53	·343173	·535513
54	·303500	·482159	54	·323008	·509212	54	·343523	·535956
55	·303818	·482613	55	·323341	·509661	55	·343874	·536400
56	·304135	·483066	56	·323675	·510109	56	·344226	·536843
57	·304452	·483519	57	·324009	·510558	57	·344578	·537287
58	·304770	·483973	58	·324343	·511005	58	·344929	·537729
59	·305089	·484426	59	·324678	·511453	59	·345281	·538172
60	·305407	·484879	60	·325013	·511901	60	·345633	·538615

42 DEGREES.			43 DEGREES.			44 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·256855	9·409681	0	0·268646	9·429181	0	0·280660	9·448181
1	·257050	·410017	1	·268845	·429502	1	·280862	·448493
2	·257245	·410346	2	·269043	·429822	2	·281064	·448806
3	·257439	·410674	3	·269242	·430142	3	·281266	·449118
4	·257634	·411003	4	·269440	·430463	4	·281469	·449431
5	·257829	·411332	5	·269639	·430783	5	·281671	·449743
6	·258024	·411660	6	·269838	·431103	6	·281874	·450055
7	·258219	·411988	7	·270037	·431423	7	·282076	·450366
8	·258414	·412317	8	·270236	·431743	8	·282279	·450678
9	·258609	·412644	9	·270434	·432061	9	·282482	·450990
10	·258805	·412972	10	·270633	·432381	10	·282684	·451301
11	·259000	·413299	11	·270833	·432701	11	·282887	·451613
12	·259196	·413628	12	·271032	·433020	12	·283090	·451924
13	·259391	·413955	13	·271231	·433339	13	·283293	·452235
14	·259586	·414282	14	·271430	·433657	14	·283495	·452545
15	·259782	·414609	15	·271629	·433976	15	·283698	·452856
16	·259977	·414935	16	·271828	·434295	16	·283901	·453167
17	·260173	·415262	17	·272028	·434613	17	·284104	·453477
18	·260369	·415589	18	·272227	·434931	18	·284307	·453788
19	·260565	·415916	19	·272427	·435250	19	·284510	·454098
20	·260760	·416242	20	·272626	·435568	20	·284713	·454408
21	·260956	·416568	21	·272826	·435886	21	·284917	·454719
22	·261152	·416894	22	·273026	·436204	22	·285120	·455028
23	·261348	·417220	23	·273225	·436521	23	·285324	·455338
24	·261544	·417546	24	·273425	·436839	24	·285527	·455647
25	·261740	·417871	25	·273625	·437156	25	·285731	·455957
26	·261937	·418197	26	·273825	·437473	26	·285934	·456266
27	·262133	·418522	27	·274025	·437790	27	·286138	·456576
28	·262330	·418847	28	·274225	·438107	28	·286342	·456885
29	·262526	·419172	29	·274425	·438424	29	·286546	·457194
30	·262722	·419497	30	·274625	·438741	30	·286750	·457503
31	·262919	·419822	31	·274825	·439057	31	·286953	·457811
32	·263116	·420147	32	·275026	·439374	32	·287157	·458120
33	·263313	·420472	33	·275226	·439690	33	·287361	·458428
34	·263510	·420796	34	·275427	·440006	34	·287565	·458736
35	·263706	·421120	35	·275628	·440323	35	·287770	·459045
36	·263903	·421444	36	·275828	·440639	36	·287974	·459353
37	·264100	·421768	37	·276029	·440954	37	·288178	·459661
38	·264297	·422092	38	·276230	·441270	38	·288383	·459969
39	·264494	·422416	39	·276430	·441585	39	·288587	·460277
40	·264691	·422739	40	·276631	·441901	40	·288792	·460585
41	·264888	·423062	41	·276832	·442216	41	·288996	·460892
42	·265086	·423386	42	·277033	·442531	42	·289200	·461199
43	·265283	·423709	43	·277234	·442846	43	·289405	·461506
44	·265480	·424032	44	·277435	·443161	44	·289610	·461813
45	·265677	·424354	45	·277636	·443475	45	·289814	·462120
46	·265875	·424677	46	·277837	·443790	46	·290019	·462427
47	·266072	·424999	47	·278039	·444105	47	·290224	·462734
48	·266270	·425322	48	·278240	·444419	48	·290429	·463041
49	·266468	·425644	49	·278441	·444734	49	·290634	·463347
50	·266666	·425967	50	·278642	·445047	50	·290839	·463653
51	·266863	·426289	51	·278844	·445361	51	·291044	·463959
52	·267061	·426611	52	·279045	·445675	52	·291249	·464265
53	·267259	·426932	53	·379247	·445989	53	·291454	·464571
54	·267457	·427254	54	·279448	·446302	54	·291660	·464877
55	·267655	·427576	55	·279650	·446615	55	·291865	·465182
56	·267853	·427897	56	·279852	·446929	56	·292070	·465488
57	·268052	·428219	57	·280054	·447242	57	·292276	·465794
58	·268250	·428539	58	·280256	·447555	58	·292482	·466099
59	·268448	·428860	59	·280458	·447868	59	·292688	·466404
60	·268646	·429181	60	·280660	·448181	60	·292893	·466709

42 DEGREES.			43 DEGREES.			44 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·345633	9·538615	0	0·367328	9·565054	0	0·390164	9·591247
1	·345985	·539057	1	·367699	·565492	1	·390554	·591681
2	·346338	·539500	2	·368070	·565930	2	·390945	·592116
3	·346691	·539942	3	·368441	·566368	3	·391336	·592550
4	·347044	·540385	4	·368813	·566807	4	·391728	·592985
5	·347399	·540828	5	·369186	·567245	5	·392121	·593420
6	·347753	·541270	6	·369559	·567684	6	·392513	·593854
7	·348107	·541712	7	·369932	·568122	7	·392905	·594288
8	·348462	·542156	8	·370305	·568560	8	·393298	·594722
9	·348817	·542597	9	·370678	·568997	9	·393692	·595157
10	·349172	·543039	10	·371052	·569435	10	·394085	·595590
11	·349528	·543482	11	·371427	·569874	11	·394479	·596024
12	·349884	·543924	12	·371801	·570311	12	·394874	·596459
13	·350240	·544366	13	·372176	·570749	13	·395269	·596893
14	·350597	·544808	14	·372551	·571186	14	·395664	·597326
15	·350954	·545249	15	·372926	·571623	15	·396059	·597760
16	·351310	·545690	16	·373302	·572061	16	·396455	·598194
17	·351667	·546132	17	·373679	·572498	17	·396851	·598627
18	·352025	·546574	18	·374055	·572935	18	·397247	·599061
19	·352384	·547016	19	·374432	·573373	19	·397644	·599495
20	·352742	·547457	20	·374809	·573810	20	·398041	·599928
21	·353100	·547898	21	·375187	·574248	21	·398439	·600362
22	·353459	·548339	22	·375565	·574685	22	·398837	·600795
23	·353818	·548780	23	·375943	·575122	23	·399235	·601229
24	·354178	·549222	24	·376321	·575559	24	·399633	·601661
25	·354538	·549663	25	·376700	·575995	25	·400032	·602095
26	·354898	·550104	26	·377079	·576432	26	·400431	·602528
27	·355258	·550544	27	·377478	·576868	27	·400831	·602962
28	·355619	·550985	28	·377838	·577306	28	·401231	·603395
29	·355980	·551425	29	·378218	·577742	29	·401631	·603828
30	·356341	·551866	30	·378599	·578179	30	·402032	·604261
31	·356703	·552307	31	·378979	·578615	31	·402433	·604698
32	·357065	·552748	32	·379360	·579052	32	·402834	·605126
33	·357427	·553189	33	·379741	·579488	33	·403235	·605559
34	·357790	·553629	34	·380123	·579924	34	·403637	·605991
35	·358153	·554069	35	·380505	·580361	35	·404040	·606425
36	·358516	·554509	36	·380888	·580797	36	·404443	·606857
37	·358880	·554949	37	·381270	·581233	37	·404846	·607290
38	·359244	·555389	38	·381653	·581669	38	·405249	·607722
39	·359608	·555829	39	·382037	·582105	39	·405653	·608155
40	·359972	·556269	40	·382420	·582541	40	·406058	·608588
41	·360337	·556709	41	·382804	·582977	41	·406462	·609020
42	·360702	·557149	42	·383188	·583412	42	·406867	·609452
43	·361068	·557589	43	·383573	·583848	43	·407272	·609884
44	·361433	·558028	44	·383958	·584284	44	·407678	·610316
45	·361799	·558467	45	·384343	·584719	45	·408084	·610748
46	·362166	·558907	46	·384729	·585155	46	·408490	·611181
47	·362532	·559346	47	·385115	·585591	47	·408896	·611613
48	·362899	·559786	48	·385501	·586026	48	·409303	·612045
49	·363266	·560225	49	·385888	·586462	49	·409710	·612477
50	·363634	·560665	50	·386275	·586896	50	·410117	·612908
51	·364002	·561104	51	·386662	·587332	51	·410525	·613340
52	·364370	·561543	52	·387050	·587767	52	·410934	·613772
53	·364739	·561982	53	·387438	·588203	53	·411342	·614203
54	·365108	·562421	54	·387826	·588637	54	·411751	·614635
55	·365477	·562860	55	·388215	·589072	55	·412160	·615066
56	·365847	·563299	56	·388604	·589507	56	·412570	·615498
57	·366217	·563738	57	·388993	·589942	57	·412981	·615930
58	·366587	·564176	58	·389383	·590377	58	·413391	·616361
59	·366957	·564615	59	·389773	·590812	59	·413802	·616793
60	·367328	·565054	60	·390164	·591247	60	·414213	·617224

45 DEGREES.			46 DEGREES.			47 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm
0	0·292893	9·466709	0	0·305342	9·484786	0	0·318001	9·502429
1	·293099	·467014	1	·305551	·485083	1	·318214	·502720
2	·293305	·467319	2	·305760	·485381	2	·318427	·503010
3	·293511	·467624	3	·305970	·485678	3	·318640	·503300
4	·293717	·467928	4	·306180	·485976	4	·318853	·503590
5	·293923	·468233	5	·306389	·486273	5	·319066	·503880
6	·294129	·468537	6	·306598	·486570	6	·319279	·504170
7	·294335	·468842	7	·306808	·486866	7	·319492	·504460
8	·294541	·469146	8	·307017	·487163	8	·319705	·504750
9	·294747	·469449	9	·307227	·487460	9	·319918	·505039
10	·294953	·469753	10	·307437	·487757	10	·320132	·505329
11	·295159	·470056	11	·307647	·488053	11	·320345	·505618
12	·295365	·470360	12	·307857	·488349	12	·320558	·505907
13	·295572	·470663	13	·308067	·488645	13	·320772	·506196
14	·295778	·470966	14	·308277	·488941	14	·320986	·506486
15	·295985	·471270	15	·308487	·489237	15	·321200	·506775
16	·296192	·471573	16	·308697	·489532	16	·321414	·507064
17	·296398	·471876	17	·308907	·489828	17	·321627	·507352
18	·296605	·472179	18	·309118	·490124	18	·321841	·507641
19	·296812	·472481	19	·309328	·490419	19	·322054	·507929
20	·297019	·472784	20	·309538	·490714	20	·322268	·508217
21	·297226	·473087	21	·309748	·491009	21	·322482	·508505
22	·297433	·473389	22	·309959	·491305	22	·322696	·508793
23	·297640	·473691	23	·310170	·491600	23	·322910	·509081
24	·297847	·473993	24	·310381	·491895	24	·323124	·509369
25	·298054	·474295	25	·310591	·492189	25	·323338	·509657
26	·298261	·474597	26	·310801	·492483	26	·323552	·509944
27	·298468	·474898	27	·311012	·492778	27	·323766	·510231
28	·298676	·475200	28	·311223	·493072	28	·323981	·510519
29	·298883	·475502	29	·311434	·493366	29	·324196	·510807
30	·299091	·475803	30	·311645	·493660	30	·324410	·511094
31	·299298	·476104	31	·311856	·493955	31	·324624	·511381
32	·299506	·476405	32	·312068	·494249	32	·324839	·511668
33	·299713	·476706	33	·312279	·494543	33	·325053	·511955
34	·299921	·477007	34	·312490	·494836	34	·325268	·512242
35	·300129	·477308	35	·312702	·495130	35	·325483	·512529
36	·300337	·477609	36	·312913	·495423	36	·325698	·512815
37	·300545	·477909	37	·313124	·495716	37	·325912	·513101
38	·300752	·478209	38	·313335	·496009	38	·326127	·513387
39	·300960	·478509	39	·313547	·496302	39	·326342	·513673
40	·301168	·478809	40	·313759	·496596	40	·326557	·513959
41	·301376	·479109	41	·313970	·496888	41	·326772	·514245
42	·301584	·479409	42	·314182	·497181	42	·326987	·514531
43	·301793	·479709	43	·314393	·497474	43	·327203	·514817
44	·302001	·480008	44	·314605	·497766	44	·327418	·515102
45	·302210	·480308	45	·314817	·498058	45	·327633	·515388
46	·302418	·480607	46	·315029	·498350	46	·327848	·515673
47	·302626	·480907	47	·315241	·498642	47	·328064	·515958
48	·302835	·481206	48	·315453	·498934	48	·328280	·516244
49	·303043	·481505	49	·315665	·499226	49	·328495	·516529
50	·303252	·481804	50	·315877	·499518	50	·328711	·516814
51	·303461	·482102	51	·316089	·499809	51	·328926	·517098
52	·303670	·482401	52	·316301	·500101	52	·329142	·517383
53	·303878	·482699	53	·316513	·500392	53	·329358	·517668
54	·304087	·482998	54	·316726	·500684	54	·329573	·517952
55	·304296	·483296	55	·316939	·500976	55	·329789	·518236
56	·304505	·483595	56	·317152	·501267	56	·330005	·518521
57	·304714	·483893	57	·317364	·501557	57	·330221	·518805
58	·304923	·484191	58	·317576	·501848	58	·330437	·519089
59	·305132	·484488	59	·317789	·502139	59	·330653	·519373
60	·305342	·484786	60	·318001	·502429	60	·330869	·519656

45 DEGREES.			46 DEGREES.			47 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·414213	9·617224	0	0·439557	9·643015	0	0·466279	9·668646
1	·414625	·617655	1	·439991	·643444	1	·466737	·669072
2	·415037	·618087	2	·440425	·643872	2	·467195	·669498
3	·415450	·618518	3	·440859	·644300	3	·467653	·669924
4	·415863	·618950	4	·441294	·644728	4	·468112	·670350
5	·416276	·619381	5	·441729	·645156	5	·468571	·670776
6	·416689	·619811	6	·442164	·645584	6	·469030	·671201
7	·417102	·620242	7	·442600	·646012	7	·469490	·671627
8	·417516	·620673	8	·443037	·646440	8	·469951	·672053
9	·417930	·621104	9	·443475	·646869	9	·470412	·672478
10	·418345	·621535	10	·443912	·647297	10	·470873	·672904
11	·418760	·621965	11	·444350	·647725	11	·471335	·673330
12	·419176	·622396	12	·444788	·648153	12	·471797	·673755
13	·419592	·622827	13	·445226	·648581	13	·472260	·674181
14	·420008	·623257	14	·445665	·649009	14	·472723	·674607
15	·420425	·623688	15	·446105	·649437	15	·473187	·675033
16	·420842	·624119	16	·446544	·649864	16	·473650	·675458
17	·421259	·624549	17	·446984	·650292	17	·474114	·675883
18	·421677	·624980	18	·447425	·650720	18	·474579	·676309
19	·422095	·625410	19	·447865	·651147	19	·475044	·676734
20	·422513	·625840	20	·448306	·651574	20	·475509	·677159
21	·422932	·626271	21	·448748	·652002	21	·475975	·677584
22	·423351	·626701	22	·449190	·652430	22	·476442	·678010
23	·423771	·627131	23	·449632	·652857	23	·476908	·678435
24	·424191	·627561	24	·450075	·653285	24	·477375	·678860
25	·424611	·627991	25	·450518	·653712	25	·477843	·679285
26	·425031	·628421	26	·450961	·654139	26	·478311	·679710
27	·425452	·628851	27	·451405	·654567	27	·478779	·680135
28	·425874	·629281	28	·451850	·654994	28	·479248	·680560
29	·426296	·629711	29	·452294	·655421	29	·479718	·680986
30	·426718	·630141	30	·452739	·655848	30	·480188	·681411
31	·427141	·630571	31	·453185	·656276	31	·480658	·681836
32	·427563	·631000	32	·453631	·656703	32	·481129	·682261
33	·427986	·631430	33	·454077	·657130	33	·481600	·682686
34	·428410	·631860	34	·454524	·657557	34	·482071	·683111
35	·428835	·632290	35	·454971	·657984	35	·482543	·683536
36	·429260	·632720	36	·455419	·658411	36	·483015	·683960
37	·429684	·633149	37	·455867	·658838	37	·483487	·684385
38	·430109	·633578	38	·456315	·659265	38	·483960	·684809
39	·430534	·634008	39	·456764	·659692	39	·484433	·685234
40	·430960	·634437	40	·457213	·660119	40	·484907	·685659
41	·431386	·634866	41	·457662	·660545	41	·485381	·686083
42	·431812	·635295	42	·458112	·660972	42	·485856	·686508
43	·432239	·635724	43	·458562	·661398	43	·486332	·686933
44	·432667	·636154	44	·459013	·661825	44	·486808	·687358
45	·433095	·636583	45	·459464	·662251	45	·487284	·687782
46	·433523	·637012	46	·459915	·662678	46	·487760	·688206
47	·433951	·637441	47	·460367	·663104	47	·488237	·688631
48	·434380	·637870	48	·460820	·663531	48	·488714	·689055
49	·434810	·638299	49	·461273	·663958	49	·489192	·689479
50	·435239	·638728	50	·461726	·664384	50	·489670	·689904
51	·435669	·639157	51	·462179	·664810	51	·490149	·690328
52	·436100	·639586	52	·462632	·665236	52	·490628	·690752
53	·436530	·640014	53	·463087	·665663	53	·491108	·691177
54	·436961	·640443	54	·463542	·666089	54	·491588	·691601
55	·437393	·640872	55	·463998	·666516	55	·492069	·692026
56	·437825	·641301	56	·464453	·666942	56	·492550	·692450
57	·438258	·641730	57	·464906	·667368	57	·493031	·692874
58	·438690	·642158	58	·465365	·667794	58	·493512	·693298
59	·439123	·642586	59	·465822	·668220	59	·493994	·693722
60	·439557	·643015	60	·466279	·668646	60	·494477	·694146

48 DEGREES.			49 DEGREES.			50 DEGREES.		
Min	Nat No.	Logarithm.	Min	Nat No.	Logarithm	Min.	Nat. No	Logarithm.
0	0·330869	9·519656	0	0·343941	9·536484	0	0·357213	9·552927
1	·331085	·519940	1	·344160	·536761	1	·357435	·553197
2	·331302	·520224	2	·344380	·537038	2	·357658	·553468
3	·331518	·520507	3	·344600	·537315	3	·357881	·553739
4	·331735	·520791	4	·344820	·537592	4	·358104	·554009
5	·331951	·521074	5	·345039	·537868	5	·358327	·554280
6	·332167	·521357	6	·345259	·538145	6	·358550	·554550
7	·332384	·521640	7	·345479	·538422	7	·358774	·554821
8	·332600	·521923	8	·345699	·538698	8	·358997	·555091
9	·332817	·522206	9	·345919	·538975	9	·359220	·555361
10	·333034	·522489	10	·346139	·539251	10	·359443	·555631
11	·333251	·522771	11	·346359	·539527	11	·359666	·555900
12	·333468	·523054	12	·346580	·539803	12	·359890	·556170
13	·333684	·523336	13	·346800	·540079	13	·360114	·556440
14	·333901	·523618	14	·347020	·540354	14	·360337	·556709
15	·334118	·523900	15	·347240	·540630	15	·360561	·556979
16	·334335	·524182	16	·347461	·540906	16	·360784	·557248
17	·334552	·524464	17	·347681	·541181	17	·361008	·557517
18	·334770	·524746	18	·347901	·541456	18	·361232	·557786
19	·334987	·525028	19	·348122	·541731	19	·361456	·558055
20	·335204	·525309	20	·348342	·542006	20	·361680	·558324
21	·335421	·525591	21	·348563	·542281	21	·361904	·558593
22	·335638	·525872	22	·348784	·542556	22	·362128	·558862
23	·335856	·526153	23	·349005	·542831	23	·362352	·559131
24	·336073	·526434	24	·349226	·543106	24	·362576	·559399
25	·336291	·526715	25	·349447	·543381	25	·362800	·559668
26	·336509	·526997	26	·349668	·543656	26	·363024	·559936
27	·336727	·527278	27	·349889	·543930	27	·363249	·560204
28	·336944	·527558	28	·350110	·544204	28	·363473	·560472
29	·337162	·527838	29	·350331	·544479	29	·363697	·560740
30	·337380	·528119	30	·350552	·544753	30	·363922	·561008
31	·337598	·528400	31	·350773	·545026	31	·364146	·561276
32	·337816	·528680	32	·350994	·545300	32	·364371	·561544
33	·338034	·528960	33	·351215	·545574	33	·364595	·561811
34	·338252	·529240	34	·351437	·545848	34	·364820	·562079
35	·338470	·529520	35	·351659	·546122	35	·365045	·562346
36	·338688	·529800	36	·351880	·546395	36	·365269	·562613
37	·338906	·530080	37	·352102	·546668	37	·365495	·562881
38	·339124	·530359	38	·352323	·546941	38	·365719	·563148
39	·339342	·530638	39	·352544	·547214	39	·365944	·563415
40	·339560	·530918	40	·352766	·547487	40	·366169	·563682
41	·339779	·531197	41	·352988	·547760	41	·366394	·563948
42	·339998	·531476	42	·353210	·548033	42	·366619	·564215
43	·340216	·531755	43	·353432	·548306	43	·366844	·564482
44	·340435	·532034	44	·353654	·548578	44	·367069	·564748
45	·340654	·532313	45	·353876	·548851	45	·367294	·565015
46	·340873	·532592	46	·354098	·549124	46	·367520	·565281
47	·341092	·532871	47	·354320	·549396	47	·367745	·565547
48	·341311	·533150	48	·354542	·549668	48	·367970	·565813
49	·341529	·533428	49	·354764	·549940	49	·368196	·566079
50	·341748	·533706	50	·354987	·550212	50	·368421	·566345
51	·341967	·533985	51	·355209	·550484	51	·368647	·566611
52	·342187	·534263	52	·355431	·550756	52	·368873	·566877
53	·342406	·534541	53	·355653	·551027	53	·369098	·567142
54	·342625	·534819	54	·355876	·551299	54	·369324	·567408
55	·342844	·535097	55	·356098	·551570	55	·369550	·567673
56	·343063	·535374	56	·356321	·551842	56	·369776	·567939
57	·343283	·535652	57	·356544	·552114	57	·370002	·568204
58	·343502	·535929	58	·356767	·552385	58	·370228	·568469
59	·343721	·536206	59	·356990	·552656	59	·370454	·568734
60	·343941	·536484	60	·357213	·552927	60	·370680	·568999

48 DEGREES.			49 DEGREES.			50 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm
0	0·494477	9·694146	0	0·524253	9·719541	0	0·555724	9·744859
1	·494960	·694570	1	·524763	·719963	1	·556263	·745280
2	·495443	·694994	2	·525274	·720386	2	·556804	·745702
3	·495927	·695418	3	·525785	·720808	3	·557344	·746123
4	·496412	·695842	4	·526297	·721231	4	·557885	·746545
5	·496896	·696266	5	·526809	·721653	5	·558427	·746966
6	·497381	·696689	6	·527322	·722076	6	·558969	·747388
7	·497867	·697113	7	·527835	·722498	7	·559511	·747809
8	·498353	·697537	8	·528348	·722920	8	·560054	·748230
9	·498840	·697961	9	·528863	·723343	9	·560598	·748652
10	·499327	·698385	10	·529378	·723766	10	·561142	·749073
11	·499814	·698808	11	·529893	·724188	11	·561687	·749494
12	·500302	·699232	12	·530408	·724610	12	·562232	·749916
13	·500790	·699656	13	·530924	·725032	13	·562778	·750337
14	·501279	·700079	14	·531440	·725454	14	·563324	·750758
15	·501768	·700503	15	·531957	·725877	15	·563871	·751180
16	·502258	·700927	16	·532475	·726299	16	·564418	·751601
17	·502749	·701351	17	·532992	·726721	17	·564966	·752022
18	·503239	·701774	18	·533510	·727143	18	·565514	·752443
19	·503730	·702198	19	·534029	·727565	19	·566063	·752865
20	·504221	·702621	20	·534548	·727987	20	·566612	·753286
21	·504713	·703045	21	·535068	·728409	21	·567162	·753707
22	·505205	·703468	22	·535589	·728832	22	·567712	·754128
23	·505698	·703891	23	·536110	·729254	23	·568263	·754549
24	·506191	·704315	24	·536631	·729676	24	·568815	·754971
25	·506685	·704738	25	·537153	·730098	25	·569367	·755392
26	·507180	·705162	26	·537675	·730520	26	·569919	·755818
27	·507674	·705585	27	·538198	·730942	27	·570472	·756214
28	·508169	·706008	28	·538721	·731364	28	·571025	·756655
29	·508664	·706431	29	·539245	·731786	29	·571579	·757076
30	·509160	·706854	30	·539769	·732208	30	·572134	·757498
31	·509657	·707278	31	·540294	·732630	31	·572689	·757919
32	·510154	·707701	32	·540819	·733052	32	·573244	·758340
33	·510651	·708124	33	·541345	·733474	33	·573800	·758761
34	·511148	·708547	34	·541871	·733896	34	·574357	·759182
35	·511646	·708970	35	·542398	·734318	35	·574914	·759603
36	·512145	·709394	36	·542925	·734740	36	·575472	·760024
37	·512645	·709817	37	·543452	·735161	37	·576030	·760445
38	·513145	·710240	38	·543980	·735583	38	·576589	·760866
39	·513645	·710663	39	·544508	·736004	39	·577148	·761287
40	·514146	·711087	40	·545037	·736426	40	·577708	·761708
41	·514647	·711509	41	·545567	·736848	41	·578268	·762129
42	·515148	·711932	42	·546097	·737270	42	·578829	·762550
43	·515650	·712355	43	·546628	·737692	43	·579390	·762971
44	·516152	·712778	44	·547159	·738114	44	·579952	·763392
45	·516655	·713200	45	·547690	·738535	45	·580514	·763813
46	·517158	·713623	46	·548222	·738957	46	·581077	·764234
47	·517662	·714046	47	·548755	·739379	47	·581641	·764655
48	·518166	·714469	48	·549288	·739800	48	·582205	·765076
49	·518670	·714892	49	·549821	·740221	49	·582770	·765497
50	·519175	·715314	50	·550355	·740643	50	·583335	·765918
51	·519681	·715737	51	·550890	·741065	51	·583900	·766339
52	·520188	·716160	52	·551426	·741487	52	·584466	·766760
53	·520695	·716583	53	·551961	·741908	53	·585033	·767181
54	·521202	·717006	54	·552497	·742330	54	·585600	·767602
55	·521709	·717428	55	·553033	·742751	55	·586168	·768022
56	·522216	·717850	56	·553571	·743173	56	·586737	·768443
57	·522725	·718273	57	·554109	·743595	57	·587306	·768864
58	·523234	·718696	58	·554647	·744017	58	·587875	·769285
59	·523744	·719118	59	·555185	·744438	59	·588445	·769706
60	·524253	·719541	60	·555724	·744859	60	·589016	·770127

51 DEGREES.			52 DEGREES.			53 DEGREES.		
Min.	Nat. No	Logarithm	Min	Nat. No	Logarithm	Min.	Nat. No	Logarithm
0	0·370680	9·568999	0	0·384339	9·584714	0	0·398185	·600085
1	·370906	·569264	1	·384568	·584973	1	·398417	·600338
2	·371132	·569528	2	·384797	·585232	2	·398650	·600591
3	·371358	·569793	3	·385027	·585491	3	·398882	·600845
4	·371584	·570057	4	·385256	·585749	4	·399115	·601098
5	·371810	·570322	5	·385485	·586008	5	·399347	·601351
6	·372037	·570586	6	·385715	·586266	6	·399580	·601603
7	·372263	·570850	7	·385944	·586525	7	·399812	·601856
8	·372490	·571114	8	·386174	·586783	8	·400045	·602109
9	·372716	·571378	9	·386404	·587041	9	·400278	·602362
10	·372943	·571642	10	·386633	·587299	10	·400510	·602614
11	·373170	·571906	11	·386863	·587557	11	·400743	·602867
12	·373396	·572170	12	·387093	·587815	12	·400976	·603119
13	·373623	·572434	13	·387323	·588073	13	·401209	·603371
14	·373850	·572697	14	·387553	·588331	14	·401442	·603623
15	·374076	·572960	15	·387783	·588588	15	·401675	·603875
16	·374303	·573224	16	·388013	·588846	16	·401908	·604127
17	·374530	·573487	17	·388243	·589103	17	·402142	·604379
18	·374757	·573750	18	·388473	·589361	18	·402375	·604631
19	·374984	·574013	19	·388703	·589618	19	·402608	·604882
20	·375211	·574276	20	·388933	·589875	20	·402841	·605134
21	·375439	·574539	21	·389164	·590132	21	·403074	·605385
22	·375666	·574802	22	·389394	·590389	22	·403308	·605637
23	·375893	·575064	23	·389624	·590646	23	·403541	·605888
24	·376120	·575327	24	·389855	·590903	24	·403775	·606139
25	·376348	·575589	25	·390085	·591160	25	·404009	·606391
26	·376575	·575852	26	·390316	·591416	26	·404242	·606642
27	·376803	·576114	27	·390547	·591673	27	·404476	·606893
28	·377030	·576376	28	·390777	·591929	28	·404710	·607144
29	·377258	·576638	29	·391008	·592186	29	·404943	·607394
30	·377485	·576900	30	·391239	·592442	30	·405177	·607645
31	·377713	·577162	31	·391469	·592698	31	·405410	·607896
32	·377941	·577424	32	·391700	·592954	32	·405645	·608146
33	·378168	·577685	33	·391931	·593210	33	·405879	·608397
34	·378396	·577947	34	·392162	·593466	34	·406113	·608647
35	·378624	·578208	35	·392393	·593721	35	·406347	·608897
36	·378852	·578470	36	·392624	·593977	36	·406581	·609147
37	·379080	·578731	37	·392855	·594233	37	·406815	·609397
38	·379308	·578992	38	·393086	·594488	38	·407049	·609647
39	·379536	·579253	39	·393317	·594743	39	·407284	·609897
40	·379764	·579514	40	·393549	·594999	40	·407518	·610147
41	·379992	·579775	41	·393780	·595254	41	·407753	·610397
42	·380221	·580036	42	·394012	·595509	42	·407987	·610646
43	·380449	·580297	43	·394243	·595764	43	·408221	·610896
44	·380677	·580557	44	·394474	·596019	44	·408456	·611145
45	·380906	·580818	45	·394706	·596274	45	·408690	·611394
46	·381134	·581078	46	·394938	·596528	46	·408925	·611644
47	·381363	·581339	47	·395169	·596783	47	·409160	·611893
48	·381592	·581599	48	·395401	·597038	48	·409394	·612142
49	·381820	·581859	49	·395632	·597292	49	·409629	·612391
50	·382049	·582119	50	·395864	·597546	50	·409864	·612640
51	·382278	·582379	51	·396096	·597801	51	·410099	·612888
52	·382506	·582639	52	·396328	·598055	52	·410334	·613137
53	·382735	·582899	53	·396560	·598309	53	·410569	·613386
54	·382964	·583158	54	·396792	·598563	54	·410804	·613634
55	·383193	·583418	55	·397024	·598817	55	·411039	·613883
56	·383422	·583677	56	·397256	·599071	56	·411274	·614131
57	·383651	·583937	57	·397488	·599324	57	·411509	·614379
58	·383880	·584196	58	·397720	·599578	58	·411744	·614627
59	·384110	·584455	59	·397953	·599832	59	·411979	·614875
60	·384339	·584714	60	·398185	·600085	60	·412215	·615124

51 DEGREES.			52 DEGREES.			53 DEGREES.		
Min.	Nat. No	Logarithm	Min	Nat No	Logarithm	Min	Nat No	Logarithm
0	0·589016	9·770127	0	0·624269	9·795872	0	0·661640	9·820622
1	·589587	·770548	1	·624875	·795793	1	·662282	·821043
2	·590159	·770969	2	·625480	·796214	2	·662924	·821464
3	·590731	·771389	3	·626086	·796634	3	·663567	·821885
4	·591303	·771810	4	·626693	·797055	4	·664211	·822306
5	·591876	·772231	5	·627300	·797476	5	·664855	·822727
6	·592450	·772652	6	·627908	·797896	6	·665500	·823148
7	·593025	·773073	7	·628517	·798317	7	666145	·823569
8	·593600	·773494	8	·629126	·798738	8	·666791	·823990
9	·594175	·773914	9	·629736	·799158	9	·667439	·824411
10	·594751	·774335	10	·630346	·799579	10	·668086	·824833
11	·595327	·774756	11	·630957	·800000	11	·668734	·825254
12	·595904	·775177	12	·631569	·800421	12	·669383	·825675
13	·596482	·775598	13	·632181	·800841	13	·670032	·826096
14	·597060	776018	14	·632794	·801262	14	·670682	·826517
15	·597639	·776439	15	·633407	·801683	15	·671333	·826938
16	·598219	·776860	16	·634021	·802104	16	·671985	·827360
17	·598799	·777281	17	·634635	·802524	17	672637	·827781
18	·599380	·777702	18	·635251	·802945	18	·673290	·828202
19	·599960	·778122	19	·635867	·803366	19	·673943	·828623
20	·600542	·778543	20	·636483	·803787	20	·674597	·829044
21	·601124	·778964	21	·637100	·804207	21	·675252	·829466
22	·601706	·779385	22	·637717	·804628	22	·675907	·829887
23	·602289	·779805	23	·638335	·805049	23	·676563	·830308
24	·602873	·780226	24	·638954	·805470	24	·677220	·830729
25	·603458	·780647	25	·639574	·805891	25	·677877	·831151
26	·604043	·781068	26	·640194	·806311	26	·678534	·831572
27	·604628	·781488	27	·640814	·806732	27	·679193	·831993
28	605214	·781909	28	·641435	·807153	28	·679852	·832415
29	·605800	·782330	29	·642057	·807574	29	·680512	·832836
30	·606387	·782750	30	·642680	·807995	30	·681173	·833257
31	·606975	·783171	31	·643303	·808415	31	·681834	·833679
32	·607564	·783592	32	·643926	·808836	32	·682496	·834100
33	·608153	·784013	33	·644550	·809257	33	·683159	·834522
34	·608742	·784433	34	·645175	·809678	34	·683822	·834943
35	·609332	·784854	35	·645801	·810099	35	·684486	·835364
36	·609923	·785275	36	·646427	·810520	36	·685150	·835786
37	·610514	·785696	37	·647054	·810940	37	·685815	·836207
38	·611106	·786116	38	·647681	·811361	38	·686481	·836629
39	·611698	·786537	39	·648309	·811782	39	·687148	·837050
40	·612291	·786958	40	·648938	·812203	40	·687815	·837472
41	·612884	·787378	41	·649567	·812624	41	·688483	·837893
42	·613478	·787799	42	·650197	·813045	42	·689152	·838315
43	·614073	·788220	43	·650827	·813466	43	·689821	·838736
44	·614668	·788640	44	·651458	·813887	44	·690491	·839158
45	·615264	·789061	45	·652090	·814307	45	·691161	·839579
46	·615860	·789482	46	·652722	·814728	46	·691832	·840001
47	·615457	·789903	47	·653355	·815149	47	·692504	·840423
48	·617054	·790323	48	·653989	·815570	48	·693177	·840844
49	·617652	·790744	49	·654623	·815991	49	·693850	·841266
50	·618251	·791165	50	·655258	·816412	50	·694524	·841688
51	·618850	·791586	51	·655893	·816833	51	·695199	·842109
52	·619450	·792006	52	·656529	·817254	52	·695874	·842531
53	·620050	·792427	53	·657166	·817675	53	·696550	·842953
54	·620651	·792848	54	·657803	·818096	54	·697227	·843374
55	·621253	·793268	55	·658441	·818517	55	·697904	·843796
56	·621855	·793689	56	·659080	·818938	56	·698582	·844218
57	·622458	·794110	57	·659719	·819359	57	·699261	·844639
58	·623061	·794531	58	·660359	·819780	58	·699941	·845061
59	·623665	·794951	59	·660999	·820201	59	·700621	·845483
60	·624269	·795372	60	·661640	·820622	60	·701302	·845905

54 DEGREES.			55 DEGREES.			56 DEGREES.		
Min	Nat No	Logarithm	Min	Nat No	Logarithm	Min.	Nat No	Logarithm.
0	0·412215	9·615124	0	0·426423	9·629841	0	0·440807	9·644249
1	·412450	·615371	1	·426662	·630084	1	·441048	·644486
2	·412685	·615619	2	·426900	·630326	2	·441289	·644724
3	·412921	·615867	3	·427139	·630569	3	·441531	·644961
4	·413156	·616114	4	·427377	·630811	4	·441772	·645198
5	·413392	·616362	5	·427616	·631054	5	·442013	·645435
6	·413628	·616610	6	·427854	·631296	6	·442255	·645673
7	·413863	·616857	7	·428093	·631538	7	·442496	·645910
8	·414099	·617104	8	·428331	·631780	8	·442738	·646147
9	·414335	·617351	9	·428570	·632022	9	·442980	·646384
10	·414571	·617599	10	·428809	·632264	10	·443221	·646620
11	·414807	·617846	11	·429047	·632505	11	·443463	·646857
12	·415042	·618092	12	·429286	·632747	12	·443704	·647094
13	·415278	·618339	13	·429525	·632989	13	·443946	·647330
14	·415514	·618586	14	·429764	·633230	14	·444188	·647567
15	·415750	·618833	15	·430003	·633472	15	·444430	·647803
16	·415986	·619079	16	·430042	·633713	16	·444672	·648040
17	·416223	·619326	17	·430481	·633954	17	·444914	·648276
18	·416459	·619572	18	·430720	·634195	18	·445156	·648512
19	·416695	·619818	19	·430960	·634437	19	·445398	·648748
20	·416931	·620065	20	·431199	·634678	20	·445640	·648984
21	·417168	·620311	21	·431438	·634919	21	·445882	·649220
22	·417404	·620557	22	·431677	·635159	22	·446224	·649456
23	·417641	·620803	23	·431917	·635400	23	·446366	·649691
24	·417877	·621049	24	·432156	·635641	24	·446608	·649927
25	·418114	·621294	25	·432396	·635881	25	·446851	·650162
26	·418350	·621540	26	·432635	·636122	26	·447093	·650398
27	·418587	·621786	27	·432875	·636362	27	·447335	·650633
28	·418823	·622031	28	·433114	·636603	28	·447578	·650869
29	·419060	·622276	29	·433354	·636843	29	·447820	·651104
30	·419297	·622522	30	·433594	·637083	30	·448063	·651339
31	·419534	·622767	31	·433833	·637323	31	·448306	·651574
32	·419771	·623012	32	·434073	·637563	32	·448548	·651809
33	·420008	·623257	33	·434313	·637803	33	·448791	·652044
34	·420245	·623502	34	·434553	·638043	34	·449034	·652279
35	·420482	·623747	35	·434793	·638283	35	·449276	·652514
36	·420719	·623992	36	·435033	·638522	36	·449519	·652748
37	·420956	·624237	37	·435273	·638762	37	·449762	·652983
38	·421193	·624481	38	·435513	·639001	38	·450005	·653217
39	·421430	·624726	39	·435753	·639241	39	·450248	·653452
40	·421668	·624970	40	·435993	·639480	40	·450491	·653686
41	·421905	·625215	41	·436234	·639719	41	·450734	·653920
42	·422143	·625459	42	·436474	·639958	42	·450977	·654155
43	·422380	·625703	43	·436714	·640197	43	·451220	·654389
44	·422617	·625947	44	·436955	·640436	44	·451463	·654623
45	·422855	·626191	45	·437195	·640675	45	·451707	·654857
46	·423092	·626435	46	·437435	·640914	46	·451950	·655090
47	·423330	·626679	47	·437676	·641153	47	·452193	·655324
48	·423568	·626923	48	·437916	·641391	48	·452437	·655558
49	·423805	·627166	49	·438157	·641630	49	·452680	·655791
50	·424043	·627410	50	·438398	·641868	50	·452924	·656025
51	·424281	·627654	51	·438639	·642107	51	·453167	·656258
52	·424519	·627897	52	·438879	·642345	52	·453411	·656492
53	·424757	·628140	53	·439120	·642583	53	·453654	·656725
54	·424995	·628384	54	·439361	·642821	54	·453898	·656958
55	·425233	·628627	55	·439602	·643060	55	·454142	·657191
56	·425471	·628870	56	·439843	·643298	56	·454385	·657424
57	·425709	·629113	57	·440084	·643535	57	·454629	·657657
58	·425947	·629356	58	·440325	·643773	58	·454873	·657890
59	·426185	·629598	59	·440566	·644011	59	·455117	·658123
60	·426423	·629841	60	·440807	·644249	60	·455361	·658356

54 DEGREES.			55 DEGREES.			56 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·701302	9·845905	0	0·743447	9·871250	0	0·788291	9·896687
1	·701983	·846327	1	·744172	·871673	1	·789063	·897112
2	·702665	·846749	2	·744897	·872096	2	·789836	·897537
3	·703348	·847170	3	·745623	·872519	3	·790609	·897962
4	·704032	·847592	4	·746350	·872942	4	·791383	·898387
5	·704716	·848014	5	·747078	·873366	5	·792158	·898812
6	·705401	·848436	6	·747806	·873789	6	·792934	·899237
7	·706087	·848858	7	·748535	·874212	7	·793710	·899662
8	·706773	·849280	8	·749265	·874635	8	·794488	·900087
9	·707460	·849702	9	·749996	·875059	9	·795266	·900512
10	·708148	·850124	10	·750727	·875482	10	·796045	·900938
11	·708836	·850546	11	·751459	·875905	11	·796825	·901363
12	·709525	·850968	12	·752192	·876329	12	·797606	·901788
13	·710215	·851390	13	·752926	·876752	13	·798387	·902213
14	·710906	·851812	14	·753661	·877176	14	·799169	·902639
15	·711597	·852234	15	·754396	·877599	15	·799952	·903064
16	·712289	·852656	16	·755132	·878023	16	·800737	·903490
17	·712982	·853078	17	·755869	·878447	17	·801521	·903915
18	·713675	·853500	18	·756606	·878870	18	·802307	·904341
19	·714369	·853922	19	·757345	·879294	19	·803094	·904766
20	·715064	·854345	20	·758084	·879717	20	·803881	·905192
21	·715760	·854767	21	·758824	·880141	21	·804669	·905617
22	·716456	·855189	22	·759564	·880564	22	·805458	·906043
23	·717153	·855612	23	·760305	·880988	23	·806248	·906469
24	·717850	·856034	24	·761048	·881412	24	·807039	·906894
25	·718548	·856456	25	·761791	·881836	25	·807830	·907320
26	·719247	·856878	26	·762535	·882260	26	·808623	·907746
27	·719947	·857301	27	·763279	·882683	27	·809416	·908172
28	·720648	·857723	28	·764024	·883107	28	·810210	·908598
29	·721349	·858145	29	·764770	·883531	29	·811005	·909024
30	·722051	·858568	30	·765517	·883955	30	·811801	·909450
31	·722753	·858990	31	·766265	·884379	31	·812598	·909876
32	·723457	·859412	32	·767013	·884803	32	·813395	·910302
33	·724161	·859835	33	·767762	·885227	33	·814193	·910728
34	·724866	·860257	34	·768512	·885651	34	·814993	·911154
35	·725571	·860680	35	·769263	·886075	35	·815793	·911580
36	·726277	·861103	36	·770014	·886499	36	·816594	·912006
37	·726984	·861525	37	·770767	·886923	37	·817396	·912432
38	·727692	·861947	38	·771520	·887347	38	·818199	·912859
39	·728401	·862370	39	·772274	·887772	39	·819002	·913285
40	·729110	·862793	40	·773029	·888196	40	·819806	·913711
41	·729820	·863216	41	·773784	·888620	41	·820611	·914138
42	·730530	·863638	42	·774540	·889044	42	·821418	·914564
43	·731241	·864061	43	·775298	·889469	43	·822225	·914991
44	·731953	·864484	44	·776056	·889893	44	·823033	·915417
45	·732666	·864906	45	·776815	·890317	45	·823842	·915844
46	·733380	·865329	46	·777574	·890742	46	·824651	·916270
47	·734094	·865752	47	·778334	·891166	47	·825462	·916697
48	·734809	·866175	48	·779095	·891591	48	·826273	·917124
49	·735525	·866597	49	·779857	·892015	49	·827085	·917550
50	·736241	·867020	50	·780620	·892440	50	·827898	·917977
51	·736958	·867443	51	·781384	·892864	51	·828712	·918404
52	·737676	·867866	52	·782148	·893289	52	·829527	·918831
53	·738395	·868289	53	·782913	·893714	53	·830343	·919258
54	·739115	·868712	54	·783679	·894138	54	·831160	·919685
55	·739835	·869135	55	·784446	·894563	55	·831977	·920112
56	·740556	·869558	56	·785213	·894988	56	·832796	·920539
57	·741277	·869981	57	·785981	·895412	57	·833615	·920966
58	·742000	·870404	58	·786750	·895837	58	·834435	·921393
59	·742723	·870827	59	·787520	·896262	59	·835256	·921820
60	·743447	·871250	60	·788291	·896687	60	·836078	·922247

57 DEGREES.			58 DEGREES.			59 DEGREES.		
Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm	Min.	Nat. No.	Logarithm
0	0·455361	9·658356	0	0·470081	9·672173	0	0·484962	9·685708
1	·455605	·658588	1	·470327	·672400	1	·485211	·685931
2	·455849	·658821	2	·470574	·672628	2	·485460	·686154
3	·456093	·659054	3	·470821	·672856	3	·485710	·686377
4	·456337	·659286	4	·471068	·673083	4	·485960	·686600
5	·456581	·659518	5	·471315	·673311	5	·486209	·686823
6	·456825	·659750	6	·471562	·673539	6	·486459	·687046
7	·457070	·659982	7	·471809	·673766	7	·486708	·687269
8	·457314	·660215	8	·472056	·673993	8	·486958	·687492
9	·457558	·660446	9	·472303	·674221	9	·487207	·687714
10	·457803	·660678	10	·472550	·674448	10	·487457	·687937
11	·458047	·660910	11	·472797	·674675	11	·487707	·688159
12	·458292	·661142	12	·473044	·674902	12	·487957	·688382
13	·458536	·661374	13	·473291	·675129	13	·488207	·688604
14	·458781	·661605	14	·473539	·675356	14	·488457	·688826
15	·459025	·661837	15	·473786	·675582	15	·488707	·689048
16	·459270	·662068	16	·474033	·675809	16	·488957	·689270
17	·459515	·662300	17	·474281	·676036	17	·489207	·689492
18	·459760	·662531	18	·474528	·676262	18	·489457	·689714
19	·460004	·662762	19	·474776	·676489	19	·489707	·689936
20	·460249	·662943	20	·475023	·676715	20	·489957	·690158
21	·460494	·663224	21	·475271	·676941	21	·490207	·690380
22	·460739	·663455	22	·475518	·677167	22	·490458	·690602
23	·460984	·663686	23	·475766	·677394	23	·490708	·690823
24	·461229	·663917	24	·476014	·677620	24	·490958	·691045
25	·461474	·664147	25	·476262	·677846	25	·491209	·691266
26	·461719	·664378	26	·476510	·678072	26	·491459	·691488
27	·461965	·664609	27	·476758	·678298	27	·491710	·691709
28	·462210	·664839	28	·477005	·678523	28	·491960	·691930
29	·462455	·665070	29	·477253	·678749	29	·492211	·692151
30	·462700	·665300	30	·477501	·678975	30	·492462	·692372
31	·462946	·665530	31	·477749	·679200	31	·492712	·692593
32	·463191	·665760	32	·477997	·679426	32	·492963	·692814
33	·463436	·665990	33	·478246	·679651	33	·493214	·693035
34	·463682	·666220	34	·478494	·679876	34	·493465	·693256
35	·463927	·666450	35	·478742	·680102	35	·493716	·693477
36	·464173	·666680	36	·478991	·680327	36	·493966	·693697
37	·464419	·666910	37	·479239	·680552	37	·494217	·693918
38	·464664	·667140	38	·479487	·680777	38	·494468	·694138
39	·464910	·667369	39	·479735	·681002	39	·494719	·694359
40	·465156	·667599	40	·479984	·681227	40	·494970	·694579
41	·465402	·667828	41	·480232	·681451	41	·495221	·694799
42	·465648	·668058	42	·480481	·681676	42	·495472	·695019
43	·465894	·668287	43	·480730	·681901	43	·495724	·695240
44	·466140	·668516	44	·480978	·682125	44	·495975	·695460
45	·466386	·668745	45	·481227	·682350	45	·496226	·695680
46	·466632	·668974	46	·481475	·682574	46	·496477	·695899
47	·466878	·669203	47	·481724	·682799	47	·496729	·696119
48	·467124	·669432	48	·481973	·683023	48	·496980	·696339
49	·467370	·669661	49	·482222	·683247	49	·497232	·696559
50	·467616	·669889	50	·482471	·683471	50	·497483	·696778
51	·467862	·670118	51	·482720	·683695	51	·467784	·696998
52	·468109	·670347	52	·482969	·683919	52	·497986	·697217
53	·468355	·670575	53	·483218	·684143	53	·498237	·697436
54	·468601	·670804	54	·483467	·684367	54	·498489	·697656
55	·468848	·671032	55	·483716	·684590	55	·498741	·697875
56	·469094	·671260	56	·483965	·684814	56	·498993	·698094
57	·469341	·671488	57	·484214	·685037	57	·499244	·698313
58	·469587	·671716	58	·484463	·685261	58	·499496	·698532
59	·469834	·671945	59	·484713	·685484	59	·499748	·698751
60	·470081	·672173	60	·484962	·685708	60	·500000	·698970

57 DEGREES.			58 DEGREES.			59 DEGREES.		
Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.	Min.	Nat. No.	Logarithm.
0	0·836078	9·922247	0	0·887080	9·947963	0	0·941604	9·973868
1	·836901	·922674	1	·887959	·948393	1	·942544	·974302
2	·837725	·923101	2	·888839	·948823	2	·943486	·974735
3	·838550	·923529	3	·889720	·949253	3	·944429	·975169
4	·839375	·923956	4	·890601	·949683	4	·945373	·975603
5	·840202	·924384	5	·891484	·950114	5	·946317	·976037
6	·841029	·924811	6	·892368	·950544	6	·947263	·976471
7	·841857	·924238	7	·893253	·950975	7	·948210	·976905
8	·842686	·925666	8	·894139	·951405	8	·949158	·977839
9	·843516	·926093	9	·895026	·951836	9	·950107	·977773
10	·844348	·926521	10	·895914	·952266	10	·951058	·978207
11	·845180	·926949	11	·896802	·952697	11	·952009	·978641
12	·846012	·927377	12	·897692	·953127	12	·952961	·979075
13	·846846	·927804	13	·898583	·953558	13	·953915	·979510
14	·847681	·928232	14	·899475	·953989	14	·954870	·979944
15	·848516	·928660	15	·900368	·954420	15	·955826	·980379
16	·849352	·929088	16	·901262	·954851	16	·956782	·980813
17	·850190	·929516	17	·902156	·955282	17	·957740	·981248
18	·851028	·929944	18	·903052	·955713	18	·958699	·981682
19	·851867	·930372	19	·903949	·956144	19	·959659	·982117
20	·852707	·930800	20	·904847	·956575	20	·960621	·982552
21	·853548	·931228	21	·905746	·957006	21	·961583	·982987
22	·854390	·931656	22	·906645	·957437	22	·962546	·983422
23	·855233	·932085	23	·907546	·957869	23	·963511	·983857
24	·856077	·932513	24	·908448	·958300	24	·964477	·984292
25	·856921	·932941	25	·909351	·958732	25	·965444	·984727
26	·857767	·933369	26	·910255	·959163	26	·966411	·985162
27	·858614	·933798	27	·911160	·959595	27	·967380	·985597
28	·859461	·934226	28	·912066	·960026	28	·968350	·986033
29	·860310	·934655	29	·912973	·960458	29	·969322	·986468
30	·861159	·935083	30	·913881	·960890	30	·970294	·986903
31	·862009	·935512	31	·914790	·961321	31	·971268	·987339
32	·862860	·935941	32	·915700	·961753	32	·972242	·987775
33	·863712	·936369	33	·916611	·962185	33	·973218	·988210
34	·864565	·936798	34	·917523	·962617	34	·974195	·988646
35	·865420	·937227	35	·918436	·963049	35	·975173	·989082
36	·866275	·937656	36	·919350	·963481	36	·976152	·989518
37	·867131	·938085	37	·920265	·963913	37	·977133	·989954
38	·867987	·938514	38	·921182	·964345	38	·978115	·990390
39	·868845	·938942	39	·922099	·964777	39	·979097	·990826
40	·869704	·939371	40	·923017	·965210	40	·980081	·991262
41	·870564	·939801	41	·923937	·965642	41	·981066	·991698
42	·871425	·940230	42	·924857	·966075	42	·982052	·992134
43	·872288	·940659	43	·925778	·966507	43	·983039	·992571
44	·873148	·941088	44	·926701	·966940	44	·984027	·993007
45	·874012	·941517	45	·927624	·967372	45	·985017	·993444
46	·874877	·941947	46	·928549	·967805	46	·986008	·993880
47	·875742	·942376	47	·929475	·968238	47	·987000	·994317
48	·876608	·942806	48	·930401	·968670	48	·987993	·994754
49	·877475	·943235	49	·931329	·969103	49	·988987	·995191
50	·878344	·943665	50	·932258	·969536	50	·989982	·995627
51	·879213	·944094	51	·933188	·969969	51	·990979	·996064
52	·880083	·944524	52	·934119	·970402	52	·991977	·996501
53	·880954	·944953	53	·935050	·970835	53	·992975	·996938
54	·881827	·945383	54	·935983	·971268	54	·993975	·997376
55	·882700	·945813	55	·936917	·971701	55	·994976	·997813
56	·883574	·946243	56	·937853	·972135	56	·995978	·998250
57	·884449	·946673	57	·938789	·972568	57	·996982	·998687
58	·885325	·947103	58	·939726	·973001	58	·997987	·999125
59	·886202	·947533	59	·940664	·973435	59	·998993	·999562
60	·887080	·947963	60	·941604	·973868	60	1·000000	10·000000

NATURAL SINES AND TANGENTS,

TO EVERY DEGREE AND MINUTE OF THE QUADRANT.

EXTENDED TO SEVEN PLACES OF DECIMALS.

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4524	034 8995	052 3360	069 7565	087 1557	104 5285	121 8693	60
1	2909	7432	035 1902	6264	070 0467	4455	8178	122 1581	59
2	5818	018 0341	4809	9169	3368	7353	105 1070	4468	58
3	8727	3249	7716	053 2074	6270	088 0251	3963	7355	57
4	001 1636	6158	036 0623	4979	9171	3148	6856	123 0241	56
5	4544	9066	3530	7883	071 2073	6046	9748	3128	55
6	7453	019 1974	6437	054 0788	4974	8943	106 2641	6015	54
7	002 0362	4883	9344	3693	7876	089 1840	5533	8901	53
8	3271	7791	037 2251	6597	072 0777	4738	8425	124 1788	52
9	6180	020 0699	5158	9502	3678	7635	107 1318	4674	51
10	9089	3608	8065	055 2406	6580	090 0532	4210	7560	50
11	003 1998	6516	038 0971	5311	9481	3429	7102	125 0446	49
12	4907	9424	3878	8215	073 2382	6326	9994	3332	48
13	7815	021 2332	6785	056 1119	5283	9223	108 2885	6218	47
14	004 0724	5241	9692	4024	8184	091 2119	5777	9104	46
15	3633	8149	039 2598	6928	074 1085	5016	8669	126 1990	45
16	6542	022 1057	5505	9832	3986	7913	109 1560	4875	44
17	9451	3965	8411	057 2736	6887	092 0809	4452	7761	43
18	005 2360	6873	040 1318	5640	9787	3706	7343	127 0646	42
19	5268	9781	4224	8544	075 2688	6602	110 0234	3531	41
20	8177	023 2690	7131	058 1448	5589	9499	3126	6416	40
21	006 1086	5598	041 0037	4352	8489	093 2395	6017	9302	39
22	3995	8506	2944	7256	076 1390	5291	8908	128 2186	38
23	6904	024 1414	5850	059 0160	4290	8187	111 1799	5071	37
24	9813	4322	8757	3064	7190	094 1083	4689	7956	36
25	007 2721	7230	042 1663	5967	077 0091	3979	7580	129 0841	35
26	5630	025 0138	4569	8871	2991	6875	112 0471	3725	34
27	8539	3046	7475	060 1775	5891	9771	3361	6609	33
28	008 1448	5954	043 0382	4678	8791	095 2666	6252	9494	32
29	4357	8862	3288	7582	078 1691	5562	9142	130 2378	31
30	7265	026 1769	6194	061 0485	4591	8458	113 2032	5262	30
31	009 0174	4677	9100	3389	7491	096 1353	4922	8146	29
32	3083	7585	044 2006	6292	079 0391	4248	7812	131 1030	28
33	5992	027 0493	4912	9196	3290	7144	114 0702	3913	27
34	8900	3401	7818	062 2099	6190	097 0039	3592	6797	26
35	010 1809	6309	045 0724	5002	9090	2934	6482	9681	25
36	4718	9216	3630	7905	080 1989	5829	9372	132 2564	24
37	7627	028 2124	6536	063 0808	4889	8724	115 2261	5447	23
38	011 0535	5032	9442	3711	7788	098 1619	5151	8330	22
39	3444	7940	046 2347	6614	081 0687	4514	8040	133 1213	21
40	6353	029 0847	5253	9517	3587	7408	116 0929	4096	20
41	9261	3755	8159	064 2420	6486	099 0303	3818	6979	19
42	012 2170	6662	047 1065	5323	9385	3197	6707	9862	18
43	5079	9570	3970	8226	082 2284	6092	9596	134 2744	17
44	7987	030 2478	6876	065 1129	5183	8986	117 2485	5627	16
45	013 0896	5385	9781	4031	8082	100 1881	5374	8509	15
46	3805	8293	048 2687	6934	083 0981	4775	8263	135 1392	14
47	6713	031 1200	5592	9836	3880	7669	118 1151	4274	13
48	9622	4108	8498	066 2739	6778	101 0563	4040	7156	12
49	014 2530	7015	049 1403	5641	9677	3457	6928	136 0038	11
50	5439	9922	4308	8544	084 2576	6351	9816	2919	10
51	8348	032 2830	7214	067 1446	5474	9245	119 2704	5801	9
52	015 1256	5737	050 0119	4349	8373	102 2138	5593	8683	8
53	4165	8644	3024	7251	085 1271	5032	8481	137 1564	7
54	7073	033 1552	5929	068 0153	4169	7925	120 1368	4445	6
55	9982	4459	8835	3055	7067	103 0819	4256	7327	5
56	016 2890	7366	051 1740	5957	9966	3712	7144	138 0208	4
57	5799	034 0274	4645	8859	086 2864	6605	121 0031	3089	3
58	8707	3181	7550	069 1761	5762	9499	2919	5970	2
59	017 1616	6088	052 0455	4663	8660	104 2392	5806	8850	1
60	4524	8995	3360	7565	087 1557	5285	8693	139 1731	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

NATURAL COSINES.

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4551	034 9208	052 4078	069 9268	087 4887	105 1042	122 7846	60
1	2909	7460	035 2120	6995	070 2191	7818	3983	123 0798	59
2	5818	018 0370	5033	9912	5115	088 0749	6925	3752	58
3	8727	3280	7945	053 2829	8038	3681	9866	6705	57
4	001 1636	6190	036 0858	5746	071 0961	6612	106 2808	9658	56
5	4544	9100	3771	8663	3885	9544	5750	124 2612	55
6	7453	019 2010	6683	054 1581	6809	089 2476	8692	5566	54
7	002 0362	4920	9596	4498	9733	5408	107 1634	8520	53
8	3271	7830	037 2509	7416	072 2657	8341	4576	125 1474	52
9	6180	020 0740	5422	055 0333	5581	090 1273	7519	4429	51
10	9089	3650	8335	3251	8505	4206	108 0462	7384	50
11	003 1998	6560	038 1248	6169	073 1430	7138	3405	126 0339	49
12	4907	9470	4161	9087	4354	091 0071	6348	3294	48
13	7816	021 2380	7074	056 2005	7279	3004	9291	6249	47
14	004 0725	5291	9988	4923	074 0203	5938	109 2234	9205	46
15	3634	8201	039 2901	7841	3128	8871	5178	127 2161	45
16	6542	022 1111	5814	057 0759	6053	092 1804	8122	5117	44
17	9451	4021	8728	3678	8979	4738	110 1066	8073	43
18	005 2360	6932	040 1641	6596	075 1904	7672	4010	128 1030	42
19	5269	9842	4555	9515	4829	093 0606	6955	3986	41
20	8178	023 2753	7469	058 2434	7755	3540	9899	6943	40
21	006 1087	5663	041 0383	5352	076 0680	6474	111 2844	9901	39
22	3996	8574	3296	8271	3606	9409	5789	129 2858	38
23	6905	024 1484	6210	059 1190	6532	094 2344	8734	5815	37
24	9814	4395	9124	4109	9458	5278	112 1680	8773	36
25	007 2723	7305	042 2038	7029	077 2384	8213	4625	130 1731	35
26	5632	025 0216	4952	9948	5311	095 1148	7571	4690	34
27	8541	3127	7866	060 2867	8237	4084	113 0517	7648	33
28	008 1450	6038	043 0781	5787	078 1164	7019	3463	131 0607	32
29	4360	8948	3695	8706	4090	9955	6410	3566	31
30	7269	026 1859	6609	061 1626	7017	096 2890	9356	6525	30
31	009 0178	4770	9524	4546	9944	5826	114 2303	9484	29
32	3087	7681	044 2438	7466	079 2871	8763	5250	132 2444	28
33	5996	027 0592	5353	062 0386	5798	097 1699	8197	5404	27
34	8905	3503	8268	3306	8726	4635	115 1144	8364	26
35	010 1814	6414	045 1183	6226	080 1653	7572	4092	133 1324	25
36	4724	9325	4097	9147	4581	098 0509	7039	4285	24
37	7633	028 2236	7012	063 2067	7509	3446	9987	7246	23
38	011 0542	5148	9927	4988	081 0437	6383	116 2936	134 0207	22
39	3451	8059	046 2842	7908	3365	9320	5884	3168	21
40	6361	029 0970	5757	064 0829	6293	099 2257	8832	6129	20
41	9270	3882	8673	3750	9221	5194	117 1781	9091	19
42	012 2179	6793	047 1588	6671	082 2150	8133	4730	135 2053	18
43	5088	9705	4503	9592	5078	100 1071	7679	5015	17
44	7998	030 2616	7419	065 2513	8007	4009	118 0628	7978	16
45	013 0907	5528	048 0334	5435	083 0936	6947	3578	136 0941	15
46	3817	8439	3250	8356	3865	9886	6528	3903	14
47	6726	031 1351	6166	066 1278	6794	101 2824	9478	6866	13
48	9635	4263	9082	4199	9723	5763	119 2428	9830	12
49	014 2545	7174	049 1997	7121	084 2653	8702	5378	137 2793	11
50	5454	032 0086	4913	067 0043	5583	102 1641	8329	5757	10
51	8364	2998	7829	2965	8512	4580	120 1279	8721	9
52	015 1273	5910	050 0746	5887	085 1442	7520	4230	138 1685	8
53	4183	8822	3662	8809	4372	103 0460	7182	4650	7
54	7093	033 1734	6578	068 1732	7302	3399	121 0133	7615	6
55	016 0002	4646	9495	4654	086 0233	6340	3085	139 0580	5
56	2912	7558	051 2411	7577	3163	9280	6036	3545	4
57	5821	034 0471	5328	069 0499	6094	104 2220	8988	6510	3
58	8731	3383	8244	3422	9025	5161	122 1941	9476	2
59	017 1641	6295	052 1161	6345	087 1956	8101	4893	140 2442	1
60	4551	9208	4078	9268	4887	105 1042	7846	5408	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

NATURAL COTANGENTS.

′	8°	9°	10°	11°	12°	13°	14°	15°	′
0	139 1731	156 4345	173 6482	190 8090	207 9117	224 9511	241 9219	258 8190	60
1	4612	7218	9346	191 0945	208 1962	225 2345	242 2041	259 1000	59
2	7492	157 0091	174 2211	3801	4807	5179	4863	3810	58
3	140 0372	2963	5075	6656	7652	8013	7685	6619	57
4	3252	5836	7939	9510	209 0497	226 0846	243 0507	9428	56
5	6132	8708	175 0803	192 2365	3341	3680	3329	260 2237	55
6	9012	158 1581	3667	5220	6186	6513	6150	5045	54
7	141 1892	4453	6531	8074	9030	9346	8971	7853	53
8	4772	7325	9395	193 0928	210 1874	227 2179	244 1792	261 0662	52
9	7651	159 0197	176 2258	3782	4718	5012	4613	3469	51
10	142 0531	3069	5121	6636	7561	7844	7433	6277	50
11	3410	5940	7984	9490	211 0405	228 0677	245 0254	9085	49
12	6289	8812	177 0847	194 2344	3248	3509	3074	262 1892	48
13	9168	160 1683	3710	5197	6091	6341	5894	4699	47
14	143 2047	4555	6573	8050	8934	9172	8713	7506	46
15	4926	7426	9435	195 0903	212 1777	229 2004	246 1533	263 0312	45
16	7805	161 0297	178 2298	3756	4619	4835	4352	3118	44
17	144 0684	3167	5160	6609	7462	7666	7171	5925	43
18	3562	6038	8022	9461	213 0304	230 0497	9990	8730	42
19	6440	8909	179 0884	196 2314	3146	3328	247 2809	264 1536	41
20	9319	162 1779	3746	5166	5988	6159	5627	4342	40
21	145 2197	4650	6607	8018	8829	8989	8445	7147	39
22	5075	7520	9469	197 0870	214 1671	231 1819	248 1263	9952	38
23	7953	163 0390	180 2330	3722	4512	4649	4081	265 2757	37
24	146 0830	3260	5191	6573	7353	7479	6899	5561	36
25	3708	6129	8052	9425	215 0194	232 0309	9716	8366	35
26	6585	8999	181 0913	198 2276	3035	3138	249 2533	266 1170	34
27	9463	164 1868	3774	5127	5876	5967	5350	3973	33
28	147 2340	4738	6635	7978	8716	8796	8167	6777	32
29	5217	7607	9495	199 0829	216 1556	233 1625	250 0984	9581	31
30	8094	165 0476	182 2355	3679	4396	4454	3800	267 2384	30
31	148 0971	3345	5215	6530	7236	7282	6616	5187	29
32	3848	6214	8075	9380	217 0076	234 0110	9432	7989	28
33	6724	9082	183 0935	200 2230	2915	2938	251 2248	268 0792	27
34	9601	166 1951	3795	5080	5754	5766	5063	3594	26
35	149 2477	4819	6654	7930	8593	8594	7879	6396	25
36	5353	7687	9514	201 0779	218 1432	235 1421	252 0694	9198	24
37	8230	167 0556	184 2373	3629	4271	4248	3508	269 2000	23
38	150 1106	3423	5232	6478	7110	7075	6323	4801	22
39	3981	6291	8091	9327	9948	9902	9137	7602	21
40	6857	9159	185 0949	202 2176	219 2786	236 2729	253 1952	270 0403	20
41	9733	168 2026	3808	5024	5624	5555	4766	3204	19
42	151 2608	4894	6666	7873	8462	8381	7579	6004	18
43	5484	7761	9524	203 0721	220 1300	237 1207	254 0393	8805	17
44	8359	169 0628	186 2382	3569	4137	4033	3206	271 1605	16
45	152 1234	3495	5240	6418	6974	6859	6019	4404	15
46	4109	6362	8098	9265	9811	9684	8832	7204	14
47	6984	9228	187 0956	204 2113	221 2648	238 2510	255 1645	272 0003	13
48	9858	170 2095	3813	4961	5485	5335	4458	2802	12
49	153 2733	4961	6670	7808	8321	8159	7270	5601	11
50	5607	7828	9528	205 0655	222 1158	239 0984	256 0082	8400	10
51	8482	171 0694	188 2385	3502	3994	3808	2894	273 1198	9
52	154 1356	3560	5241	6349	6830	6633	5705	3997	8
53	4230	6425	8098	9195	9666	9457	8517	6794	7
54	7104	9291	189 0954	206 2042	223 2501	240 2280	257 1328	9592	6
55	9978	172 2156	3811	4888	5337	5104	4139	274 2390	5
56	155 2851	5022	6667	7734	8172	7927	6950	5187	4
57	5725	7887	9523	207 0580	224 1007	241 0751	9760	7984	3
58	8598	173 0752	190 2379	3426	3842	3574	258 2570	275 0781	2
59	156 1472	3617	5234	6272	6676	6396	5381	3577	1
60	4345	6482	8090	9117	9511	9219	8190	6374	0
′	81°	80°	79°	78°	77°	76°	75°	74°	′

'	8°		9°		10°		11°		12°		13°		14°		15°		'
0	140	5408	158	3844	176	3270	194	3803	212	5566	230	8682	249	3280	267	9492	60
1		8375		6826		6269		6822		8606	231	1746		6370	268	2610	59
2	141	1342		9809		9269		9841	213	1647		4811		9460		5728	58
3		4308	159	2791	177	2269	195	2861		4688		7876	250	2551		8847	57
4		7276		5774		5270		5881		7730	232	0941		5642	269	1967	56
5	142	0243		8757		8270		8901	214	0772		4007		8734		5087	55
6		3211	160	1740	178	1271	196	1922		3814		7073	251	1826		8207	54
7		6179		4724		4273		4943		6857	233	0140		4919	270	1328	53
8		9147		7708		7274		7964		9900		3207		8012		4449	52
9	143	2115	161	0692	179	0276	197	0986	215	2944		6274	252	1106		7571	51
10		5084		3677		3279		4008		5988		9342		4200	271	0694	50
11		8053		6662		6281		7031		9032	234	2410		7294		3817	49
12	144	1022		9647		9284	198	0053	216	2077		5479	253	0389		6940	48
13		3991	162	2632	180	2287		3076		5122		8548		3484	272	0064	47
14		6961		5618		5291		6100		8167	235	1617		6580		3188	46
15		9931		8603		8295		9124	217	1213		4687		9676		6313	45
16	145	2901	163	1590	181	1299	199	2148		4259		7758	254	2773		9438	44
17		5872		4576		4303		5172		7306	236	0829		5870	273	2564	43
18		8842		7563		7308		8197	218	0353		3900		8968		5690	42
19	146	1813	164	0550	182	0313	200	1222		3400		6971	255	2066		8817	41
20		4784		3537		3319		4248		6448	237	0044		5165	274	1945	40
21		7756		6525		6324		7274		9496		3116		8264		5072	39
22	147	0727		9513		9330	201	0300	219	2544		6189	256	1363		8201	38
23		3699	165	2501	183	2337		3327		5593		9262		4463	275	1330	37
24		6672		5489		5343		6354		8643	238	2336		7564		4459	36
25		9644		8478		8350		9381	220	1692		5410	257	0664		7589	35
26	148	2617	166	1467	184	1358	202	2409		4742		8485		3766	276	0719	34
27		5590		4456		4365		5437		7793	239	1560		6868		3850	33
28		8563		7446		7373		8465	221	0844		4635		9970		6981	32
29	149	1536	167	0436	185	0382	203	1494		3895		7711	258	3073	277	0113	31
30		4510		3426		3390		4523		6947	240	0788		6176		3245	30
31		7484		6417		6399		7552		9999		3864		9280		6378	29
32	150	0458		9407		9409	204	0582	222	3051		6942	259	2384		9512	28
33		3433	168	2398	186	2418		3612		6104	241	0019		5488	278	2646	27
34		6408		5390		5428		6643		9157		3097		8593		5780	26
35		9383		8381		8439		9674	223	2211		6176	260	1699		8915	25
36	151	2358	169	1373	187	1449	205	2705		5265		9255		4805	279	2050	24
37		5333		4366		4460		5737		8319	242	2334		7911		5186	23
38		8309		7358		7471		8769	224	1374		5414	261	1018		8322	22
39	152	1285	170	0351	188	0483	206	1801		4429		8494		4126	280	1459	21
40		4262		3344		3495		4834		7485	243	1575		7234		4597	20
41		7238		6338		6507		7867	225	0541		4656	262	0342		7735	19
42	153	0215		9331		9520	207	0900		3597		7737		3451	281	0873	18
43		3192	171	2325	189	2533		3934		6654	244	0819		6560		4012	17
44		6170		5320		5546		6968		9711		3902		9670		7152	16
45		9147		8314		8559	208	0003	226	2769		6984	263	2780	282	0292	15
46	154	2125	172	1309	190	1573		3038		5827	245	0068		5891		3432	14
47		5103		4304		4587		6073		8885		3151		9002		6573	13
48		8082		7300		7602		9109	227	1944		6236	264	2114		9715	12
49	155	1061	173	0296	191	0617	209	2145		5003		9320		5226	283	2857	11
50		4040		3292		3632		5181		8063	246	2405		8339		5999	10
51		7019		6288		6648		8218	228	1123		5491	265	1452		9143	9
52		9998		9285		9664	210	1255		4184		8577		4566	284	2286	8
53	156	2978	174	2282	192	2680		4293		7244	247	1663		7680		5430	7
54		5958		5279		5696		7331	229	0306		4750	266	0794		8575	6
55		8939		8277		8713	211	0369		3367		7837		3909	285	1720	5
56	157	1919	175	1275	193	1731		3407		6429	248	0925		7025		4866	4
57		4900		4273		4748		6446		9492		4018	267	0141		8012	3
58		7881		7272		7766		9486	230	2555		7102		3257	286	1159	2
59	158	0863	176	0271	194	0784	212	2525		5618	249	0191		6374		4306	1
60		3844		3270		3803		5566		8682		3280		9492		7454	0
'	81°		80°		79°		78°		77°		76°		75°		74°		'

NATURAL COTANGENTS.

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	275 6374	292 3717	309 0170	325 5682	342 0201	358 3679	374 6066	390 7311	60
1	9170	6499	2936	8432	2935	6395	8763	9989	59
2	276 1965	9280	5702	326 1182	5668	9110	375 1459	391 2666	58
3	4761	293 2061	8468	3932	8400	359 1825	4156	5343	57
4	7556	4842	310 1234	6681	343 1133	4540	6852	8019	56
5	277 0352	7623	3999	9430	3865	7254	9547	392 0695	55
6	3147	294 0403	6764	327 2179	6597	9968	376 2243	3371	54
7	5941	3183	9529	4928	9329	360 2682	4938	6047	53
8	8736	5963	311 2294	7676	344 2060	5395	7632	8722	52
9	278 1530	8743	5058	328 0424	4791	8108	377 0327	393 1397	51
10	4324	295 1522	7822	3172	7521	361 0821	3021	4071	50
11	7118	4302	312 0586	5919	345 0252	3534	5714	6745	49
12	9911	7081	3349	8666	2982	6246	8408	9419	48
13	279 2704	9859	6112	329 1413	5712	8958	378 1101	394 2093	47
14	5497	296 2638	8875	4160	8441	362 1669	3794	4766	46
15	8290	5416	313 1638	6906	346 1171	4380	6486	7439	45
16	280 1083	8194	4400	9653	3900	7091	9178	395 0111	44
17	3875	297 0971	7163	330 2398	6628	9802	379 1870	2783	43
18	6667	3749	9925	5144	9357	363 2512	4562	5455	42
19	9459	6526	314 2686	7889	347 2085	5222	7253	8127	41
20	281 2251	9303	5448	331 0634	4812	7932	9944	396 0798	40
21	5042	298 2079	8209	3379	7540	364 0641	380 2634	3468	39
22	7833	4856	315 0969	6123	348 0267	3351	5324	6139	38
23	282 0624	7632	3730	8867	2994	6059	8014	8809	37
24	3415	299 0408	6490	332 1611	5720	8768	381 0704	397 1479	36
25	6205	3184	9250	4355	8447	365 1476	3393	4148	35
26	8995	5959	316 2010	7098	349 1173	4184	6082	6818	34
27	283 1785	8734	4770	9841	3898	6891	8770	9486	33
28	4575	300 1509	7529	333 2584	6624	9599	382 1459	398 2155	32
29	7364	4284	317 0288	5326	9349	366 2306	4147	4823	31
30	284 0153	7058	3047	8069	350 2074	5012	6834	7491	30
31	2942	9832	5805	334 0810	4798	7719	9522	399 0158	29
32	5731	301 2606	8563	3552	7523	367 0425	383 2209	2825	28
33	8520	5380	318 1321	6293	351 0246	3130	4895	5492	27
34	285 1308	8153	4079	9034	2970	5836	7582	8158	26
35	4096	302 0926	6836	335 1775	5693	8541	384 0268	400 0825	25
36	6884	3699	9593	4516	8416	368 1246	2953	3490	24
37	9671	6471	319 2350	7256	352 1139	3950	5639	6156	23
38	286 2458	9244	5106	9996	3862	6654	8324	8821	22
39	5246	303 2016	7863	336 2735	6584	9358	385 1008	401 1486	21
40	8032	4788	320 0619	5475	9306	369 2061	3693	4150	20
41	287 0819	7559	3374	8214	353 2027	4765	6377	6814	19
42	3605	304 0331	6130	337 0953	4748	7468	9060	9478	18
43	6391	3102	8885	3691	7469	370 0170	386 1744	402 2141	17
44	9177	5872	321 1640	6429	354 0190	2872	4427	4804	16
45	288 1963	8643	4395	9167	2910	5574	7110	7467	15
46	4748	305 1413	7149	338 1905	5630	8276	9792	403 0129	14
47	7533	4183	9903	4642	8350	371 0977	387 2474	2791	13
48	289 0318	6953	322 2657	7379	355 1070	3678	5156	5453	12
49	3103	9723	5411	339 0116	3789	6379	7837	8114	11
50	5887	306 2492	8164	2852	6508	9079	388 0518	404 0775	10
51	8671	5261	323 0917	5589	9226	372 1780	3199	3436	9
52	290 1455	8030	3670	8325	356 1944	4479	5880	6096	8
53	4239	307 0798	6422	340 1060	4662	7179	8560	8756	7
54	7022	3566	9174	3796	7380	9878	389 1240	405 1416	6
55	9805	6334	324 1926	6531	357 0097	373 2577	3919	4075	5
56	291 2588	9102	4678	9265	2814	5275	6598	6734	4
57	5371	308 1869	7429	341 2000	5531	7973	9277	9393	3
58	8153	4636	325 0180	4734	8248	374 0671	390 1955	406 2051	2
59	292 0935	7403	2931	7468	358 0964	3369	4633	4709	1
60	3717	309 0170	5682	342 0201	3679	6066	7311	7366	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

NATURAL COSINES.

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	286 7454	305 7807	324 9197	344 8276	363 9702	383 8640	404 0262	424 4748	60
1	287 0602	306 0488	325 2418	6530	364 2997	384 1978	8646	8182	59
2	3751	3670	5630	9785	6292	5317	7031	425 1616	58
3	6900	6852	8848	345 3040	9588	8656	405 0417	5051	57
4	288 0050	307 0034	326 2066	6296	365 2885	385 1996	3804	8487	56
5	3201	3218	5284	9553	6182	5337	7191	426 1924	55
6	6352	6402	8504	346 2810	9480	8679	406 0579	5361	54
7	9503	9586	327 1724	6068	366 2779	386 2021	3968	8800	53
8	289 2655	308 2771	4944	9327	6079	5364	7358	427 2239	52
9	5808	5957	8165	347 2586	9379	8708	407 0748	5680	51
10	8961	9143	328 1387	5846	367 2680	387 2053	4139	9121	50
11	290 2114	309 2330	4610	9107	5981	5398	7531	428 2563	49
12	5269	5517	7833	348 2368	9284	8744	408 0924	6005	48
13	8423	8705	329 1056	5630	368 2587	388 2091	4318	9449	47
14	291 1578	310 1893	4281	8893	5890	5439	7718	429 2894	46
15	4734	5083	7505	349 2156	9195	8787	409 1108	6339	45
16	7890	8272	330 0731	5420	369 2500	389 2136	4504	9785	44
17	292 1047	311 1462	3957	8685	5806	5486	7901	430 3232	43
18	4205	4653	7184	350 1950	9112	8837	410 1299	6680	42
19	7363	7845	331 0411	5216	370 2420	390 2189	4697	431 0129	41
20	293 0521	312 1036	3639	8483	5728	5541	8097	3579	40
21	3680	4229	6868	351 1750	9036	8894	411 1497	7030	39
22	6839	7422	332 0097	5018	371 2346	391 2247	4898	432 0481	38
23	9999	313 0616	3327	8287	5656	5602	8300	3933	37
24	294 3160	3810	6557	352 1556	8967	8957	412 1703	7386	36
25	6321	7005	9788	4826	372 2278	392 2313	5106	433 0840	35
26	9483	314 0200	333 3020	8096	5590	5670	8510	4295	34
27	295 2645	3396	6252	353 1368	8903	9027	413 1915	7751	33
28	5808	6593	9485	4640	373 2217	393 2386	5321	434 1208	32
29	8971	9790	334 2719	7912	5532	5745	8728	4665	31
30	296 2135	315 2988	5953	354 1186	8847	9105	414 2136	8124	30
31	5299	6186	9188	4460	374 2163	394 2465	5544	435 1583	29
32	8464	9385	335 2424	7734	5479	5827	8953	5043	28
33	297 1630	316 2585	5660	355 1010	8797	9189	415 2363	8504	27
34	4796	5785	8896	4286	375 2115	395 2552	5774	436 1966	26
35	7962	8986	336 2134	7562	5433	5916	9186	5429	25
36	298 1129	317 2187	5372	356 0840	8753	9280	416 2598	8893	24
37	4297	5389	8610	4118	376 2073	396 2645	6012	437 2357	23
38	7465	8591	337 1850	7397	5394	6011	9426	5823	22
39	299 0634	318 1794	5090	357 0676	8716	9378	417 2841	9289	21
40	3803	4998	8330	3956	377 2038	397 2746	6257	438 2756	20
41	6973	8202	338 1571	7237	5361	6114	9673	6224	19
42	300 0144	319 1407	4813	358 0518	8685	9483	418 3091	9693	18
43	3315	4613	8056	3801	378 2010	398 2853	6509	439 3163	17
44	6486	7819	339 1299	7083	5335	6224	9928	6634	16
45	9658	320 1025	4543	359 0367	8661	9595	419 3348	440 0105	15
46	301 2831	4232	7787	3651	379 1988	399 2968	6769	3578	14
47	6004	7440	340 1032	6936	5315	6341	420 0190	7051	13
48	9178	321 0649	4278	360 0222	8644	9715	3613	441 0526	12
49	302 2352	3858	7524	3508	380 1973	400 3089	7036	4001	11
50	5527	7067	341 0771	6795	5302	6465	421 0460	7477	10
51	8703	322 0278	4019	361 0082	8633	9841	3885	442 0954	9
52	303 1879	3489	7267	3371	381 1964	401 3218	7311	4432	8
53	5055	6700	342 0516	6660	5296	6596	422 0738	7910	7
54	8232	9912	3765	9949	8629	9974	4165	443 1390	6
55	304 1410	323 3125	7015	362 3240	382 1962	402 3354	7594	4871	5
56	4588	6338	343 0266	6531	5296	6734	423 1023	8352	4
57	7767	9552	3518	9823	8631	403 0115	4453	444 1834	3
58	305 0946	324 2766	6770	363 3115	383 1967	3496	7884	5318	2
59	4126	5981	344 0023	6408	5303	6879	424 1316	8802	1
60	7307	9197	3276	9702	8640	404 0262	4748	445 2287	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

NATURAL COTANGENTS.

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	406 7366	422 6183	438 3711	453 9905	469 4716	484 8096	500 0000	515 0381	60
1	407 0024	8819	6326	454 2497	7284	485 0640	2519	2874	59
2	2681	423 1455	8940	5088	9852	3184	5037	5367	58
3	5337	4090	439 1553	7679	470 2419	5727	7556	7859	57
4	7993	6725	4166	455 0269	4986	8270	501 0073	516 0351	56
5	408 0649	9360	6779	2859	7553	486 0812	2591	2842	55
6	3305	424 1994	9392	5449	471 0119	3354	5107	5333	54
7	5960	4628	440 2004	8038	2685	5895	7624	7824	53
8	8615	7262	4615	456 0627	5250	8436	502 0140	517 0314	52
9	409 1269	9895	7227	3216	7815	487 0977	2655	2804	51
10	3923	425 2528	9838	5804	472 0380	3517	5170	5293	50
11	6577	5161	441 2448	8392	2944	6057	7685	7782	49
12	9230	7793	5059	457 0979	5508	8597	503 0199	518 0270	48
13	410 1883	426 0425	7668	3566	8071	488 1136	2713	2758	47
14	4536	3056	442 0278	6153	473 0634	3674	5227	5246	46
15	7189	5687	2887	8739	3197	6212	7740	7733	45
16	9841	8318	5496	458 1325	5759	8750	504 0252	519 0219	44
17	411 2492	427 0949	8104	3910	8321	489 1288	2765	2705	43
18	5144	3579	443 0712	6496	474 0882	3825	5276	5191	42
19	7795	6208	3319	9080	3443	6361	7788	7676	41
20	412 0445	8838	5927	459 1665	6004	8897	505 0298	520 0161	40
21	3096	428 1467	8534	4248	8564	490 1433	2809	2646	39
22	5745	4095	444 1140	6832	475 1124	3968	5319	5130	38
23	8395	6723	3746	9415	3683	6503	7828	7613	37
24	413 1044	9351	6352	460 1998	6242	9038	506 0338	521 0096	36
25	3693	429 1979	8957	4580	8801	491 1572	2846	2579	35
26	6342	4606	445 1562	7162	476 1359	4105	5355	5061	34
27	8990	7233	4167	9744	3917	6638	7863	7543	33
28	414 1638	9859	6771	461 2325	6474	9171	507 0370	522 0024	32
29	4285	430 2485	9375	4906	9031	492 1704	2877	2505	31
30	6932	5111	446 1978	7486	477 1588	4236	5384	4986	30
31	9579	7736	4581	462 0066	4144	6767	7890	7466	29
32	415 2226	431 0361	7184	2646	6700	9298	508 0396	9945	28
33	4872	2986	9786	5225	9255	493 1829	2901	523 2424	27
34	7517	5610	447 2388	7804	478 1810	4359	5406	4903	26
35	416 0163	8234	4990	463 0382	4364	6889	7910	7381	25
36	2808	432 0857	7591	2960	6919	9419	509 0414	9859	24
37	5453	3481	448 0192	5538	9472	494 1948	2918	524 2336	23
38	8097	6105	2792	8115	479 2026	4476	5421	4813	22
39	417 0741	8726	5392	464 0692	4579	7005	7924	7290	21
40	3385	433 1348	7992	3269	7131	9532	510 0426	9766	20
41	6028	3970	449 0591	5845	9683	495 2060	2928	525 2241	19
42	8671	6591	3190	8420	480 2235	4587	5429	4717	18
43	418 1313	9212	5789	465 0996	4786	7113	7930	7191	17
44	3956	434 1832	8387	3571	7337	9639	511 0431	9665	16
45	6597	4453	450 0984	6145	9888	496 2165	2931	526 2139	15
46	9239	7072	3582	8719	481 2438	4690	5431	4613	14
47	419 1880	9692	6179	466 1293	4987	7215	7930	7085	13
48	4521	435 2311	8775	3866	7537	9740	512 0429	9558	12
49	7161	4930	451 1372	6439	482 0086	497 2264	2927	527 2030	11
50	9801	7548	3967	9012	2634	4787	5425	4502	10
51	420 2441	436 0166	6563	467 1584	5182	7310	7923	6973	9
52	5080	2784	9158	4156	7730	9833	513 0420	9443	8
53	7719	5401	452 1753	6727	483 0277	498 2355	2916	528 1914	7
54	421 0358	8018	4347	9298	2824	4877	5413	4383	6
55	2996	437 0634	6941	468 1869	5370	7399	7908	6853	5
56	5634	3251	9535	4439	7916	9920	514 0404	9322	4
57	8272	5866	453 2128	7009	484 0462	499 2441	2899	529 1790	3
58	422 0909	8482	4721	9578	3007	4961	5393	4258	2
59	3546	438 1097	7313	469 2147	5552	7481	7887	6726	1
60	6183	3711	9905	4716	8096	500 0000	515 0381	9193	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	445 2287	466 3077	487 7326	509 5254	531 7094	554 3091	577 3503	600 8606	60
1	5773	6618	488 0927	8919	532 0826	6894	7382	601 2566	59
2	9260	467 0161	4530	510 2585	4559	555 0698	578 1262	6527	58
3	446 2747	3705	8133	6252	8293	4504	5144	602 0490	57
4	6236	7250	489 1737	9919	533 2029	8311	9027	4454	56
5	9726	468 0796	5343	511 3588	5765	556 2119	579 2912	8419	55
6	447 3216	4342	8949	7259	9503	5929	6797	603 2386	54
7	6708	7890	490 2557	512 0930	534 3242	9739	580 0681	6354	53
8	448 0200	469 1439	6166	4602	6981	557 3551	4573	604 0323	52
9	3693	4988	9775	8275	535 0723	7364	8462	4294	51
10	7187	8539	491 3386	513 1950	4465	558 1179	581 2353	8266	50
11	449 0682	470 2090	6997	5625	8208	4994	6245	605 2240	49
12	4178	5643	492 0610	9302	536 1953	8811	582 0139	6215	48
13	7675	9196	4224	514 2980	5699	559 2629	4034	606 0192	47
14	450 1173	471 2751	7838	6658	9446	6449	7930	4170	46
15	4672	6306	493 1454	515 0338	537 3194	560 0269	583 1828	8149	45
16	8171	9863	5071	4019	6943	4091	5726	607 2130	44
17	451 1672	472 3420	8689	7702	538 0694	7914	9627	6112	43
18	5173	6978	494 2308	516 1385	4445	561 1738	584 3528	608 0095	42
19	8676	473 0538	5928	5069	8198	5564	7431	4080	41
20	452 2179	4098	9549	8755	539 1952	9391	585 1335	8067	40
21	5683	7659	495 3171	517 2441	5707	562 3219	5241	609 2054	39
22	9188	474 1222	6794	6129	9464	7048	9148	6043	38
23	453 2694	4785	496 0418	9818	540 3221	563 0879	586 3056	610 0034	37
24	6201	8349	4043	518 3508	6980	4710	6965	4026	36
25	9709	475 1914	7669	7199	541 0740	8543	587 0876	8019	35
26	454 3218	5481	497 1297	519 0891	4501	564 2378	4788	611 2014	34
27	6728	9048	4925	4584	8263	6213	8702	6011	33
28	455 0238	476 2616	8554	8278	542 2027	565 0050	588 2616	612 0008	32
29	3750	6185	498 2185	520 1974	5791	3888	6533	4007	31
30	7263	9755	5816	5671	9557	7728	589 0450	8008	30
31	456 0776	477 3326	9449	9368	543 3324	566 1568	4369	613 2010	29
32	4290	6899	499 3082	521 3067	7092	5410	8289	6013	28
33	7806	478 0472	6717	6767	544 0862	9254	590 2211	614 0018	27
34	457 1322	4046	500 0352	522 0468	4632	567 3098	6134	4024	26
35	4839	7621	3989	4170	8404	6944	591 0058	8032	25
36	8357	479 1197	7627	7874	545 2177	568 0791	3984	615 2041	24
37	458 1877	4774	501 1266	523 1578	5951	4639	7910	6052	23
38	5397	8352	4906	5284	9727	8488	592 1839	616 0064	22
39	8918	480 1932	8547	8990	546 3503	569 2339	5768	4077	21
40	459 2439	5512	502 2189	524 2698	7281	6191	9699	8092	20
41	5962	9093	5832	6407	547 1060	570 0045	593 3632	617 2108	19
42	9486	481 2675	9476	525 0117	4840	3899	7565	6126	18
43	460 3011	6258	503 3121	3829	8621	7755	594 1501	618 0145	17
44	6537	9842	6768	7541	548 2404	571 1612	5437	4166	16
45	461 0063	482 3427	504 0415	526 1255	6188	5471	9375	8188	15
46	3591	7014	4063	4969	9973	9331	595 3314	619 2211	14
47	7119	483 0601	7713	8685	549 3759	572 3192	7255	6236	13
48	462 0649	4189	505 1363	527 2402	7547	7054	596 1196	620 0263	12
49	4179	7778	5015	6120	550 1335	573 0918	5140	4291	11
50	7710	484 1368	8668	9839	5125	4783	9084	8322	10
51	463 1243	4959	506 2322	528 3560	8916	8649	597 3030	621 2351	9
52	4776	8552	5977	7281	551 2708	574 2516	6978	6383	8
53	8310	485 2145	9633	529 1004	6502	6385	598 0926	622 0417	7
54	464 1845	5739	507 3290	4727	552 0297	575 0255	4877	4452	6
55	5382	9334	6948	8452	4093	4126	8828	8488	5
56	8919	486 2931	508 0607	530 2178	7890	7999	599 2781	623 2527	4
57	465 2457	6528	4267	5906	553 1688	576 1873	6735	6566	3
58	5996	487 0126	7929	9634	5488	5748	600 0691	624 0607	2
59	9536	3726	509 1591	531 3364	9288	9625	4648	4650	1
60	466 3077	7326	5254	7094	554 3091	577 3503	8606	8694	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	529 9193	544 6390	559 1929	573 5764	587 7853	601 8150	615 6615	629 3204	60
1	530 1659	8830	4340	8147	588 0206	602 0473	8907	5464	59
2	4125	545 1269	6751	574 0529	2558	2795	616 1198	7724	58
3	6591	3707	9162	2911	4910	5117	3489	9983	57
4	9057	6145	560 1572	5292	7262	7439	5780	630 2242	56
5	531 1521	8583	3981	7672	9613	9760	8069	4500	55
6	3986	546 1020	6390	575 0053	589 1964	603 2080	617 0359	6758	54
7	6450	3456	8798	2432	4314	4400	2648	9015	53
8	8913	5892	561 1206	4811	6663	6719	4936	631 1272	52
9	532 1376	8328	3614	7190	9012	9038	7224	3528	51
10	3839	547 0763	6021	9568	590 1361	604 1356	9511	5784	50
11	6301	3198	8428	576 1946	3709	3674	618 1798	8039	49
12	8763	5632	562 0834	4323	6057	5991	4084	632 0293	48
13	533 1224	8066	3239	6700	8404	8308	6370	2547	47
14	3685	548 0499	5645	9076	591 0750	605 0624	8655	4800	46
15	6145	2932	8049	577 1452	3096	2940	619 0939	7053	45
16	8605	5365	563 0453	3827	5442	5255	3224	9306	44
17	534 1065	7797	2857	6202	7787	7570	5507	633 1557	43
18	3523	549 0228	5260	8576	592 0132	9884	7790	3809	42
19	5982	2659	7663	578 0950	2476	606 2198	620 0073	6059	41
20	8440	5090	564 0066	3323	4819	4511	2355	8310	40
21	535 0898	7520	2467	5696	7163	6824	4636	634 0559	39
22	3355	9950	4869	8069	9505	9136	6917	2808	38
23	5812	550 2379	7270	579 0440	593 1847	607 1447	9198	5057	37
24	8268	4807	9670	2812	4189	3758	621 1478	7305	36
25	536 0724	7236	565 2070	5183	6530	6069	3757	9553	35
26	3179	9663	4469	7553	8871	8379	6036	635 1800	34
27	5634	551 2091	6868	9923	594 1211	608 0689	8314	4046	33
28	8089	4518	9267	580 2292	3550	2998	622 0592	6292	32
29	537 0543	6944	566 1665	4661	5889	5306	2870	8537	31
30	2996	9370	4062	7030	8228	7614	5146	636 0782	30
31	5449	552 1795	6459	9397	595 0566	9922	7423	3026	29
32	7902	4220	8856	581 1765	2904	609 2229	9698	5270	28
33	538 0354	6645	567 1252	4132	5241	4535	623 1974	7513	27
34	2806	9069	3648	6498	7577	6841	4248	9756	26
35	5257	553 1492	6043	8864	9913	9147	6522	637 1998	25
36	7708	3915	8437	582 1230	596 2249	610 1452	8796	4240	24
37	539 0158	6338	568 0832	3595	4584	3756	624 1069	6481	23
38	2608	8760	3225	5959	6918	6060	3342	8721	22
39	5058	554 1182	5619	8323	9252	8363	5614	638 0961	21
40	7507	3603	8011	583 0687	597 1586	611 0666	7885	3201	20
41	9955	6024	569 0403	3050	3919	2969	625 0156	5440	19
42	540 2403	8444	2795	5412	6251	5270	2427	7678	18
43	4851	555 0864	5187	7774	8583	7572	4696	9916	17
44	7298	3283	7577	584 0136	598 0915	9873	6966	639 2153	16
45	9745	5702	9968	2497	3246	612 2173	9235	4390	15
46	541 2191	8121	570 2357	4857	5577	4473	626 1503	6626	14
47	4637	556 0539	4747	7217	7906	6772	3771	8862	13
48	7082	2956	7136	9577	599 0236	9071	6038	640 1097	12
49	9527	5373	9524	585 1936	2565	613 1369	8305	3332	11
50	542 1971	7790	571 1912	4294	4893	3666	627 0571	5566	10
51	4415	557 0206	4299	6652	7221	5964	2837	7799	9
52	6859	2621	6686	9010	9549	8260	5102	641 0032	8
53	9302	5036	9073	586 1367	600 1876	614 0556	7366	2264	7
54	543 1744	7451	572 1459	3724	4202	2852	9631	4496	6
55	4187	9865	3844	6080	6528	5147	628 1894	6728	5
56	6628	558 2279	6229	8435	8854	7442	4157	8958	4
57	9069	4692	8614	587 0790	601 1179	9736	6420	642 1189	3
58	544 1510	7105	573 0998	3145	3503	615 2029	8682	3418	2
59	3951	9517	3381	5499	5827	4322	629 0943	5647	1
60	6390	559 1929	5764	7853	8150	6615	3204	7876	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

NATURAL COSINES.

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	624 8694	649 4076	674 5085	700 2075	726 5425	753 5541	781 2856	809 7840	60
1	625 2739	8212	9318	6411	9871	754 0102	7542	810 2658	59
2	6786	650 2350	675 3553	701 0749	727 4318	4666	782 2229	7478	58
3	626 0834	6490	7790	5089	8767	9232	6919	811 2300	57
4	4884	651 0631	676 2028	9430	728 3218	755 3799	783 1611	7124	56
5	8935	4774	6268	702 3773	7671	8369	6305	812 1951	55
6	627 2938	8918	677 0509	8118	729 2125	756 2941	784 1002	6780	54
7	7042	652 3064	4752	703 2464	6582	7514	5700	813 1611	53
8	628 1098	7211	8997	6813	730 1041	757 2090	785 0400	6444	52
9	5155	653 1360	678 3243	704 1163	5501	6668	5103	814 1280	51
10	9214	5511	7402	5515	9963	758 1248	9808	6113	50
11	629 3274	9663	679 1741	9869	731 4428	5829	786 4515	815 0958	49
12	7336	654 3817	5993	705 4224	8894	759 0418	9224	5801	48
13	630 1399	7972	680 0246	8581	732 3362	4999	787 3935	816 0646	47
14	5464	655 2129	4501	706 2940	7832	9587	8649	5493	46
15	9530	6287	8758	7301	733 2303	760 4177	788 3364	817 0343	45
16	631 3593	656 0447	681 3016	707 1664	6777	8769	8082	5195	44
17	7667	4609	7276	6028	734 1253	761 3363	789 2802	818 0049	43
18	632 1733	8772	682 1537	708 0395	5730	7959	7524	4905	42
19	5810	657 2937	5801	4763	735 0210	762 2557	790 2248	9764	41
20	9833	7103	683 0066	9133	4691	7157	6975	819 4625	40
21	633 3959	658 1271	4333	709 3504	9174	763 1759	791 1703	9488	39
22	8035	5441	8601	7878	736 3660	6363	6434	820 4354	38
23	634 2113	9312	684 2871	710 2253	8147	764 0969	792 1167	9222	37
24	6193	659 3785	7148	6630	737 2636	5577	5902	821 4093	36
25	635 0274	7930	685 1416	711 1009	7127	765 0188	793 0640	8965	35
26	4357	660 2136	5692	5390	738 1620	4800	5379	822 3840	34
27	8441	6318	9969	9772	6115	9414	794 0121	8718	33
28	636 2527	661 0492	686 4247	712 4157	739 0611	766 4031	4865	823 3597	32
29	6314	4678	8528	8543	5110	8649	9611	8479	31
30	637 0703	8856	687 2810	713 2931	9611	767 3270	795 4359	824 3364	30
31	4793	662 3040	7093	7320	740 4113	7893	9110	8251	29
32	8885	7225	688 1379	714 1712	8618	768 2517	796 3862	825 3140	28
33	638 2978	663 1413	5666	6106	741 3124	7144	8617	8031	27
34	7078	5601	9955	715 0501	7633	769 1778	797 3374	826 2925	26
35	639 1169	9792	689 4246	4893	742 2143	6404	8134	7821	25
36	5267	664 3934	8533	9297	6655	770 1037	798 2895	827 2719	24
37	9366	8178	690 2832	716 3693	743 1170	5672	7659	7620	23
38	640 3467	665 2373	7123	8100	5686	771 0309	799 2425	828 2523	22
39	7569	6570	691 1425	717 2505	744 0204	4948	7193	7429	21
40	641 1673	666 0769	5725	6911	4724	9589	800 1963	829 2337	20
41	5779	4959	692 0026	718 1319	9246	772 4233	6736	7247	19
42	9886	9171	4328	5729	745 3770	8878	801 1511	830 2160	18
43	642 3994	667 3374	8633	719 0141	8296	773 3526	6288	7075	17
44	8105	7580	693 2939	4554	746 2824	8176	802 1067	831 1992	16
45	643 2216	668 1783	7247	8970	7354	774 2827	5849	6912	15
46	6329	5995	694 1557	720 3337	747 1886	7481	803 0632	832 1834	14
47	644 0444	669 0205	5868	7806	6420	775 2137	5418	6759	13
48	4560	4417	695 0181	721 2227	748 0956	6795	804 0206	833 1686	12
49	8678	8630	4498	6650	5494	776 1455	4997	6615	11
50	645 2797	670 2845	8813	722 1075	749 0038	6118	9790	834 1547	10
51	6918	7061	696 3131	5502	4575	777 0782	805 4584	6481	9
52	646 1041	671 1280	7451	9930	9119	5448	9382	835 1418	8
53	5165	5500	697 1773	723 4361	750 3665	778 0117	806 4181	6357	7
54	9290	9721	6097	8793	8212	4788	8983	836 1298	6
55	647 3417	672 3944	698 0422	724 3227	751 2762	9460	807 3787	6242	5
56	7546	8169	4749	7663	7814	779 4135	8593	837 1188	4
57	648 1676	673 2396	9078	725 2101	752 1867	8812	808 3401	6136	3
58	5808	6624	699 3409	6540	6423	780 3492	8212	838 1087	2
59	9941	674 0854	7741	726 098[illegible]	753 0981	8173	809 3025	6041	1
60	649 4076	5085	700 2075	542[illegible]	5541	781 2856	7840	839 0996	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	642 7876	656 0590	669 1806	681 9934	694 6584	707 1068	719 3398	731 8537	60
1	643 0104	2785	3468	682 2111	8676	3124	5418	5521	59
2	2332	4980	5628	4287	695 0767	5180	7438	7503	58
3	4559	7174	7789	6368	2858	7236	9457	9486	57
4	6785	9367	9948	8489	4949	9291	720 1476	732 1467	56
5	9011	657 1560	670 2108	683 0618	7089	708 1345	3494	3449	55
6	644 1236	3752	4266	2738	9128	3398	5511	5429	54
7	3461	5944	6424	4861	696 1217	5451	7528	7409	53
8	5685	8135	8582	6984	3305	7504	9544	9388	52
9	7909	658 0826	671 0739	9107	5392	9556	721 1559	733 1367	51
10	645 0132	2516	2895	684 1229	7479	709 1607	3574	3345	50
11	2355	4706	5051	3350	9565	3657	5589	5322	49
12	4577	6895	7206	5471	697 1651	5707	7602	7299	48
13	6798	9083	9361	7591	3736	7757	9615	9275	47
14	9019	659 1271	672 1515	9711	5821	9806	722 1628	734 1250	46
15	646 1240	3458	3668	685 1830	7905	710 1854	3640	3225	45
16	3460	5645	5821	3948	9988	3901	5651	5199	44
17	5679	7831	7973	6066	698 2071	5948	7661	7173	43
18	7898	660 0017	673 0125	8184	4153	7995	9671	9146	42
19	647 0116	2202	2276	686 0300	6234	711 0041	723 1681	735 1118	41
20	2334	4386	4427	2416	8315	2086	3690	3090	40
21	4551	6570	6577	4532	699 0396	4130	5693	5061	39
22	6767	8754	8727	6647	2476	6174	7705	7032	38
23	8984	661 0936	674 0876	8761	4555	8218	9712	9002	37
24	648 1199	3119	3024	687 0375	6633	712 0260	724 1719	736 0971	36
25	3414	5300	5172	2988	8711	2308	3724	2940	35
26	5628	7482	7319	5101	700 0789	4344	5729	4908	34
27	7842	9662	9466	7213	2866	6385	7734	6875	33
28	649 0056	662 1842	675 1612	9325	4942	8426	9738	8842	32
29	2268	4022	3757	688 1435	7018	713 0465	725 1741	737 0808	31
30	4480	6200	5902	3546	9098	2504	3744	2778	30
31	6692	8379	8046	5655	701 1167	4543	5746	4738	29
32	8903	663 0557	676 0190	7765	3241	6581	7747	6703	28
33	650 1114	2734	2333	9878	5314	8618	9748	8666	27
34	3324	4910	4476	689 1981	7387	714 0655	726 1748	738 0629	26
35	5533	7087	6618	4089	9459	2691	3748	2592	25
36	7742	9262	8760	6195	702 1531	4727	5747	4553	24
37	9951	664 1437	677 0901	8302	3601	6762	7745	6515	23
38	651 2158	3612	3041	690 0407	5672	8796	9743	8475	22
39	4366	5785	5181	2512	7741	715 0830	727 1740	739 0435	21
40	6572	7959	7320	4617	9811	2863	3736	2394	20
41	8778	665 0181	9459	6721	703 1879	4895	5732	4353	19
42	652 0984	2304	678 1597	8824	3947	6927	7728	6311	18
43	3189	4475	3734	691 0927	6014	8959	9722	8268	17
44	5394	6646	5871	3029	8081	716 0989	728 1716	740 0225	16
45	7598	8817	8007	5131	704 0147	3019	3710	2181	15
46	9801	666 0987	679 0143	7232	2213	5049	5703	4137	14
47	653 2004	3156	2278	9332	4278	7078	7695	6092	13
48	4206	5325	4413	692 1432	6342	9106	9686	8046	12
49	6408	7493	6547	3581	8406	717 1134	729 1677	741 0000	11
50	8609	9661	8681	5630	705 0469	3161	3668	1953	10
51	654 0810	667 1828	680 0818	7728	2532	5187	5657	3935	9
52	3010	3994	2946	9825	4594	7213	7646	5857	8
53	5209	6160	5078	693 1922	6655	9238	9635	7818	7
54	7408	8326	7209	4018	8716	718 1263	730 1623	9758	6
55	9607	668 0490	9339	6114	706 0776	3287	3610	742 1708	5
56	655 1804	2655	681 1469	8209	2835	5310	5597	3658	4
57	4002	4818	3599	694 0304	4894	7333	7583	5606	3
58	6198	6981	5728	2398	6953	9355	9568	7554	2
59	8395	9144	7856	4491	9011	719 1377	731 1553	9502	1
60	656 0590	669 1806	9934	6584	707 1068	3398	8537	743 1448	0
′	49°	48°	47°	46°	45°	44°	43°	42°	′

NATURAL COSINES.

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	839 0996	869 2867	900 4040	932 5151	965 6888	1·00 00000	1·03 55303	1·07 23687	60
1	5955	7976	9309	933 0591	966 2511	05819	61333	29943	59
2	840 0915	870 3087	901 4580	6034	8137	11642	67367	36203	58
3	5878	8200	9854	934 1479	967 3767	17469	73404	42467	57
4	841 0844	871 3316	902 5131	6928	9399	23298	79445	48734	56
5	5812	8435	903 0411	935 2380	968 5035	29131	85489	55006	55
6	842 0782	872 3556	5693	7834	969 0674	34968	91538	61282	54
7	5755	8680	904 0979	936 3292	6316	40807	97589	67561	53
8	843 0730	873 3806	6267	8753	970 1962	46651	1·04 03645	73845	52
9	5708	8935	905 1557	937 4216	7610	52497	09704	80132	51
10	844 0688	874 4067	6851	9683	971 3262	58348	15767	86423	50
11	5670	9201	906 2147	938 5153	8917	64201	21833	92718	49
12	845 0655	875 4338	7446	939 0625	972 4575	70058	27904	99018	48
13	5643	9478	907 2748	6101	973 0236	75918	33977	1·08 05321	47
14	846 0633	876 4620	8053	940 1579	5901	81782	40055	11628	46
15	5625	9765	908 3360	7061	974 1569	87649	46136	17939	45
16	847 0620	877 4912	8671	941 2545	7240	93520	52221	24254	44
17	5617	878 0062	909 3984	8033	975 2914	99394	58310	30573	43
18	848 0617	5215	9300	942 3523	8591	1·01 05272	64402	36896	42
19	5619	879 0370	910 4619	9017	976 4272	11153	70498	43223	41
20	849 0624	5528	9940	943 4513	9956	17033	76598	49554	40
21	5631	880 0688	911 5265	944 0013	977 5643	22925	82702	55889	39
22	850 0640	5852	912 0592	5516	978 1333	28817	88809	62228	38
23	5653	881 1017	5922	945 1021	7027	34712	94920	68571	37
24	851 0667	6186	913 1255	6530	979 2724	40610	1·05 01034	74918	36
25	5684	882 1357	6591	946 2042	8424	46512	07153	81269	35
26	852 0704	6531	914 1929	7556	980 4127	52418	13275	87624	34
27	5726	883 1707	7270	947 3074	9833	58326	19401	93984	33
28	853 0750	6886	915 2615	8595	981 5543	64239	25531	1·09 00347	32
29	5777	884 2068	7962	948 4119	982 1256	70155	31664	06714	31
30	854 0807	7253	916 3312	9646	6973	76074	37801	13085	30
31	5839	885 2440	8665	949 5176	983 2692	81997	43942	19460	29
32	855 0873	7630	917 4020	950 0709	8415	87923	50087	25840	28
33	5910	886 2822	9379	6245	984 4141	93853	56235	32223	27
34	856 0950	8017	918 4740	951 1784	9871	99786	62388	38610	26
35	5992	887 3215	919 0104	7326	985 5603	1·02 05723	68544	45002	25
36	857 1037	8415	5471	952 2871	986 1339	11664	74704	51397	24
37	6084	888 3619	920 0841	8420	7079	17608	80867	57797	23
38	858 1133	8825	6214	953 3971	987 2821	23555	87035	64201	22
39	6185	889 4033	921 1590	9526	8567	29506	93206	70609	21
40	859 1240	9244	6969	954 5083	988 4316	35461	99381	77020	20
41	6297	890 4458	922 2350	955 0644	989 0069	41419	1·06 05560	83436	19
42	860 1357	9675	7734	6208	5825	47381	11742	89857	18
43	6419	891 4894	923 3122	956 1774	990 1584	53346	17929	96281	17
44	861 1484	892 0116	8512	7344	7346	59315	24119	1·10 02709	16
45	6551	5341	924 3905	957 2917	991 3112	65287	30313	09141	15
46	862 1621	893 0569	9301	8494	8881	71263	36511	15578	14
47	6694	5799	925 4700	958 4073	992 4654	77243	42713	22019	13
48	863 1768	894 1032	926 0102	9655	993 0429	83226	48918	28463	12
49	6846	6268	5506	959 5241	6208	89212	55128	34912	11
50	864 1926	895 1506	927 0914	960 0829	994 1991	95203	61341	41365	10
51	7009	6747	6324	6421	7777	1·03 01196	67558	47823	9
52	865 2094	896 1991	928 1738	961 2016	995 3566	07194	73779	54284	8
53	7181	7238	7154	7614	9358	13195	80004	60750	7
54	866 2272	897 2487	929 2573	962 3215	996 5154	19199	86233	67219	6
55	7365	7739	7996	8819	997 0953	25208	92466	73693	5
56	867 2460	898 2994	930 3421	963 4427	6756	31220	98702	80171	4
57	7558	8251	8849	964 0037	998 2562	37235	1·07 04943	86653	3
58	868 2659	899 3512	931 4280	5651	8371	43254	11187	93140	2
59	7762	8775	9714	965 1268	999 4184	49277	17435	99630	1
60	869 2867	900 4040	932 5151	6888	1·000 0000	55303	23687	1·11 06125	0
′	49°	48°	47°	46°	45°	44°	43°	42°	′

NATURAL COTANGENTS.

′	48°	49°	50°	51°	52°	53°	54°	′
0	743 1448	754 7096	766 0444	777 1460	788 0108	798 6355	809 0170	60
1	3394	9004	2314	3290	1898	8105	1879	59
2	5340	755 0911	4183	5120	3688	9855	3588	58
3	7285	2818	6051	6949	5477	799 1604	5296	57
4	9229	4724	7918	8777	7266	3352	7004	56
5	744 1173	6630	9785	778 0604	9054	5100	8710	55
6	3115	8535	767 1652	2431	789 0841	6847	810 0416	54
7	5058	756 0439	3517	4258	2627	8593	2122	53
8	6999	2343	5382	6084	4413	800 0338	3826	52
9	8941	4246	7246	7909	6198	2083	5530	51
10	745 0881	6148	9110	9733	7983	3827	7234	50
11	2821	8050	768 0973	779 1557	9767	5571	8936	49
12	4760	9951	2835	3380	790 1550	7314	811 0638	48
13	6699	757 1851	4697	5202	3333	9056	2339	47
14	8636	6751	6558	7024	5115	801 0797	4040	46
15	746 0574	5650	8418	8845	6896	2538	5740	45
16	2510	7548	769 0278	780 0665	8676	4278	7439	44
17	4446	9446	2137	2485	791 0456	6018	9137	43
18	6382	758 1343	3996	4304	2235	7756	812 0835	42
19	8317	3240	5853	6123	4014	9495	2532	41
20	747 0251	5136	7710	7940	5792	802 1232	4229	40
21	2184	7031	9567	9757	7569	2969	5925	39
22	4117	8926	770 1423	781 1574	9345	4705	7620	38
23	6049	759 0820	3278	3390	792 1121	6440	9314	37
24	7981	2713	5132	5205	2896	8175	813 1008	36
25	9912	4606	6986	7019	4671	9909	2701	35
26	748 1842	6498	8840	8833	6445	803 1642	4393	34
27	3772	8389	771 0692	782 0646	8218	3375	6084	33
28	5701	760 0280	2544	2459	9990	5107	7775	32
29	7629	2170	4395	4270	793 1762	6838	9466	31
30	9557	4060	6246	6082	3533	8569	814 1155	30
31	749 1484	5949	8096	7892	5304	804 0299	2844	29
32	3411	7837	9945	9702	7074	2028	4532	28
33	5337	9724	772 1794	783 1511	8843	3756	6220	27
34	7262	761 1611	3642	3320	794 0611	5484	7906	26
35	9187	3497	5489	5127	2379	7211	9593	25
36	750 1111	5383	7336	6935	4146	8938	815 1278	24
37	3034	7268	9182	8741	5913	805 0664	2963	23
38	4957	9152	773 1027	784 0547	7678	2389	4647	22
39	6879	762 1036	2872	2352	9444	4113	6330	21
40	8800	2919	4716	4157	795 1208	5837	8013	20
41	751 0721	4802	6559	5961	2972	7560	9695	19
42	2641	6683	8402	7764	4735	9283	816 1376	18
43	4561	8564	774 0244	9566	6497	806 1005	3056	17
44	6480	763 0445	2086	785 1368	8259	2726	4736	16
45	8398	2325	3926	3169	796 0020	4446	6416	15
46	752 0316	4204	5767	4970	1780	6166	8094	14
47	2233	6082	7606	6770	3540	7885	9772	13
48	4149	7960	9445	8569	5299	9603	817 1449	12
49	6065	9838	775 1283	786 0367	7058	807 1321	3125	11
50	7980	764 1714	3121	2165	8815	3038	4801	10
51	9894	3590	4957	3963	797 0572	4754	6476	9
52	753 1808	5465	6794	5759	2329	6470	8151	8
53	3721	7340	8629	7555	4084	8185	9824	7
54	5634	9214	776 0464	9350	5839	9899	818 1497	6
55	7546	765 1087	2298	787 1145	7594	808 1612	3169	5
56	9457	2960	4132	2939	9347	3325	4841	4
57	754 1368	4832	5965	4732	798 1100	5037	6512	3
58	3278	6704	7797	6524	2853	6749	8182	2
59	5187	8574	9629	8316	4604	8460	9852	1
60	7096	766 0444	777 1460	788 0108	6355	809 0170	819 1520	0
′	41°	40°	39°	38°	37°	36°	35°	′

′	48°	49°	50°	51°	52°	53°	54°	′
0	1·11 06125	1·15 03684	1·19 17536	1·23 48972	1·27 99416	1·32 70448	1·37 63819	60
1	12624	10445	24579	56319	1·28 07094	78483	72242	59
2	19127	17210	31626	63672	14776	86524	80672	58
3	25635	23979	38679	71030	22465	94571	89108	57
4	32146	30754	45736	78393	30160	1·33 02624	97551	56
5	38662	37532	52799	85762	37860	10684	1·38 06001	55
6	45182	44316	59866	93136	45566	18750	14458	54
7	51706	51104	66938	1·24 00515	53277	26822	22922	53
8	58235	57896	74015	07900	60995	34900	31392	52
9	64768	64693	81097	15290	68718	42984	39869	51
10	71305	71495	88184	22685	76447	51075	48358	50
11	77846	78301	95276	30086	84182	59172	56844	49
12	84391	85112	1·20 02373	37492	91922	67276	65342	48
13	90941	91927	09475	44903	99669	75386	73847	47
14	97495	98747	16581	52320	1·29 07421	83502	82358	46
15	1·12 04053	1·16 05571	23693	59742	15179	91624	90876	45
16	10616	12400	30810	67169	22943	99753	99401	44
17	17183	19234	37932	74602	30713	1·34 07888	1·39 07934	43
18	23754	26073	45058	82040	38488	16029	16478	42
19	30329	32916	52190	89484	46270	24177	25019	41
20	36909	39763	59327	96933	54057	32331	33571	40
21	43493	46615	66468	1·25 04388	61850	40492	42131	39
22	50081	53472	73615	11848	69649	48658	50698	38
23	56674	60334	80767	19318	77454	56832	59272	37
24	63271	67200	87924	26784	85265	65011	67852	36
25	69872	74071	95085	34260	93081	73198	76440	35
26	76478	80947	1·21 02252	41742	1·30 00904	81390	85034	34
27	83088	87827	09424	49229	08733	89589	93636	33
28	89702	94712	16601	56721	16567	97794	1·40 02245	32
29	96321	1·17 01601	23783	64219	24407	1·35 06006	10860	31
30	1·13 02944	08496	30970	71723	32254	14224	19483	30
31	09571	15395	38162	79232	40106	22449	28113	29
32	16203	22298	45359	86747	47964	30680	36749	28
33	22839	29207	52562	94267	55828	38918	45393	27
34	29479	36120	59769	1·26 01792	63699	47162	54044	26
35	36124	43038	66982	09323	71575	55413	62702	25
36	42773	49960	74199	16860	79457	63670	71367	24
37	49427	56888	81422	24402	87345	71934	80039	23
38	56085	63820	88650	31950	95239	80204	88718	22
39	62747	70756	95883	39503	1·31 03140	88481	97405	21
40	69414	77698	1·22 03121	47062	11046	96764	1·41 06098	20
41	76086	84644	10364	54626	18958	1·36 05054	14799	19
42	82761	91595	17613	62196	26876	13350	23506	18
43	89441	98551	24866	69772	34801	21653	32221	17
44	96126	1·18 05512	32125	77353	42731	29963	40943	16
45	1·14 02815	12477	39389	84940	50668	38279	49673	15
46	09508	19447	46658	92532	58610	46602	58409	14
47	16206	26422	53932	1·27 00130	66559	54931	67158	13
48	22908	33402	61211	07733	74518	63267	75904	12
49	29615	40387	68496	15342	82474	71610	84662	11
50	36326	47376	75786	22957	90441	79959	93427	10
51	43041	54370	83081	30578	98414	88315	1·42 02200	9
52	49762	61369	90381	38204	1·32 06393	96678	10979	8
53	56486	68373	97687	45835	14379	1·37 05047	19766	7
54	63215	75382	1·23 04997	53473	22370	13423	28561	6
55	69949	82395	12313	61116	30368	21806	37362	5
56	76687	89414	19634	68765	38371	30195	46171	4
57	83429	96437	26961	76419	46381	38591	54988	3
58	90176	1·19 03465	34292	84079	54397	46994	63811	2
59	96928	10498	41629	91745	62420	55403	72642	1
60	1·15 03684	17536	48972	99416	70448	63819	81480	0
′	41°	40°	39°	38°	37°	36°	35°	′

NATURAL COTANGENTS.

′	55°	56°	57°	58°	59°	60°	61°	′
0	819 1520	829 0376	838 6706	848 0481	857 1673	866 0254	874 6197	60
1	3189	2002	8290	2022	3171	1708	7607	59
2	4856	3628	9873	3562	4668	3161	9016	58
3	6523	5252	839 1455	5102	6164	4614	875 0425	57
4	8189	6877	3037	6641	7660	6066	1832	56
5	9854	8500	4618	8179	9155	7517	3239	55
6	820 1519	830 0123	6199	9717	858 0649	8967	4645	54
7	3183	1745	7778	849 1254	2143	867 0417	6051	53
8	4846	3366	9357	2790	3635	1866	7455	52
9	6509	4987	840 0936	4325	5127	3314	8859	51
10	8170	6607	2513	5860	6619	4762	876 0263	50
11	9832	8226	4090	7394	8109	6209	1665	49
12	821 1492	9845	5666	8927	9599	7655	3067	48
13	3152	831 1463	7241	850 0459	859 1088	9100	4468	47
14	4811	3080	8816	1991	2576	868 0544	5868	46
15	6469	4696	841 0390	3522	4064	1988	7268	45
16	8127	6312	1963	5053	5551	3431	8666	44
17	9784	7927	3536	6582	7037	4874	877 0064	43
18	822 1440	9541	5108	8111	8523	6315	1462	42
19	3096	832 1155	6679	9639	860 0007	7756	2858	41
20	4751	2768	8249	851 1167	1491	9196	4254	40
21	6405	4380	9819	2693	2975	869 0636	5649	39
22	8059	5991	842 1388	4219	4457	2074	7043	38
23	9712	7602	2956	5745	5939	3512	8437	37
24	823 1364	9212	4524	7269	7420	4949	9830	36
25	3015	833 0822	6091	8793	8901	6386	878 1222	35
26	4666	2430	7657	852 0316	861 0380	7821	2613	34
27	6316	4038	9222	1839	1859	9256	4004	33
28	7965	5646	843 0787	3360	3337	870 0691	5394	32
29	9614	7252	2351	4881	4815	2124	6783	31
30	824 1262	8858	3914	6402	6292	3557	8171	30
31	2909	834 0463	5477	7921	7768	4989	9559	29
32	4556	2068	7039	9440	9243	6420	879 0946	28
33	6202	3672	8600	853 0958	862 0717	7851	2332	27
34	7847	5275	844 0161	2475	2191	9281	3717	26
35	9491	6877	1720	3992	3664	871 0710	5102	25
36	825 1135	8479	3279	5508	5137	2138	6486	24
37	2778	835 0080	4838	7023	6608	3566	7869	23
38	4420	1680	6395	8538	8079	4993	9251	22
39	6062	3279	7952	854 0051	9549	6419	880 0633	21
40	7703	4878	9508	1564	863 1019	7844	2014	20
41	9343	6476	845 1064	3077	2488	9269	3394	19
42	826 0983	8074	2618	4588	3956	872 0693	4774	18
43	2622	9670	4172	6099	5423	2116	6152	17
44	4260	836 1266	5726	7609	6889	3538	7530	16
45	5897	2862	7278	9119	8355	4960	8907	15
46	7534	4456	8830	855 0627	9820	6381	881 0284	14
47	9170	6050	846 0381	2135	864 1284	7801	1660	13
48	827 0806	7643	1932	3643	2748	9221	3035	12
49	2440	9236	3481	5149	4211	873 0640	4409	11
50	4074	837 0827	5030	6655	5673	2058	5782	10
51	5708	2418	6579	8160	7134	3475	7155	9
52	7340	4009	8126	9664	8595	4891	8527	8
53	8972	5598	9673	856 1168	865 0055	6307	9898	7
54	828 0603	7187	847 1219	2671	1514	7722	882 1269	6
55	2234	8775	2765	4173	2973	9137	2638	5
56	3864	838 0363	4309	5674	4430	874 0550	4007	4
57	5493	1950	5853	7175	5887	1963	5376	3
58	7121	3536	7397	8675	7344	3375	6743	2
59	8749	5121	8939	857 0174	8799	4786	8110	1
60	829 0376	6706	848 0481	1673	866 0254	6197	9476	0
′	34°	33°	32°	31°	30°	29°	28°	′

′	55°	56°	57°	58°	59°	60°	61°	′
0	1·42 81480	1·48 25610	1·53 98650	1·60 03345	1·66 42795	1·73 20508	1·80 40478	60
1	90326	84916	1·54 08460	13709	53766	32149	52860	59
2	99178	44231	18280	24082	64748	43803	65256	58
3	1·43 08039	53554	28108	34465	75741	55468	77664	57
4	16906	62884	37946	44858	86744	67144	90086	56
5	25781	72228	47792	55260	97758	78833	1·81 02521	55
6	34664	81570	57647	65672	1·67 08782	90538	14969	54
7	43554	90925	67510	76094	19818	1·74 02245	27430	53
8	52451	1·49 00288	77388	86525	30864	13909	39904	52
9	61356	09659	87264	96966	41921	25705	52391	51
10	70268	19039	97155	1·61 07417	52988	37458	64892	50
11	79187	28426	1·55 07054	17873	64067	49213	774 5	49
12	88114	37822	16963	28349	75156	60984	89932	48
13	97049	47225	26880	38829	86256	72768	1·82 02478	47
14	1·44 05991	56637	36806	49320	97367	84564	15026	46
15	14940	66058	46741	59820	1·68 08189	96371	27593	45
16	23897	75486	56685	70330	19621	1·75 08191	40178	44
17	32862	84923	66639	80850	30765	20023	52767	43
18	41834	94367	76601	91380	41919	31866	65374	42
19	50814	1·50 03821	86572	1·62 01920	53085	43722	77994	41
20	59801	13282	96552	12469	64261	55590	90628	40
21	68796	22751	1·56 06542	23029	75449	67470	1·83 03275	39
22	77798	32229	16540	33599	86647	79362	15936	38
23	86808	41716	26548	44178	97856	91267	28610	37
24	95825	51210	36564	54768	1·69 09077	1·76 03183	41297	36
25	1·45 04850	60718	46590	65368	20808	15112	53999	35
26	13883	70224	56625	75977	31550	27053	66713	34
27	22923	79743	66669	86597	42804	39007	79442	33
28	31971	89271	76722	97227	54069	50972	92184	32
29	41027	98807	86784	1·63 07867	65344	62950	1·84 04940	31
30	50090	1·51 08352	96856	18517	76681	74940	17709	30
31	59161	17905	1·57 06936	29177	87929	86948	30492	29
32	68240	27466	17026	39847	99238	98958	43289	28
33	77326	37086	27126	5 528	1·70 10559	1·77 10985	56 99	27
34	86420	46614	37234	61218	21890	23024	68923	26
35	95522	56201	47352	71919	33233	35076	81761	25
36	1·46 04632	65796	57479	82630	44587	47141	94613	24
37	13749	75400	67615	93351	55953	59218	1·85 07479	23
38	22874	85012	77760	1·64 04082	67329	71307	2 358	22
39	32007	94632	87915	14824	78717	83409	33252	21
40	41147	1·52 04261	98079	25576	90116	95524	46159	20
41	50296	13899	1·58 08258	36338	1·71 01527	1·78 07651	59080	19
42	59452	23545	18436	47111	12949	19790	72015	18
43	68616	33200	28628	57893	24382	31948	84965	17
44	77788	42868	38880	68687	35827	44107	97928	16
45	86967	52535	49041	79490	47283	56285	1·86 10905	15
46	96155	62215	59261	90304	58751	68475	23896	14
47	1·47 05350	71904	69491	1·65 01128	70230	80678	36902	13
48	14553	81602	79731	11963	81720	92898	49921	12
49	23764	91308	89979	22808	93222	1·79 05121	62955	11
50	32983	1·53 01023	1·59 00238	33663	1·72 04736	17362	76003	10
51	42210	10746	10505	44529	16261	29616	89065	9
52	51445	20479	20783	55405	27797	41883	1·87 02141	8
53	60688	3 219	31070	66292	39346	54162	15231	7
54	69938	39969	41366	77189	50965	66454	28336	6
55	79197	49727	51672	88097	62477	78759	41455	5
56	88463	59491	61987	99016	74000	91077	54588	4
57	97738	69270	72312	1·66 09945	85654	1·80 03408	67736	3
58	1·48 07021	79054	82647	20884	97260	15751	80898	2
59	16311	88848	92991	31834	1·73 08878	28108	94074	1
60	25610	98650	1·60 03345	42795	2 508	40478	1·88 07265	0
′	34°	33°	32°	31°	30°	29°	28°	′

NATURAL COTANGENTS.

′	62°	63°	64°	65°	66°	67°	68°	′
0	882 9476	891 0065	898 7940	906 3078	913 5455	920 5049	927 1839	60
1	883 0841	1385	9215	4307	6637	6185	2928	59
2	2206	2705	899 0489	5535	7819	7320	4016	58
3	3569	4024	1763	6762	9001	8455	5104	57
4	4933	5342	3035	7989	914 0181	9589	6191	56
5	6295	6659	4307	9215	1361	921 0722	7277	55
6	7656	7975	5578	907 0440	2540	1854	8363	54
7	9017	9291	6848	1665	3718	2986	9447	53
8	884 0377	892 0606	8117	2888	4895	4116	928 0531	52
9	1736	1920	9386	4111	6072	5246	1614	51
10	3095	3234	900 0654	5333	7247	6375	2696	50
11	4453	4546	1921	6554	8422	7504	3778	49
12	5810	5858	3188	7775	9597	8632	4858	48
13	7166	7169	4458	8995	915 0770	9758	5938	47
14	8522	8480	5718	908 0214	1943	922 0884	7017	46
15	9876	9789	6982	1432	3115	2010	8096	45
16	885 1230	893 1098	8246	2649	4286	3134	9173	44
17	2584	2406	9508	3866	5456	4258	929 0250	43
18	3936	3714	901 0770	5082	6626	5381	1326	42
19	5288	5021	2031	6297	7795	6503	2401	41
20	6639	6326	3292	7511	8963	7624	3475	40
21	7989	7632	4551	8725	916 0130	8745	4549	39
22	9339	8936	5810	9938	1297	9865	5622	38
23	886 0688	894 0240	7068	909 1150	2462	923 0984	6694	37
24	2036	1542	8325	2361	3627	2102	7765	36
25	3383	2844	9582	3572	4791	3220	8835	35
26	4730	4146	902 0838	4781	5955	4336	9905	34
27	6075	5446	2092	5990	7118	5452	930 0974	33
28	7420	6746	3347	7199	8279	6567	2042	32
29	8765	8045	4600	8406	9440	7682	3109	31
30	887 0108	9344	5853	9613	917 0601	8795	4176	30
31	1451	895 0641	7105	910 0819	1760	9908	5241	29
32	2793	1938	8356	2024	2919	924 1020	6306	28
33	4134	3234	9606	3228	4077	2131	7370	27
34	5475	4529	903 0856	4432	5234	3242	8434	26
35	6815	5824	2105	5635	6391	4351	9496	25
36	8154	7118	3353	6837	7546	5460	931 0558	24
37	9492	8411	4600	8038	8701	6568	1619	23
38	888 0830	9703	5847	9238	9855	7676	2679	22
39	2166	896 0994	7098	911 0438	918 1009	8782	3739	21
40	3503	2285	8338	1637	2161	9888	4797	20
41	4838	3575	9582	2835	3313	925 0993	5855	19
42	6172	4864	904 0825	4033	4464	2097	6912	18
43	7506	6153	2068	5229	5614	3201	7969	17
44	8839	7440	3310	6425	6763	4303	9024	16
45	889 0171	8727	4551	7620	7912	5405	932 0079	15
46	1503	897 0014	5792	8815	9060	6506	1133	14
47	2834	1299	7032	912 0008	919 0207	7606	2186	13
48	4164	2584	8271	1201	1353	8706	3238	12
49	5493	3868	9509	2393	2499	9805	4290	11
50	6822	5151	905 0746	3584	3644	926 0902	5340	10
51	8149	6433	1983	4775	4788	2000	6390	9
52	9476	7715	3219	5965	5931	3096	7439	8
53	890 0803	8996	4454	7154	7073	4192	8488	7
54	2128	898 0276	5688	8342	8215	5286	9535	6
55	3453	1555	6922	9529	9356	6380	933 0582	5
56	4777	2834	8154	913 0716	920 0496	7474	1628	4
57	6100	4112	9386	1902	1635	8566	2673	3
58	7423	5389	906 0618	3087	2774	9658	3718	2
59	8744	6665	1848	4271	3912	927 0748	4761	1
60	891 0065	7940	3078	5455	5049	1839	5804	0
′	27°	26°	25°	24°	23°	22°	21°	′

′	62°	63°	64°	65°	66°	67°	68°	′
0	1·88 07265	1·96 26105	2·05 03038	2·14 45069	2·24 60368	2·35 58524	2·47 50869	60
1	20470	40227	18185	61366	77962	77590	71612	59
2	33690	54364	33349	77688	95580	96683	92386	58
3	46924	68518	48581	94021	2·25 13221	2·36 15801	2·48 13190	57
4	60172	82688	63732	2·15 10378	30885	34916	34023	56
5	73436	96874	78950	26757	48572	54118	54887	55
6	86713	1·97 11077	94187	43156	66283	73316	75781	54
7	1·89 00006	25296	2·06 09442	59575	84016	92540	96706	53
8	13318	39531	24716	76015	2·26 01773	2·37 11791	2·49 17660	52
9	26635	53782	40008	92476	19554	31068	38645	51
10	39971	68050	55318	2·16 08958	37357	50372	59661	50
11	53322	82334	70646	25460	55184	69708	80707	49
12	66688	96635	85994	41983	73035	89060	2·50 01784	48
13	80068	1·98 10952	2·07 01359	58527	90909	2·38 08444	22891	47
14	93464	25286	16743	75091	2·27 08807	27855	44029	46
15	1·90 06874	39636	32146	91677	26729	47293	65193	45
16	20299	54008	47567	2·17 08283	44674	66758	86398	44
17	33738	68387	63007	24911	62643	86250	2·51 07629	43
18	47193	82787	78465	41559	80636	2·39 05769	28891	42
19	60663	97204	93942	58229	98653	25316	50183	41
20	74147	1·99 11637	2·08 09438	74920	2·28 16693	44839	71507	40
21	87647	26087	24953	91631	34758	64490	92863	39
22	1·91 01162	40554	40487	2·18 08364	52846	84118	2·52 14249	38
23	14691	55038	56039	25119	70959	2·40 03774	35667	37
24	28236	69539	71610	41894	89096	23457	57117	36
25	41795	84056	87200	58691	2·29 07257	43168	78593	35
26	55370	98590	2·09 02809	75510	25442	62906	2·53 00111	34
27	68960	2·00 13142	18437	92349	43651	82672	21655	33
28	82565	27710	34085	2·19 09210	61885	2·41 02465	43231	32
29	96186	42295	49751	26093	80143	22286	64839	31
30	1·92 09821	56897	65436	42997	98425	42136	86479	30
31	23472	71516	81140	59923	2·30 16732	62018	2·54 08151	29
32	37138	86153	96864	76871	35064	81918	29855	28
33	50819	2·01 00806	2·10 12607	93840	53420	2·42 01851	51591	27
34	64516	15477	28369	2·20 10831	71801	21812	73359	26
35	78228	30164	44150	27843	90206	41801	95160	25
36	91956	44869	59951	44878	2·31 08637	61819	2·55 16992	24
37	1·93 05699	59592	75771	61934	27092	81864	38858	23
38	19457	74331	91611	79012	45571	2·43 01938	60756	22
39	33231	89088	2·11 07470	96112	64076	22041	82686	21
40	47020	2·02 03862	23348	2·21 13234	82606	42172	2·56 04649	20
41	60825	18654	39246	30379	2·32 01160	62331	26645	19
42	74645	33462	55164	47545	19740	82519	48674	18
43	88481	48289	71101	64733	38345	2·44 02736	70735	17
44	1·94 02333	63133	87057	81944	56975	22982	92830	16
45	16200	77994	2·12 03034	99177	75630	43256	2·57 14957	15
46	30083	92873	19030	2·22 16432	94311	63559	37118	14
47	43981	2·03 07769	35046	33709	2·33 13017	83891	59312	13
48	57896	22683	51082	51009	31748	2·45 04252	81539	12
49	71826	37615	67137	68331	50505	24642	2·58 03800	11
50	85772	52565	83213	85676	69287	45061	26094	10
51	99733	67532	99338	2·23 03043	88095	65510	48421	9
52	1·95 13711	82517	2·13 15423	20433	2·34 06928	85987	70782	8
53	27704	97519	31559	37845	25787	2·46 06494	93177	7
54	41713	2·04 12540	47714	55280	44672	27030	2·59 15606	6
55	55739	27578	63890	72738	63582	47596	38068	5
56	69780	42634	80085	90213	82519	68191	60564	4
57	83837	57708	96301	2·24 07721	2·35 01481	88816	83095	3
58	97910	72800	2·14 12537	25247	20469	2·47 09470	2·60 05659	2
59	1·96 12000	87910	28793	42796	39488	30155	28258	1
60	26105	2·05 03038	45069	60368	58524	50869	50891	0
′	27°	26°	25°	24°	23°	22°	21°	′

′	69°	70°	71°	72°	73°	74°	75°	′
0	933 5804	939 6926	945 5186	951 0565	956 3048	961 2617	965 9258	60
1	6846	7921	6132	1464	3898	3418	966 0011	59
2	7888	8914	7078	2361	4747	4219	0762	58
3	8928	9907	8023	3258	5595	5019	1513	57
4	9968	940 0899	8968	4154	6443	5818	2263	56
5	934 1007	1891	9911	5050	7290	6616	3012	55
6	2045	2881	946 0854	5944	8136	7413	3761	54
7	3082	3871	1795	6838	8981	8210	4508	53
8	4119	4860	2736	7731	9825	9005	5255	52
9	5154	5848	3677	8623	957 0669	9800	6001	51
10	6189	6835	4616	9514	1512	962 0594	6746	50
11	7223	7822	5555	952 0404	2354	1387	7490	49
12	8257	8808	6493	1294	3195	2180	8234	48
13	9289	9793	7430	2183	4035	2972	8977	47
14	935 0321	941 0777	8366	3071	4875	3762	9718	46
15	1352	1760	9301	3958	5714	4552	967 0459	45
16	2382	2743	947 0236	4844	6552	5342	1200	44
17	3412	3724	1170	5730	7389	6130	1939	43
18	4440	4705	2103	6615	8225	6917	2678	42
19	5468	5686	3035	7499	9061	7704	3415	41
20	6495	6665	3966	8382	9895	8490	4152	40
21	7521	7644	4897	9264	958 0729	9275	4888	39
22	8547	8621	5827	953 0146	1562	963 0060	5624	38
23	9571	9598	6756	1027	2394	0843	6358	37
24	936 0595	942 0575	7684	1907	3226	1626	7092	36
25	1618	1550	8612	2786	4056	2408	7825	35
26	2641	2525	9538	3664	4886	3189	8557	34
27	3662	3498	948 0464	4542	5715	3969	9288	33
28	4683	4471	1389	5418	6543	4748	968 0018	32
29	5703	5444	2313	6294	7371	5527	0748	31
30	6722	6415	3237	7170	8197	6305	1476	30
31	7740	7386	4159	8044	9023	7081	2204	29
32	8758	8355	5081	8917	9848	7858	2931	28
33	9774	9324	6002	9790	959 0672	8633	3658	27
34	937 0790	943 0293	6922	954 0662	1496	9407	4383	26
35	1806	1260	7842	1533	2318	964 0181	5108	25
36	2820	2227	8760	2403	3140	0954	5832	24
37	3833	3192	9678	3273	3961	1726	6555	23
38	4846	4157	949 0595	4141	4781	2497	7277	22
39	5858	5122	1511	5009	5600	3268	7998	21
40	6869	6085	2426	5876	6418	4037	8719	20
41	7880	7048	3341	6743	7236	4806	9438	19
42	8889	8010	4255	7608	8053	5574	969 0157	18
43	9898	8971	5168	8473	8869	6341	0875	17
44	938 0906	9931	6080	9336	9684	7108	1593	16
45	1913	944 0890	6991	955 0199	960 0499	7873	2309	15
46	2920	1849	7902	1062	1312	8638	3025	14
47	3925	2807	8812	1923	2125	9402	3740	13
48	4930	3764	9721	2784	2937	965 0165	4453	12
49	5934	4720	950 0629	3643	3748	0927	5167	11
50	6938	5675	1536	4502	4558	1689	5879	10
51	7940	6630	2443	5361	5368	2449	6591	9
52	8942	7584	3348	6218	6177	3209	7301	8
53	9943	8537	4253	7074	6984	3968	8011	7
54	939 0943	9489	5157	7930	7792	4726	8720	6
55	1942	945 0441	6061	8785	8598	5484	9428	5
56	2940	1391	6963	9639	9403	6240	970 0136	4
57	3938	2341	7865	956 0492	961 0208	6996	0842	3
58	4935	3290	8766	1345	1012	7751	1548	2
59	5931	4238	9666	2197	1815	8505	2253	1
60	6926	5186	951 0565	3048	2617	9258	2957	0
′	20°	19°	18°	17°	16°	15°	14°	′

NATURAL COSINES.

′	69°	70°	71°	72°	73°	74°	75°	′
0	2·60 50891	2·74 74774	2·90 42109	3·07 76835	3·27 08526	3·48 74144	3·73 20508	60
1	73558	99661	69576	3·08 07325	42588	3·49 12470	63980	59
2	96259	2·75 24588	97[illegible]9	37869	76715	50874	3·74 07546	58
3	2·61 18995	49554	2·91 24649	68468	3·28 10907	89856	51207	57
4	41766	74561	52276	99122	45164	3·50 27916	94963	56
5	64571	99608	799[illegible]9	3·09 29831	79487	66555	3·75 38815	55
6	87411	2·76 24695	2·92 07610	60596	3·29 13876	3·51 05273	82763	54
7	2·62 10286	49822	35358	91416	48330	44070	3·76 26807	53
8	83196	74990	63152	3·10 22291	82851	82946	70947	52
9	56141	2·77 00199	9[illegible]995	59223	3·30 17438	3·52 21902	3·77 15185	51
10	79121	25448	2·93 18885	84210	52091	60988	59519	50
11	2·63 02136	50788	46822	3·11 15254	86811	3·53 00054	3·78 08951	49
12	25136	76069	74307	46353	3·31 21598	89251	48481	48
13	48271	2·78 01440	2·94 02840	77509	56452	78528	93109	47
14	71393	26853	30921	3·12 08722	91373	3·54 17886	3·79 37835	46
15	94549	52307	59050	39991	3·32 26362	57325	82661	45
16	2·64 17741	77802	87227	71317	61419	96846	3·80 27585	44
17	40969	2·79 03339	2·95 15453	3·13 02701	96543	3·55 36449	72609	43
18	64232	28917	43727	34141	3·33 31736	76133	3·81 17733	42
19	87531	54537	72050	65639	66997	3·56 15900	62957	41
20	2·65 10867	80193	2·96 00422	97194	3·34 02326	55749	3·82 08281	40
21	34238	2·80 05931	28842	3·14 28367	87724	95681	53767	39
22	57645	31646	57312	60478	78191	3·57 35096	99233	38
23	81089	57433	85831	92207	3·35 08728	75794	3·83 44861	37
24	2·66 04569	83263	2·97 14399	3·15 23994	44333	3·58 15975	90591	36
25	28085	2·81 09134	43016	55840	80068	56241	3·84 36424	35
26	51633	35048	71683	87744	3·36 15753	96590	82358	34
27	75227	61004	2·98 00400	3·16 19706	51568	3·59 37024	3·85 28396	33
28	98853	87003	29167	51728	87453	77543	74537	32
29	2·67 22516	2·82 13045	57988	838[illegible]8	3·37 234[illegible]8	3·60 18146	3·86 20792	31
30	46215	39129	86850	3·17 15948	59434	58835	67131	30
31	69951	65256	2·99 15766	48147	95531	99609	3·87 13564	29
32	93725	91426	44734	80406	3·38 81699	3·61 40469	60142	28
33	2·68 17535	2·83 17639	73751	3·18 12724	67938	81415	3·88 06805	27
34	41883	43896	3·00 02820	45102	3·39 04249	3·62 22447	53574	26
35	65267	70196	31939	77540	40631	63566	3·89 00448	25
36	89190	96539	61109	3·19 10039	77085	3·63 04771	47429	24
37	2·69 13149	2·84 22926	9[illegible]330	42598	3·40 13612	46064	94516	23
38	37147	49356	3·01 19603	75217	50210	87444	3·90 41710	22
39	61131	75831	48926	3·20 07897	86882	3·64 28911	89011	21
40	85254	2·85 02349	78301	40638	3·41 23626	70467	3·91 36420	20
41	2·70 09364	28911	3·02 07728	73440	60443	3·65 12111	83937	19
42	33513	55517	37207	3·21 06304	97333	53844	3·92 31563	18
43	57699	82168	66737	39228	3·42 34297	95665	79297	17
44	81923	2·86 08863	96320	72215	71334	3·66 87575	3·93 27141	16
45	2·71 06186	35602	3·03 25934	3·22 05263	3·43 08446	79575	75[illegible]94	15
46	30487	62386	55641	38373	45631	3·67 21665	3·94 23157	14
47	54826	89215	85331	71546	82891	63845	71331	13
48	79204	2·87 16088	3·04 15173	3·23 04780	3·44 20226	3·68 06115	3·95 19615	12
49	2·72 03620	43007	45018	38078	57635	48475	68011	11
50	28076	69970	74915	71438	95120	90927	3·96 16518	10
51	52569	96979	3·05 04866	3·24 04860	3·45 32679	3·69 33469	65137	9
52	77102	2·88 24033	34370	38346	70315	76104	3·97 13868	8
53	2·73 01674	51132	64928	71895	3·46 08026	3·70 18830	62712	7
54	26284	78277	95038	3·25 05508	45813	61648	3·98 11669	6
55	50934	2·89 05467	3·06 25203	39184	83676	3·71 04558	60739	5
56	75623	32704	55421	72924	3·47 21616	47561	3·99 09924	4
57	2·74 00352	59985	85694	3·26 06728	59632	90658	59223	3
58	25120	87314	3·07 16020	40596	97726	3·72 33847	4·00 08636	2
59	49927	2·90 14688	46400	74529	3·48 35896	77131	58165	1
60	74774	42109	76835	3·27 08526	74144	3·73 20508	4·01 07809	0
′	20°	19°	18°	17°	16°	15°	14°	

′	76°	77°	78°	79°	80°	81°	82°	′
0	970 2957	974 3701	978 1476	981 6272	9848 078	9876 883	9902 681	60
1	3661	4355	2080	6826	582	9877 338	9903 085	59
2	4363	5008	2684	7380	9849 086	792	489	58
3	5065	5660	3287	7933	589	9878 245	891	57
4	5766	6311	3889	8485	9850 091	697	9904 293	56
5	6466	6962	4490	9037	593	9879 148	694	55
6	7165	7612	5090	9587	9851 093	599	9905 095	54
7	7863	8261	5689	982 0137	593	9880 048	494	53
8	8561	8909	6288	0686	9852 092	497	893	52
9	9258	9556	6886	1234	590	945	9906 290	51
10	9953	975 0203	7483	1781	9853 087	9881 392	687	50
11	971 0649	0849	8079	2327	583	838	9907 083	49
12	1343	1494	8674	2873	9854 079	9882 284	478	48
13	2036	2138	9268	3417	574	728	873	47
14	2729	2781	9862	3961	9855 068	9883 172	9908 266	46
15	3421	3423	979 0455	4504	561	615	659	45
16	4112	4065	1047	5046	9856 053	9884 057	9909 051	44
17	4802	4706	1638	5587	544	498	442	43
18	5491	5345	2228	6128	9857 035	939	832	42
19	6180	5985	2818	6668	524	9885 378	9910 221	41
20	6867	6623	3406	7206	9858 013	817	610	40
21	7554	7260	3994	7744	501	9886 255	997	39
22	8240	7897	4581	8282	988	692	9911 384	38
23	8926	8533	5167	8818	9859 475	9887 128	770	37
24	9610	9168	5752	9353	960	564	9912 155	36
25	972 0294	9802	6337	9888	9860 445	998	540	35
26	0976	976 0435	6921	983 0422	929	9888 432	923	34
27	1658	1068	7504	0955	9861 412	865	9913 306	33
28	2339	1699	8086	1487	894	9889 297	688	32
29	3020	2330	8668	2019	9862 375	728	9914 069	31
30	3699	2960	9247	2549	856	9890 159	449	30
31	4378	3589	9827	3079	9863 336	588	828	29
32	5056	4218	980 0405	3608	815	9891 017	9915 206	28
33	5733	4845	0983	4136	9864 293	445	584	27
34	6409	5472	1560	4663	770	872	961	26
35	7084	6098	2136	5189	9865 246	9892 298	9916 337	25
36	7759	6723	2712	5715	722	723	712	24
37	8432	7347	3286	6239	9866 196	9893 148	9917 087	23
38	9105	7970	3860	6763	670	572	459	22
39	9777	8593	4433	7286	9867 143	994	832	21
40	973 0449	9215	5005	7808	615	9894 416	9918 204	20
41	1119	9836	5576	8330	9868 087	838	574	19
42	1789	977 0456	6147	8850	557	9895 258	944	18
43	2458	1075	6716	9370	9869 027	677	9919 314	17
44	3125	1693	7285	9889	496	9896 096	682	16
45	3793	2311	7853	984 0407	964	514	9920 049	15
46	4459	2928	8420	0924	9870 431	931	416	14
47	5124	3544	8986	1441	897	9897 347	782	13
48	5789	4159	9552	1956	9871 363	762	9921 147	12
49	6453	4773	981 0116	2471	827	9898 177	511	11
50	7116	5387	0680	2985	9872 291	590	874	10
51	7778	5999	1243	3498	754	9899 003	9922 237	9
52	8439	6611	1805	4010	9873 216	415	599	8
53	9100	7222	2366	4521	678	826	959	7
54	9760	7832	2927	5032	9874 138	9900 237	9923 319	6
55	974 0419	8442	3486	5542	598	643	679	5
56	1077	9050	4045	6050	9875 057	9901 055	9924 037	4
57	1734	9658	4603	6558	514	462	394	3
58	2390	978 0265	5160	7066	972	869	751	2
59	3046	0871	5716	7572	9876 428	9902 275	9925 107	1
60	3701	1476	6272	8078	883	681	462	0
′	13°	12°	11°	10°	9°	8°	7°	′

′	76°	77°	78°	79°	80°	81°	82°	′
0	4·01 07809	4·33 14759	4·70 46301	5·14 45540	5·6 712818	6·3 137515	7·1 153697	60
1	57570	72316	4·71 18686	5·15 25557	809446	256601	304190	59
2	4·02 07446	4·34 30018	81256	5·16 05813	906394	376126	455308	58
3	57440	87866	4·72 49012	86311	5·7 003663	496092	607056	57
4	4·03 07550	4·35 45861	4·73 16954	5·17 67051	101256	616502	759437	56
5	57779	4·36 04003	85083	5·18 48035	199178	737859	912456	55
6	4·04 08125	62293	4·74 53401	5·19 29264	297416	858665	7·2 066116	54
7	58590	4·37 20731	4·75 21907	5·20 10738	395988	980422	220422	53
8	4·05 09174	79817	90603	92459	494889	6·4 102633	375378	52
9	59877	4·38 38054	4·76 59490	5·21 74428	594122	225301	530987	51
10	4·06 10700	96940	4·77 28568	5·22 56647	693688	348428	687255	50
11	61643	4·39 55977	97837	5·23 39116	793588	472017	844184	49
12	4·07 12707	4·40 15164	4·78 67300	5·24 21836	893825	596070	7·3 001780	48
13	63892	74504	4·79 36957	5·25 04809	994400	720591	160047	47
14	4·08 15199	4·41 33996	4·80 06808	88035	5·8 095315	845581	318989	46
15	66627	93641	76854	5·26 71517	196572	971043	478610	45
16	4·09 18178	4·42 53439	4·81 47096	5·27 55255	298172	6·5 096981	638916	44
17	69852	4·43 13392	4·82 17536	5·28 39251	400117	223396	799909	43
18	4·10 21649	73500	88174	5·29 23565	502410	350293	961595	42
19	73569	4·44 33762	4·83 59010	5·30 08018	605051	477672	7·4 123978	41
20	4·11 25614	94181	4·84 30045	92793	708042	605538	287064	40
21	77784	4·45 54756	4·85 01282	5·31 77839	811386	733892	451855	39
22	4·12 30079	4·46 15489	72719	5·32 63131	915084	862739	615357	38
23	82499	76379	4·86 44359	5·33 48696	5·9 019138	992080	780576	37
24	4·13 35046	4·47 37428	4·87 16201	5·34 34527	123550	6·6 121919	946514	36
25	87719	98636	88248	5·35 20626	228322	252258	7·5 113178	35
26	4·14 40519	4·48 60004	4·88 60499	5·36 06993	333455	383100	280571	34
27	93446	4·49 21532	4·89 32956	93630	438952	514449	448699	33
28	4·15 47301	83221	4·90 05620	5·37 80538	544815	646307	617567	32
29	99685	4·50 45072	78491	5·38 67718	651045	778677	787179	31
30	4·16 52993	4·51 07085	4·91 51570	5·39 55172	757644	911562	957541	30
31	4·17 06440	69261	4·92 24859	5·40 42901	864614	6·7 044966	7·6 128657	29
32	60011	4·52 31601	98358	5·41 30906	971957	178891	300533	28
33	4·18 13713	94105	4·93 72068	5·42 19188	6·0 079676	313341	473174	27
34	67546	4·53 56773	4·94 45990	5·43 07750	187772	448318	646584	26
35	4·19 21510	4·54 19608	4·95 20125	96592	296247	583826	820769	25
36	75666	82608	94474	5·44 85715	405103	719867	995735	24
37	4·20 29835	4·55 45776	4·96 69037	5·45 75121	514343	856446	7·7 171486	23
38	84196	4·56 09111	4·97 43817	5·46 64812	623967	993565	348028	22
39	4·21 38690	72615	4·98 18813	5·47 54788	733979	6·8 131227	525366	21
40	93318	4·57 36287	94027	5·48 45052	844381	269437	703506	20
41	4·22 48080	4·58 00129	4·99 69459	5·49 35604	955174	408196	882458	19
42	4·23 02977	64141	5·00 45111	5·50 26446	6·1 066369	547508	7·8 062212	18
43	58009	4·59 28325	5·01 20984	5·51 17579	177948	687878	242790	17
44	4·24 13177	92689	97078	5·52 09305	289923	827867	424191	16
45	68482	4·60 57207	5·02 73895	5·53 00724	402303	968799	606428	15
46	4·25 23923	4·61 21908	5·03 49935	92749	515085	6·9 110359	789489	14
47	79501	86783	5·04 26700	5·54 85052	628272	252489	973896	13
48	4·26 35218	4·62 51882	5·05 03690	5·55 77663	741865	395192	7·9 158151	12
49	91072	4·63 17056	80907	5·56 70574	855867	538478	343758	11
50	4·27 47066	82457	5·06 58352	5·57 63786	970279	682335	530224	10
51	4·28 03199	4·64 48034	5·07 36025	5·58 57302	6·2 085106	826781	717555	9
52	59472	4·65 13738	5·08 13926	5·59 51121	200347	971806	905756	8
53	4·29 15835	79721	92061	5·60 45247	316007	7·0 117441	8·0 094835	7
54	72440	4·66 45832	5·09 70426	5·61 39630	432086	263662	284796	6
55	4·30 29136	4·67 12124	5·10 49024	5·62 34421	548588	410482	475647	5
56	85974	78595	5·11 27855	5·63 29474	665515	557905	667894	4
57	4·31 42955	4·68 45248	5·12 06921	5·64 24888	782868	705934	860042	3
58	4·32 00079	4·69 12083	86224	5·65 20516	900651	854573	8·1 053599	2
59	57347	79100	5·13 65763	5·66 16509	6·3 018866	7·1 003826	248071	1
60	4·33 14759	4·70 46301	5·14 45540	5·67 12818	137515	153697	443464	0
′	13°	12°	11°	10°	9°	8°	7°	′

NATURAL COTANGENTS.

′	83°	84°	85°	86°	87°	88°	89°	′
0	9925 462	9945 219	9961 647	9975 641	9986 295	9993 908	9998 477	60
1	816	523	9962 200	843	447	9994 009	527	59
2	9926 169	825	452	9976 045	598	110	577	58
3	521	9946 127	704	245	748	209	625	57
4	873	428	954	445	898	308	673	56
5	9927 224	729	9963 204	645	9987 046	405	720	55
6	573	9947 028	453	843	194	502	766	54
7	922	327	701	9977 040	340	598	812	53
8	9928 271	625	948	237	486	693	856	52
9	618	921	9964 195	433	631	788	900	51
10	965	9948 217	440	627	775	881	942	50
11	9929 310	518	685	821	919	974	984	49
12	655	807	929	9978 015	9988 061	9995 066	9999 025	48
13	999	9949 101	9965 172	207	203	157	065	47
14	9930 342	398	414	399	344	247	105	46
15	685	685	655	589	484	336	143	45
16	9931 026	976	895	779	623	424	181	44
17	367	9950 266	9966 135	968	761	512	218	43
18	706	556	374	9979 156	899	599	254	42
19	9932 045	844	612	343	9989 035	684	289	41
20	384	9951 132	849	530	171	770	323	40
21	721	419	9967 085	716	306	854	357	39
22	9933 057	705	321	900	440	937	389	38
23	393	990	555	9980 084	573	9996 020	421	37
24	728	9952 274	789	267	706	101	452	36
25	9934 062	557	9968 022	450	837	182	482	35
26	395	840	254	631	968	262	511	34
27	727	9953 122	485	811	9990 098	341	539	33
28	9935 058	403	715	991	227	419	567	32
29	389	683	945	9981 170	355	497	593	31
30	719	962	9969 173	348	482	573	619	30
31	9936 047	9954 240	401	525	609	649	644	29
32	375	518	628	701	734	724	668	28
33	703	795	854	877	859	798	692	27
34	9937 029	9955 070	9970 080	9982 052	983	871	714	26
35	355	345	304	225	9991 106	943	736	25
36	679	620	528	398	228	9997 015	756	24
37	9938 003	893	750	570	350	086	776	23
38	326	9956 165	972	742	470	156	795	22
39	648	437	9971 193	912	590	224	813	21
40	969	708	413	9983 082	709	292	831	20
41	9939 290	978	633	250	827	360	847	19
42	610	9957 247	851	418	944	426	863	18
43	928	515	9972 069	485	9992 060	492	878	17
44	9940 246	783	286	751	176	556	892	16
45	563	9958 049	502	917	290	620	905	15
46	880	315	717	9984 081	404	683	917	14
47	9941 195	580	931	245	517	745	928	13
48	510	844	9973 145	408	629	807	939	12
49	823	9959 107	357	570	740	867	949	11
50	9942 136	370	569	731	851	927	958	10
51	448	631	780	891	960	986	966	9
52	760	892	990	9985 050	9993 069	9998 044	973	8
53	9943 070	9960 152	9974 199	209	177	101	979	7
54	379	411	408	367	284	157	983	6
55	688	669	615	524	390	213	989	5
56	996	926	822	680	495	267	993	4
57	9944 303	9961 183	9975 028	835	600	321	996	3
58	609	438	233	989	704	374	993	2
59	914	693	437	9986 143	806	426	1·0000 000	1
60	9945 219	947	641	295	908	477	000	0
′	6°	5°	4°	3°	2°	1°	0°	′

′	83°	84°	85°	86°	87°	88°	89°	
0	8·1 443464	9·5 143645	11·430052	14·300666	19·081137	28·636253	57·289962	60
1	639786	410618	468474	360696	187930	877089	58·261174	59
2	837941	679068	507154	421230	295922	29·122005	59·265872	58
3	8·2 035239	949022	546093	482273	405133	371106	60·305820	57
4	234384	9·6 220486	585294	543833	515584	624499	61·382905	56
5	434485	493475	624761	605916	627296	882299	62·499154	55
6	635547	768000	664495	668529	740291	30·144619	63·656741	54
7	837579	9·7 044075	704500	731679	854591	411580	64·858008	53
8	8·3 040586	321713	744779	795372	970219	683307	66·105473	52
9	244577	600927	785333	859616	20·087199	959928	67·401854	51
10	449558	881732	826167	924417	205553	31·241577	68·750087	50
11	655536	9·8 164140	867282	989784	325308	528392	70·153346	49
12	862519	448166	908682	15·055723	446486	820516	71·615070	48
13	8·4 070515	733823	950370	122242	569115	32·118099	73·138991	47
14	279531	9·9 021125	992349	189349	693220	421295	74·729165	46
15	489578	310088	12·034622	257052	818828	730264	76·390009	45
16	700651	600724	077192	325358	945966	33·045173	78·126342	44
17	912772	893050	120062	394276	21·074664	366194	79·943430	43
18	8·5 125948	10·018708	163236	463814	204949	693509	81·847041	42
19	340172	048283	206716	533981	336851	34·027303	83·843507	41
20	555468	078031	250505	604784	470401	367771	85·939791	40
21	771838	107954	294609	676233	605630	715115	88·143572	39
22	989290	138054	339028	748337	742569	35·069546	90·463336	38
23	8·6 207833	168332	383768	821105	881251	431282	92·908487	37
24	427475	198789	428831	894545	22·021710	800553	95·489475	36
25	648223	229428	474221	968667	163980	36·177596	98·217943	35
26	870088	260249	519942	16·043482	308097	562659	101·10690	34
27	8·7 093077	291255	565997	118998	454096	956001	104·17094	33
28	317198	322447	612390	195225	602015	37·357892	107·42648	32
29	542461	353827	659125	272174	751892	768613	110·89205	31
30	768874	385397	706205	349855	903766	38·188459	114·58865	30
31	996446	417158	753634	428279	23·057677	617738	118·54018	29
32	8·8 225186	449112	801417	507456	213666	39·056771	122·77396	28
33	455103	481261	849557	587396	371777	505895	127·32134	27
34	686206	513607	898058	668112	532052	965460	132·21851	26
35	918505	546151	946924	749614	694537	40·435837	137·50745	25
36	8·9 152009	578895	996160	831915	859277	917412	143·23712	24
37	386726	611841	13·045769	915025	24·026320	41·410588	149·46502	23
38	622668	644992	095757	998957	195714	915790	156·25908	22
39	859843	678348	146127	17·083724	367509	42·433464	163·70019	21
40	9·0 098261	711913	196883	169337	541758	964077	171·88540	20
41	337933	745687	248031	255809	718512	43·508122	180·93220	19
42	578867	779673	299574	343155	897826	44·066113	190·98419	18
43	821074	813872	351518	431385	25·079757	638593	202·21875	17
44	9·1 064564	848283	403867	520516	264361	45·226141	214·85762	16
45	309348	882921	456625	610559	451700	829351	229·18166	15
46	555436	917775	509799	701529	641832	46·448862	245·55198	14
47	802838	952850	563391	793442	834823	47·085343	264·44080	13
48	9·2 051564	988150	617409	886310	26·030736	739501	286·47773	12
49	301627	11·023676	671856	980150	229638	48·412084	312·52137	11
50	553035	059431	726738	18·074977	431600	49·103881	343·77371	10
51	805802	095416	782060	170807	636690	815726	381·97099	9
52	9·3 059930	131635	837827	267654	844984	50·548506	429·71757	8
53	315450	168089	894045	365537	27·056557	51·303157	491·10600	7
54	572355	204780	950719	464471	271486	52·080673	572·95721	6
55	830663	241712	14·007856	564473	489853	882109	687·54887	5
56	9·4 090384	278885	065459	665562	711740	53·708587	859·43630	4
57	351531	316304	123536	767754	937233	54·561300	1145·9153	3
58	614116	353970	182092	871068	28·166422	55·441517	1718·8732	2
59	878149	391885	241134	975523	399397	56·350590	3437·7467	1
60	9·5 143645	430052	300666	19·081137	636253	57·289962	Infinite.	0
′	6°	5°	4°	3°	2°	1°	0°	′

NATURAL COTANGENTS.

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0·000000	26	1·414973	51	1·707570	76	1·880814
2	0·301030	27	1·431364	52	1·716003	77	1·886491
3	0·477121	28	1·447158	53	1·724276	78	1·892095
4	0·602060	29	1·462398	54	1·732394	79	1·897627
5	0·698970	30	1·477121	55	1·740363	80	1·903090
6	0·778151	31	1·491362	56	1·748188	81	1·908485
7	0·845098	32	1·505150	57	1·755875	82	1·913814
8	0·903090	33	1·518514	58	1·763428	83	1·919078
9	0·954243	34	1·531479	59	1·770852	84	1·924279
10	1·000000	35	1·544068	60	1·778151	85	1·929419
11	1·041393	36	1·556303	61	1·785330	86	1·934498
12	1·079181	37	1·568202	62	1·792392	87	1·939519
13	1·113943	38	1·579784	63	1·799341	88	1·944483
14	1·146128	39	1·591065	64	1·806180	89	1·949390
15	1·176091	40	1·602060	65	1·812913	90	1·954243
16	1·204120	41	1·612784	66	1·819544	91	1·959041
17	1·230449	42	1·623249	67	1·826075	92	1·963788
18	1·255273	43	1·633468	68	1·832509	93	1·968483
19	1·278754	44	1·643453	69	1·838849	94	1·973128
20	1·301030	45	1·653213	70	1·845098	95	1·977724
21	1·322219	46	1·662758	71	1·851258	96	1·982271
22	1·342423	47	1·672098	72	1·857332	97	1·986772
23	1·361728	48	1·681241	73	1·863323	98	1·991226
24	1·380211	49	1·690196	74	1·869232	99	1·995635
25	1·397940	50	1·698970	75	1·875061	100	2·000000

No.	0	1	2	3	4	5	6	7	8	9	Diff.
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272
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1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	255
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	98 5	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198
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1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
9	460898	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9828	9959	520090	520221	520353	520484	520615	520745	520876	521007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	1351	128
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
1	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036	108
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
6	9486	9586	9686	9785	9885	9984	640084	640183	640283	640382	99
7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
7	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	68
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
9	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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No.	0	1	2	3	4	5	6	7	8	9	Dif
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	960042	960090	960138	960185	960233	960280	960328	960376	960423	48
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	48
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
No.	0	1	2	3	4	5	6	7	8	9	Dif

No.	0	1	2	3	4	5	6	7	8	9	Diff.
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE *of the Lengths of Circular Arcs, radius being unity.*

Sec.	Length of arc.	Min.	Length of arc.	Deg.	Length of arc.	Deg.	Length of arc.
1	0·0000048	1	0·0002909	1	0·0174533	61	1·0646509
2	·0000097	2	·0005818	2	·0349066	62	·0821042
3	·0000145	3	·0008727	3	·0523599	63	·0995575
4	·0000194	4	·0011636	4	·0698132	64	·1170108
5	·0000242	5	·0014544	5	·0872665	65	·1344641
6	·0000291	6	·0017453	6	·1047198	66	·1519174
7	·0000339	7	·0020362	7	·1221730	67	·1693707
8	·0000388	8	·0023271	8	·1396263	68	·1868240
9	·0000436	9	·0026181	9	·1570796	69	·2042773
10	·0000485	10	·0029089	10	·1745329	70	·2217305
11	·0000533	11	·0031998	11	·1919862	71	·2391839
12	·0000582	12	·0034907	12	·2094395	72	·2566372
13	·0000630	13	·0037816	13	·2268928	73	·2740905
14	·0000679	14	·0040725	14	·2443461	74	·2915438
15	·0000727	15	·0043634	15	·2617994	75	·3089970
16	·0000776	16	·0046543	16	·2792527	76	·3264502
17	·0000824	17	·0049452	17	·2967060	77	·3439034
18	·0000873	18	·0052361	18	·3141593	78	·3613567
19	·0000921	19	·0055270	19	·3316186	79	·3788100
20	·0000970	20	·0058178	20	·3490659	80	·3962634
21	·0001018	21	·0061087	21	·3665192	81	·4137167
22	·0001067	22	·0063996	22	·3839725	82	·4311700
23	·0001115	23	·0066905	23	·4014258	83	·4486233
24	·0001164	24	·0069814	24	·4188791	84	·4660766
25	·0001212	25	·0072723	25	·4363324	85	·4835299
26	·0001261	26	·0075632	26	·4537857	86	·5009832
27	·0001309	27	·0078540	27	·4712390	87	·5184365
28	·0001358	28	·0081449	28	·4886923	88	·5358898
29	·0001406	29	·0084358	29	·5061456	89	·5533431
30	·0001454	30	·0087266	30	·5235988	90	·5707963
31	·0001502	31	·0090175	31	·5410521	91	·5882496
32	·0001551	32	·0093084	32	·5585054	92	·6057029
33	·0001599	33	·0095993	33	·5759587	93	·6231562
34	·0001648	34	·0098902	34	·5934120	94	·6406095
35	·0001693	35	·0101811	35	·6108653	95	·6580628
36	·0001745	36	·0104720	36	·6283186	96	·6755161
37	·0001793	37	·0107629	37	·6457719	97	·6929694
38	·0001842	38	·0110538	38	·6632252	98	·7104227
39	·0001890	39	·0113447	39	·6806785	99	·7278760
40	·0001939	40	·0116355	40	·6981317	100	·7453293
41	·0001987	41	·0119264	41	·7155850	1	·7627826
42	·0002036	42	·0122173	42	·7330383	2	·7802359
43	·0002084	43	·0125082	43	·7504916	3	·7976892
44	·0002133	44	·0127991	44	·7679449	4	·8154125
45	·0002181	45	·0130900	45	·7853982	5	·8325958
46	·0002230	46	·0133809	46	·8028515	6	·8500491
47	·0002278	47	·0136718	47	·8203048	7	·8675024
48	·0002327	48	·0139627	48	·8377581	8	·8849557
49	·0002375	49	·0142536	49	·8552113	9	·9024090
50	·0002424	50	·0145444	50	·8726646	110	·9198622
51	·0002472	51	·0148353	51	·8901179	11	·9373155
52	·0002521	52	·0151262	52	·9075712	12	·9547688
53	·0002569	53	·0154171	53	·9250245	13	·9722221
54	·0002618	54	·0157080	54	·9424778	14	·9896754
55	·0002666	55	·0159989	55	·9599311	15	2·0071287
56	·0002715	56	·0162898	56	·9773844	16	·0245820
57	·0002763	57	·0165807	57	·9948377	17	·0420353
58	·0002812	58	·0168716	58	1·0122910	18	·0594886
59	·0002860	59	·0171625	59	·0297443	19	·0769419
60	·0002909	60	·0174533	60	·0471976	120	·0943951

TABLE *of the Lengths of Circular Arcs, radius being unity.*

Deg.	Length of arc	Deg.	Length of arc	Deg.	Length of arc.	Deg.	Length of arc.
121	2·1118484	181	3·1590460	241	4·2062435	301	5·2534411
2	·1293017	2	·1764993	2	·2236968	2	·2708944
3	·1467550	3	·1939526	3	·2411501	3	·2883477
4	·1642083	4	·2114059	4	·2586034	4	·3058010
5	·1816616	5	·2288592	5	·2760567	5	·3232542
6	·1991149	6	·2463125	6	·2935100	6	·3407075
7	·2165682	7	·2637658	7	·3109633	7	·3581608
8	·2340215	8	·2812191	8	·3284166	8	·3756141
9	·2514748	9	·2986724	9	·3458699	9	·3930674
130	·2689280	190	·3161256	250	·3633231	310	·4105207
1	·2863813	1	·3335789	1	·3807764	1	·4279740
2	·3038346	2	·3510322	2	·3982297	2	·4454273
3	·3212879	3	·3684855	3	·4156830	3	·4628806
4	·3387412	4	·3859388	4	·4331363	4	·4803339
5	·3561945	5	·4033921	5	·4505896	5	·4977872
6	·3736478	6	·4208454	6	·4680429	6	·5152405
7	·3911011	7	·4382987	7	·4854962	7	·5326938
8	·4085544	8	·4557520	8	·5029495	8	·5501471
9	·4260077	9	·4732053	9	·5204028	9	·5676004
140	·4434609	200	·4906585	260	·5378560	320	·5850536
1	·4609143	1	·5081118	1	·5553093	1	·6025069
2	·4783676	2	·5255651	2	·5727626	2	·6199202
3	·4958209	3	·5430184	3	·5902160	3	·6374135
4	·5132742	4	·5604717	4	·6076693	4	·6548668
5	·5307274	5	·5779250	5	·6251225	5	·6723201
6	·5481807	6	·5953783	6	·6425758	6	·6897744
7	·5656340	7	·6128316	7	·6600291	7	7072267
8	·5830873	8	·6302849	8	·6774824	8	·7246800
9	·6005406	9	·6477382	9	·6949357	9	·7421333
150	·6179939	210	·6651914	270	·7123890	330	·7595865
1	·6354472	1	·6826447	1	·7298423	1	·7770398
2	·6529005	2	·7000980	2	·7472956	2	·7944931
3	·6703538	3	·7175513	3	·7647489	3	·8119464
4	·6878071	4	·7350046	4	·7822022	4	·8293997
5	·7052604	5	·7524579	5	·7996554	5	·8468530
6	·7227137	6	·7699112	6	·8171087	6	·8643063
7	·7401670	7	·7873645	7	·8345620	7	·8817596
8	·7576203	8	·8048178	8	·8520153	8	·8992129
9	·7750736	9	·8222711	9	·8694686	9	·9166661
160	·7925268	220	·8397243	280	·8869219	340	·9341194
1	·8099801	1	·8571776	1	·9043752	1	·9515727
2	·8274334	2	·8746309	2	·9218285	2	·9690260
3	·8448867	3	·8920842	3	·9392818	3	·9864793
4	·8623400	4	·9095375	4	·9567351	4	6·0039326
5	·8797933	5	·9269908	5	·9741883	5	·0213859
6	·8972466	6	·9444441	6	·9916416	6	·0388392
7	·9146999	7	·9618974	7	5·0090949	7	·0562955
8	·9321532	8	·9793507	8	·0264582	8	·0737458
9	·9496065	9	·9968040	9	·0440015	9	·0911990
170	·9670597	230	4·0142572	290	·0614548	350	·1086523
1	·9845130	1	·0317106	1	·0789081	1	·1261056
2	3·0019667	2	·0491639	2	·0963614	2	·1435589
3	·0194196	3	·0666172	3	·1138147	3	1610122
4	·0368729	4	·0840705	4	·1312680	4	·1784655
5	·0543262	5	·1015237	5	·1487213	5	·1959188
6	·0707795	6	·1189770	6	·1661746	6	·2133721
7	·0892328	7	·1364303	7	·1836279	7	·2308254
8	·1066861	8	·1538836	8	·2010812	8	·2482787
9	·1241394	9	·1713369	9	·2185345	9	·2657320
180	·1415927	240	1887902	300	·2359878	360	·2831853

EXPLANATION OF THE USES AND APPLICATIONS OF THE TABLE OF LONG CHORDS.

PROBLEM.

Required to find the distances or abscissas on the chord from which, if ordinates or perpendiculars be drawn, they will pass through the station points on the curve.

EXAMPLE.—Let the given curve be 1000 ft. long of 5° curvature, or 1146 ft radius.

For the first station from the beginning we have

$$\frac{\text{chord for 1000 ft.} - \text{chord for 800 ft.}}{2} = \text{1st distance,}$$

$$\frac{\text{chord } 800 - \text{chord } 600}{2} = \text{2nd distance, etc.}$$

Then by table we have,

	Intermediate Distance.	Distance.			Total.
$\frac{968{\cdot}87 - 784{\cdot}10}{2}$	$= 92{\cdot}385$	1st	dist.	=	92·385
$\frac{784{\cdot}10 - 593{\cdot}36}{2}$	$= 95{\cdot}370$	2nd	"	=	187·755
$\frac{593{\cdot}36 - 398{\cdot}10}{2}$	$= 97{\cdot}630$	3rd	"	=	285·385
$\frac{398{\cdot}10 - 198{\cdot}81}{2}$	$= 99{\cdot}645$	4th	"	=	385·030
$\frac{198{\cdot}81 - 0000}{2}$	$= 99{\cdot}405$	5th	"	=	484·435

$$484{\cdot}435 = \text{half length} = \frac{968{\cdot}87}{2}$$

Thus for any given station we take from the length of the whole chord the length of a chord of twice as many stations less than the one under consideration; that is, 1st station from beginning 2 less; 2 from beginning, 4 less, etc., and take half the difference.

If the chord had been for 900 ft of curve, we should have,

$$\frac{877{\cdot}32 - 689{\cdot}39}{2} = 93{\cdot}965 = \text{1st distance.}$$

$$\frac{689{\cdot}39 - 496{\cdot}20}{2} = 96{\cdot}595 = \text{2nd } ''$$

$$\frac{496{\cdot}20 - 299{\cdot}24}{2} = 98{\cdot}480 = \text{3rd } ''$$

$$\frac{299{\cdot}24 - 100}{2} = 99{\cdot}620 = \text{4th } ''$$

388·660

Add 50·—

$$438{\cdot}66 = \frac{877{\cdot}32}{2} = \text{half length of chord.}$$

In like manner we may find the ordinates connecting these abscissas with their points on the curve.

Let the length of chord and radius be as already given. Then we have,

Mid. ordinate 1000 ft. — mid. ordinate 800 ft. curve = ordinate at 1st station.

Mid. ordinate 1000 ft. — mid. ordinate 600 ft. = ordinate at 2nd station.

For this purpose we have calculated a table of middle ordinates corresponding to that of long chords. From this we have,

107·39 — 69·13 = 38·62 = 1st ordinate.
107·39 — 39·06 = 68·33 = 2nd "
107·39 — 17·41 = 89·98 = 3rd "
107·39 — 4·36 = 103·03 = 4th "
107·39 — 0·00 = 107·39 = 5th or middle ordinate.

Were the chord for 900 ft. of curve we should have by tables,

87·25 — 53·05 = 34·20 = 1st ordinate.
87·25 — 27·17 = 60·08 = 2nd "
87·25 — 9·81 = 77·44 = 3rd "
87·25 — 1·09 = 86·16 = 4th "
87·25 — 0·00 = 87·25 = middle "

This will sufficiently demonstrate how the ordinates can be obtained for any other length of chord or curve. The same principle obtains in regard to any other rate of curvature. After passing the middle ordinate, their lengths will be repeated inversely; as will also be the intermediate lengths of abscissas. Then from end of first abscissa erect first ordinate, and so on in regular rotation.

TABLE

Of Middle Ordinates from Chords subtending Curves of from 100 *to* 1000 *feet in length; calculated to every* 15′ *of Curvature from* 15′ *to* 8°. *Radius of* 1° *being* 5730 *feet.*

	LENGTHS OF ARCS.									
	100	200	300	400	500	600	700	800	900	1000
	MIDDLE ORDINATES.									
Curvature.										
0° 15′	0·06	0·22	0·49	0·87	1·36	1·96	2·67	3·49	4·42	5·45
30	0·11	0·44	0·98	1·75	2·73	3·93	5·34	6·98	8·83	10·90
45	0·16	0·65	1·47	2·62	4·09	5·89	8·01	10·47	13·25	16·35
1° 00′	0·22	0·87	1·96	3·49	5·45	7·85	10·69	13·96	17·67	21·80
15	0·27	1·09	2·45	4·36	6·82	9·81	13·36	17·44	22·07	27·24
30	0·33	1·31	2·94	5·23	8·18	11·77	16·03	20·93	26·48	32·68
45	0·38	1·53	3·43	6·11	9·54	13·73	18·70	24·41	30·88	38·11
2° 00′	0·44	1·75	3·92	6·98	10·90	15·68	21·35	27·88	35·27	43·52
15	0·49	1·96	4·41	7·85	12·26	17·64	24·02	31·35	39·66	48·93
30	0·55	2·18	4·91	8·72	13·62	19·60	26·68	34·82	44·04	54·33
45	0·60	2·40	5·40	9·59	14·98	21·56	29·33	38·29	48·41	59·71
3° 00′	0·65	2·62	5·89	10·46	16·34	23·52	31·98	41·74	52·78	65·08
15	0·71	2·84	6·38	11·33	17·70	25·47	34·68	45·19	57·18	70·44
30	0·76	3·05	6·87	12·20	19·06	27·42	37·28	48·63	61·47	75·78
45	0·82	3·27	7·36	13·07	20·41	29·36	39·92	52·07	65·80	81·10
4° 00′	0·87	3·49	7·85	13·94	21·77	31·31	42·56	55·50	70·12	86·40
15	0·93	3·71	8·34	14·81	23·12	33·25	45·19	58·92	74·48	91·68
30	0·98	3·93	8·82	15·68	24·47	35·19	47·82	62·34	78·72	96·94
45	1·04	4·14	9·32	16·55	25·82	37·13	50·44	65·74	82·99	102·18
5° 00′	1·09	4·36	9·81	17·41	27·17	39·06	53·05	69·13	87·25	107·39
15	1·15	4·58	10·30	18·28	28·52	40·99	55·67	72·51	90·50	112·58
30	1·20	4·80	10·79	19·15	29·87	42·92	58·27	75·88	95·73	117·75
45	1·25	5·01	11·27	20·01	31·21	44·84	60·86	79·25	99·94	122·89
6° 00′	1·31	5·23	11·76	20·88	32·55	46·76	63·45	82·60	104·13	127·99
15	1·36	5·45	12·25	21·74	33·89	48·67	66·04	85·93	108·30	133·07
30	1·42	5·67	12·74	22·60	35·23	50·59	68·62	89·26	112·45	138·12
45	1·47	5·89	13·23	23·47	36·57	52·50	71·18	92·57	116·58	143·13
7° 00′	1·53	6·10	13·71	24·33	37·91	54·40	73·74	95·87	120·69	148·12
15	1·58	6·32	14·20	25·19	39·24	56·30	76·30	99·15	124·78	153·07
30	1·64	6·54	14·69	26·05	40·57	58·19	78·84	102·42	128·84	157·98
45	1·69	6·76	15·18	26·91	41·90	60·08	81·37	105·68	132·88	162·86
8° 00′	1·75	6·98	15·66	27·77	43·23	61·97	83·90	108·92	136·89	167·70

On the principles by which the following tables are calculated.

Let $m =$ linear opening of switch rail, $s =$ angular opening of rail, $f =$ angle of frog, $g =$ gauge of track.

Let $x =$ length of chord from opening of switch rail to point of frog. Then will the amount of curvature between the opening of rail where curve commences and point of frog $= f - s$; therefore the instrument setting over the open end of switch rail with a back-sight on the fixed end of it, the instrumental deflection to the point

of frog will be $=\frac{f-s}{2}$. But if the backsight be taken on a point (say 5 inches distant) parallel with the main track, the deflection will then be $=\frac{f-s}{2}+s=\frac{f+s}{2}$. Making the value of x, radius, $g-m$ will be homologous to the sine of $\frac{f+s}{2}$ Then we have,

$$\text{Sin}\left(\frac{f+s}{2}\right) : R :: g-m : x = \frac{R(g-m)}{\sin\left(\frac{f+s}{2}\right)}$$

EXAMPLE:

Calling $s = 1^\circ\ 15'$, $f = 6^\circ\ 45'$, $g = 4{\cdot}70$, $m = 0{\cdot}42$, and $g-m = 4{\cdot}28$, we have sin. $4^\circ : R :: 4{\cdot}28 : x = 61{\cdot}36$ft.

When a double opening of a switch rail for a double turnout occurs, we have,

$$\sin.\left(\frac{f+2s}{2}\right) : R :: g - 2\times m : x = \text{distance to } \textit{nearest} \text{ frog.}$$

The linear and angular opening of rail being the same, this table may be adapted to any other gauge by increasing the value of x as given in this table, and the length of radius of turnout 2 per cent. for every additional inch in the gauge. This is a little too much; the correction for a 6 ft. gauge being about 30 per cent. Thus 100 ft. chord of turnout on this track will give 130 ft. on 6 ft. gauge, and 1000 ft. radius will give 1300 ft. This is for a straight line. When on a curve going the same way as turnout, it is sufficiently accurate for practice to add rate of curve of main track to that of the table; but when going in opposite direction, subtract it; thus making relative departure from main track the same as on a straight line.

EXAMPLE:

Thus a 5° frog for a 4ft. 8½ inch gauge gives a distance of 78·5 ft. curvature 4° 46'. If the main track were a 4° curve and going the same way, distance being the same, the rate of curvature would be 4° 46' + 4° = 8° 46', radius 653 ft.; but going the other way 4° 46' — 4° = 0° 46', radius 7473 ft.

TABLE

Of distances on chord from opening of switch rail where the curve commences, to point of frog, radius of curvature and rate per 100 ft., calculated to every 15 minutes of frog angle, from 3° to 15°. Constant data: opening of switch rail 5 inches = ·42 ft., average angular opening say 1° 15′, rails being from 18 to 20 ft. long. Variable data gauges of road.

Gauge 4ft. 8½ inches = 4·70 ft.

Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.	Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.
3°	115·43	3779·3	1° 31′	9°	47 99	355·0	16° 09′
15′	109·02	3023·3	1 50	15′	46·78	335·3	17 07
30′	103·28	2613·2	2 11	30′	45·69	317·6	18 04
45′	98·12	2249·0	2 33	45′	44·66	301·3	19 02
4°	93·45	1947·2	2 56½	10°	43·67	286·2	20 03
15′	89·21	1704·0	3 22	15′	42 72	272·2	21 05
30′	85·33	1508·0	3 48	30′	41·80	259·3	22 08
45′	81·78	1339·0	4 17	45′	40·95	247·2	23 13
5°	78·51	1199·8	4 46½	11°	40·11	236·0	24 20
15′	75·50	1081·6	5 18	15′	39·36	225·4	25 28
30′	72·70	980·3	5 51	30′	38·55	215·8	26 37
45′	70·01	892·9	6 25	45′	37 81	206·6	27 48
6°	67·69	816 8	7 01	12°	37·10	198·0	29° —
15′	65·44	750·1	7 39	15	36·41	189·5	30 13
30′	63·33	690·4	8 18	30′	35·75	182·4	31 27
45′	61·36	639·4	8 58	45′	35·12	175·3	32 43
7°	59·50	593·0	9 40	13°	34·51	168·6	34 02
15′	57·75	550·8	10 24	15′	33·91	162·2	35 23
30′	56·01	514·6	11 09	30′	33·34	156 3	36 45
45′	54·55	481·1	11 56	45′	32·79	150·6	38 08
8°	53·08	451·8	12 44	14°	32·26	145·3	39 32
15′	51·69	423·3	13 35	15′	31·74	140·2	40 58
30′	50·36	398·3	14 25½	30′	31·24	135·4	42 26
45′	49·11	375·4	15 17	45′	30·75	130·8	43 56
				15°	30·28	126·5	45 26

TABLE

Of distances on chord from opening of switch rail to point of frog, radius of curvature and rate per 100 ft.

Gauge 4ft. 10 inches.

Angle of frog	Distances.	Length of radius.	Rate of curve per 100 ft.	Angle of frog.	Distances.	Length of radius	Rate of curve per 100 ft.
3°	118·89	3892·	1° 28′	9°	49·42	365·7	15° 41
15′	112·29	3217·0	1 47	15′	48·18	345 3	16 36
30′	106·37	2709·	2 07	30′	47 06	327·1	17 32
45′	101·06	2316·	2 28½	45′	46·00	310 3	18 29
4°	96·25	2006·	2 51½	10°	44·98	294·7	19 28
15′	91 88	1755·	3 16	15′	44·00	280·3	20 27
30′	87·88	1553·	3 41½	30′	43·06	267·	21 28½
45′	84·23	1379·	4 09½	45′	42 17	254 6	22 31½
5°	80 86	1235·	4 38½	11°	41·31	243·	23 36
15′	77·76	1134·	5 03	15′	40 48	232 3	24 42
30′	74·88	1009·	5 40½	30′	39·70	222·2	25 49
45′	72·21	919·	6 14	45′	38·94	212·7	26 58
6°	69·72	841·	6 49	12°	38·21	203·9	28 09
15′	67·40	772·	7 25½	15′	37 50	195·5	29 21
30′	65·22	712·	8 03	30′	36 82	187 8	30 33
45′	63·20	658·	8 42½	45′	36·17	180 5	31 46
7°	61·28	610	9 23½	13°	35·54	173 6	33 00
15′	59·48	568·	10 06	15′	34·92	167·	34 18
30′	57·79	530·	10 50	30′	34·34	160·9	35 39
45′	56·18	495·5	11 35	45′	33·77	155·	37 00
8°	54·67	464·3	12 21	14°	33 22	149·6	38 20
15′	53·24	436·	13 09	15′	32·69	144·4	39 44
30′	51·87	410·2	13 59	30′	32·17	139·4	41 10
45′	50·58	386·6	14 50	45′	31·67	134·7	42 36
				15°	31·18	130·2	44 04

TABLE

Of distances on chord from opening of switch rail to point of frog, radius of curvature and rate per 100 ft.

Gauge 5 feet.

Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.	Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.
3°	128·51	4036·	1°25½′	9°	51·24	379·9	15° 05′
15′	116·65	3436·	1 40	15′	50·00	358·7	15 58
30′	110·50	2810·	2 02	30′	48·88	339·8	16 52
45′	104·98	2408·	2 28	45′	47·78	322·3	17 48
4°	100·00	2080·	2 45	10°	46·72	306·2	18 44
15′	95·45	1820·	3 08¾	15′	45·71	291·2	19 42
30′	91·30	1611·	3 33	30′	44·73	277·4	20 40
45′	87·50	1430·	4 00	45′	43·81	264·5	21 40
5°	84·	1281·	4 28	11°	42·91	252·5	22 42
15′	80·78	1156·	4 57	15′	42·00	241·2	23 46
30′	77·78	1047·	5 27	30′	41·24	230·9	24 52
45′	75·00	965·	5 58	45′	40·45	221·0	26 01
6°	72·32	873·	6 33½	12°	39·69	211·8	27 10
15′	70·00	802·	7 09	15′	38·95	202·7	28 20
30′	67·76	739·	7 45	30′	38·25	195·1	29 30
45′	65·65	684·	8 23	45′	37·57	187·5	30 40
7°	63·66	634·	9 02	13°	36·92	180·2	31 50
15′	61·78	590·	9 43	15′	36·28	173·5	33 02
30′	60·00	550·	10 25	30′	35·67	167·2	34 17
45′	58·36	514·	11 09	45′	35·08	161·1	35 35
8°	56·79	482·	11 54	14°	34·51	155·4	36 55
15′	55·30	452·	12 40	15′	33·96	150·0	38 16
30′	53·88	426·	13 27	30′	33·42	144·8	39 38
45′	52·54	401·	14 17	45′	32·90	139·9	41 00
				15°	32·39	135·3	42 23

TABLE

Of distances on chord from opening of switch rail to point of frog, radius of curvature and rate per 100 feet.

Gauge 5 feet 6 inches

Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.	Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.
3°	136·78	4478·	1° 17′	9°	56·87	420·7	13° 39′
15′	129·19	3750·	1 32	15′	55·40	397·4	14 27
30′	122·38	3116·	1 50	30′	54·14	376·4	15 14
45′	116·27	2664·	2 09	45′	52·92	357·0	16 04
4°	110·75	2307·	2 29	10°	51·74	339·1	16 55
15′	105·71	2019·	2 50	15′	50·62	322·5	17 47
30′	101·11	1786·	3 12	30′	49·54	307·2	18 40
45′	96·90	1586·	3 37	45	48·52	292·9	19 35
5°	93·03	1421·0	4 02	11°	47·52	280·0	20 30
15′	89·46	1281·	4 28	15	46·52	267·2	21 28
30′	86·14	1161·	4 56	30	45·68	255·7	22 26
45′	83·15	1062·	5 24	45′	44·80	244·8	23 26
6°	80·16	967·	5 56	12°	43·96	234·2	24 30
15′	77·58	888·8	6 27	15	43·14	224·7	25 33
30′	75·04	819·	7 00	30	42·36	215·9	26 36
45	72·71	757·6	7 34	45′	41·61	207·7	27 40
7°	70·50	702·8	8 10	13°	40·89	199·7	28 46
15′	68·43	653·8	8 46	15′	40·18	192·2	29 54
30′	66·47	609·8	9 24	30′	39·50	185·2	31 02
45′	64·64	570·0	10 04	45′	38·85	178·4	32 11
8°	62·89	534·	10 45	14°	38·22	172·1	33 21
15′	61·25	501·6	11 27	15′	37·61	166·1	34 33
30′	59·67	471·9	12 10	30′	37·01	160·4	35 47
45′	58·19	444·8	12 54½	45′	36·44	154·9	37 03
				15°	31·87	150·0	38 18

TABLE

Of distances on chord from opening of switch rail to point of frog, radius of curvature and rate per 100 ft.

Gauge 6 feet.

Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.	Angle of frog.	Distances.	Length of radius.	Rate of curve per 100 ft.
3°	150·06	4913·1	1° 10′	9°	62·40	461 6	12° 26′
15′	141·73	4060·3	1 24⅜	15′	60·81	435 9	13 10
30′	134·26	3419·3	1 40½	30′	59 40	412·9	13 55
45′	127·56	2923·7	1 57½	45′	58·06	391·7	14 40
4°	121·50	2531·4	2 16	10°	56·77	372·1	15 25
15′	115·97	2215·2	2 35	15′	55·54	353 9	16 12
30′	110·93	1960 4	2 55½	30′	54·35	337 1	17 00
45′	106·31	1740·7	3 17½	45′	53·24	321·4	17 50
5°	102·06	1560·0	3 40½	11°	52·14	306·8	18 42
15′	98·15	1406·1	4 04½	15′	51 04	293·2	19 34
30′	94·51	1274·4	4 30	30′	50·12	280 5	20 27
45′	91·14	1160·8	4 56	45′	49 15	268 6	21 22
6°	88·00	1061·8	5 24	12°	48·23	257·4	22 18
15′	85·07	975·0	5 53	15′	47·33	246·0	23 15
30′	82 33	898·8	6 23	30′	46·47	237·1	24 12
45′	79·77	831·2	6 54	45′	45·66	227·9	25 12
7°	77·35	771·0	7 26	13°	44·86	219·2	26 12
15′	75·08	717·3	8 00	15′	44·08	210·9	27 14
30′	72·94	669·0	8 34	30′	43·34	203·2	28 17
45′	70·92	625·4	9 10	45′	42·63	195·8	29 20
8°	69·00	586·0	9 47½	14°	41·94	188·9	30 23
15′	67·20	550·3	10 25½	15′	41·26	182·3	31 28
30′	65·47	517·8	11 05	30′	40 61	176·0	32 36
45′	63·84	488·0	11 46	45′	39·98	170·0	33 45
				15°	39·36	164·5	33 54

MISCELLANEOUS NOTES AND EXAMPLES.

SUPPOSE a curve to contain 57° 24′ curvature; distance between centres of inner and outer track 5ft. Required difference in length between inside and outside of track. By table of circular arcs:

$$\begin{array}{lr} 57^\circ \text{ gives} & 0{\cdot}9948377 \\ 24^\circ \quad `` & \underline{0{\cdot}0069814} \\ & 1{\cdot}0018191 \\ \text{Multiply} & \underline{5} \\ & 5{\cdot}0090955 \end{array}$$

Ans. 5 *ft.*

To find the length of any circular arc, multiply tabular arc of given number of degrees by the radius. Half of this *tabular* length gives the tabular area of a section of some number of degrees, and this tabular area multiplied by the square of radius, gives the required area of sector; or this tabular area, multiplied by the difference of the squares of the two radii, gives the area of a ring. Thus if inner radius $= 3$ ft., outer $= 4$, thickness being 1, we have $4^2 - 3^2 = 7$, which multiplied by tabular area gives area required. Suppose the radius of the intrados of an arch containing 134° 46′ is 6·5 ft., the thickness of voussoirs $= 1{\cdot}5$.

Then $8^2 - 6{\cdot}5^2 = 21{\cdot}75$.

$$\begin{array}{lr} 134^\circ \text{ gives} & 2{\cdot}3387412 \\ 46' \quad `` & \underline{0{\cdot}0133809} \\ 134^\circ\, 46' \quad `` & 2{\cdot}3521221 \times 21{\cdot}75 = 51{\cdot}16 \text{ nearly,} \end{array}$$

and $\dfrac{51{\cdot}16}{2} = 25{\cdot}08 =$ area.

When the span and rise are given to find the curvature of arc, make $\dfrac{\text{rise}}{\text{half span}} =$ nat. tang. $\frac{1}{4}$ curvature.

EXAMPLE.—Suppose span $= 18$ ft., rise $= 6$ ft., then $\frac{6}{9} = 0{\cdot}666667$ $=$ nat. tang. 33° 41½′, and 33° 41½′ × 4 = 134° 46′ of curvature.

Let it be required to find radius, we would then have,

$$\frac{(\frac{1}{2}\text{ span})^2 + (\text{rise})^2}{2 \times \text{rise}} = \text{radius. Thus } \frac{9^2 + 6^2}{2 \times 6} = 9{\cdot}75 = \text{radius of arc.}$$

Had it been a 12 ft. span and 4 ft. rise, radius would have been 6·5 feet.

Analogous to this last example, and derived from the same proposition of geometry, is an easy method of determining the distance across a river or ravine.

Let the instrument be at B with a foresight upon C across river; from B lay off a right angle to D Set the instrument over D and

lay off from D C a right angle D A meeting C B produced in Then by similar triangles,

$$AB : BD :: BD : BC; \text{ or } \frac{BD^2}{AB} = BC.$$ Suppose that B D 50 ft. and A B = 3 ft., then $\frac{2500}{3} = 833{\cdot}3$ ft.

To Triangulate round an Obstruction on a Curve.

EXAMPLE.—Suppose in running a 3° curve, I find the point f sta. 2645 to be occupied by a house; I find, however, that 2644 + 7 and 2645 + 25 are clear of the house; also, that I have sufficie room for an equilateral triangle whose sides are 50 ft. each. E tablish 2644 + 75 and set the instrument over it. Now suppose t last reliable point on curve to be at sta. 2640. The instrument deflection from 2640 to 2645 + 25 = 525 ft. is 7° 52½'. Set t vernier to this reading, and clamp the instrument with a backsig on 2640, so that, when the vernier is at 0, the telescope may poi towards 2645 + 25. Unclamp vernier, set the reading at 60°, a measure 50 ft. in line of telescope. Set instrument over this poi and turn the interior angle = 60°, measuring 50 ft. as before. S the transit over this last point, sta. 2645 + 25, with the vernier 60° so that the zero line shall coincide with the chord from 2644 75 to 2045 + 25. Clamp the instrument with a sight on the secor point or vertex of triangle. Then set the vernier at 1° 52½', t instrumental deflection for 125 ft., and the telescope will point direction of sta. 2646, from whence continue the curve, if require as before.

This was an expedient applied to advantage by a former associa in making the final location of the Ohio and Mississippi R. I Ripley County, Indiana.

Similar examples and corollaries to previous propositions mig be added indefinitely, but this would transcend the proper limi of the work. To an adept practitioner possessing ordinary faculti of generalization, it is believed the rules and formulas already give will be suggestive of the means of solving most of the other proble which may occur in practice.

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